Journal of Quality Measurement and Analysis *JQMA* 21(4) 2025, 249-275
e-ISSN: 2600-8602 *http://www.ukm.my/jqma*
https://doi.org/10.17576/jqma.2104.2025.13

A MULTI-CRITERIA DECISION-MAKING FRAMEWORK FOR ANALYSING RIVER WATER POLLUTION USING GENERALISED L-R INTUITIONISTIC FUZZY VIKOR

(*Rangka Kerja Pembuatan Keputusan Pelbagai Kriteria untuk Menganalisis Pencemaran Air Sungai Menggunakan L-R Intutionistik Fuzzy VIKOR Umum*)

MUHAMMAD ASYRAN SHAFIE, NOR HANIMAH KAMIS*, DAUD MOHAMAD & SERIPAH AWANG KECHIL

ABSTRACT

Effective decision-making under uncertainty is a critical challenge in multi-criteria decision-making problems, particularly when dealing with ambiguity, vagueness, inconsistency, and imprecise data. This study proposes a novel mathematical framework based on generalised trapezoidal L-R intuitionistic fuzzy numbers (GTrLRIFNs) integrated with the VIKOR method to address uncertainty in decision-making. The proposed generalised trapezoidal L-R intuitionistic fuzzy VIKOR (GTrLRIF VIKOR) method extends traditional intuitionistic fuzzy numbers by incorporating non-linear left and right membership and non-membership functions, as well as confidence levels, to better capture human judgment in the evaluation of VIKOR method. A generalised aggregation operator, generalised trapezoidal L-R intuitionistic fuzzy weighted average (GTrLRIF-WA), is developed to facilitate the combination of uncertain criteria values, enhancing the model's capability to process complex linguistic and numerical data. The proposed method of GTrLRIF VIKOR is applied to classify alternatives in a real-world case involving water quality assessment for five rivers in Johor, Malaysia, based on six evaluation parameters. The results demonstrate that GTrLRIF VIKOR produces consistent rankings with traditional methods which are Water Quality Index (WQI) method and Fuzzy Complex Index (FCI) method while offering a more robust representation of uncertainty. The proposed GTrLRIF VIKOR method addresses uncertainty by incorporating degrees of confidence, making it a more suitable, flexible, and realistic approach as it captures greater uncertainty compared to the traditional WQI method and FCI method.

Keywords: confidence level; intuitionistic fuzzy number; L-R type; MCDM; VIKOR; river water pollution; weighted average

ABSTRAK

Pembuatan keputusan yang berkesan di bawah ketidakpastian ialah cabaran kritikal dalam masalah membuat keputusan berbilang kriteria, terutamanya apabila menangani kekaburan, kekaburan, ketidakkonsistenan dan data yang tidak tepat. Kajian ini mencadangkan rangka kerja matematik baru berdasarkan nombor kabur intuisi L-R trapezoid umum (GTrLRIFNs) yang disepadukan dengan kaedah VIKOR untuk menangani ketidakpastian dalam membuat keputusan. Kaedah VIKOR kabur intuisi umum trapezoid L-R (GTrLRIF VIKOR) yang dicadangkan memanjangkan nombor kabur intuisi tradisional dengan menggabungkan fungsi keahlian dan bukan keahlian kiri dan kanan bukan linear, serta tahap keyakinan, untuk menangkap pertimbangan manusia dengan lebih baik dalam penilaian kaedah VIKOR. Pengendali pengagregatan umum, purata berwajaran kabur intuisi L-R trapezoid umum (GTrLRIF-WA), dibangunkan untuk memudahkan gabungan nilai kriteria yang tidak pasti, meningkatkan keupayaan model untuk memproses data linguistik dan berangka yang kompleks. Kaedah GTrLRIF VIKOR yang dicadangkan digunakan untuk mengklasifikasikan alternatif dalam kes dunia sebenar yang melibatkan penilaian kualiti air untuk lima sungai di Johor, Malaysia, berdasarkan enam parameter penilaian. Keputusan menunjukkan bahawa

GTrLRIF VIKOR menghasilkan penarafan yang konsisten dengan kaedah tradisional iaitu kaedah Indeks Kualiti Air (WQI) dan kaedah Fuzzy Complex Index (FCI) sambil menawarkan perwakilan ketidakpastian yang lebih mantap. Kaedah GTrLRIF VIKOR yang dicadangkan menangani ketidakpastian dengan memasukkan darjah keyakinan, menjadikannya pendekatan yang lebih sesuai, fleksibel dan realistik kerana ia menangkap ketidakpastian yang lebih besar berbanding kaedah WQI tradisional dan kaedah FCI.

Kata kunci: tahap keyakinan; nombor kabur intuisi; jenis L-R; MCDM; VIKOR; pencemaran air sungai; purata wajaran

1. Introduction

Multiple-Criteria Decision Making (MCDM) is a vital approach for addressing complex decision problems involving conflicting criteria (Sahoo & Goswami 2023). It enables decision-makers to systematically evaluate and prioritise alternatives by considering multiple quantitative and qualitative factors simultaneously. Recent literature highlights the rapid growth of MCDM method, driven by its ability to handle complex problems. Popular methods such as Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), VIKOR, and Analytic Hierarchy Process (AHP) method are widely applied across fields like environmental (Deng *et al*. 2024; Gaćina *et al*. 2024; Shinde *et al*. 2024).

One critical area where MCDM can play an important role is in addressing water pollution, particularly river pollution, which is a growing environmental and public health concern (Garai 2024; Karbasi Ahvazi *et al*. 2024; Ostad-Ali-Askari & Kianmehr 2024). River pollution, caused by the presence of harmful substances and contaminants, affects not only water quality but also has far-reaching impacts on biodiversity, agriculture, and industrial productivity (Liu *et al*. 2013). Water resources in Malaysia originate from rivers, lakes, and groundwater, according to the Department of Environment Malaysia (DOE Malaysia) (2019).

Rivers are natural streams of flowing water that are naturally clean at their source. However, the water quality of the river is adversely affected by point and non-point pollutant sources as the water moves downstream (Chowdhury *et al*. 2018). The primary point sources in Malaysia have been determined to be industrial locations, sewage treatment plants, and residential sullage. Markets, eateries, workshops, homes, solid waste disposal facilities, aquaculture, gas stations, more are additional point sources of pollution in the river basin (Afroz *et al*. 2014). A water body is impacted by diffuse sources of non-point source pollution, which can build up from multiple sources. Controlling pollution from non-point sources is more difficult than controlling pollution from point sources. While some pollution originates from natural sources, human activity has been the primary cause of river pollution in Malaysia (Afroz *et al*. 2014).

As we examine the various sources of river pollution in Malaysia, it is crucial to note that the issue extends to different regions. For instance, in Johor, Malaysia, where the main economic activity is agriculture, and oil palm plantation is the most dominant land use for agriculture in Johor, Malaysia, which is 71% (Pak *et al*. 2021). The high nutrient concentration in fertilizers used on oil palm farms can frequently be carried into surrounding rivers during a storm event, leading to river pollution. There are possible pollution sources for the Johor River: mineral components, industrial and man activity, agriculture, sewage, paint/ rubber/ plastic industry, abundant elements in the earth's crust, and livestock manure (Samsudin *et al*. 2017). Various pollution sources, ranging from industrial and human activities to agricultural practices, contribute to the pollution of the Johor River, highlighting the need for a comprehensive river water pollution classification method. Classifying river water pollution is therefore necessary to accurately and effectively to determine river pollution while maintaining water resources and permitting targeted remediation efforts.

Department of Environment Malaysia (2019) stated that different water quality criteria need to be assessed to determine the health of the river water ensuring its safety for any purpose. This is due to the water quality level being overly detailed and technical, displaying monitoring information on particular substances without offering a holistic and comprehensible picture of water quality (Khan 2017). Although the two are closely related, it is important to distinguish between water quality and water pollution when talking about both of them. "Water quality" is the overall condition of the water, taking into account all of its physical, chemical, biological, and radiological properties. Contrarily, "water pollution" mostly refers to the degradation or contamination of water due to dangerous substances.

There are several ways to assess the level of pollution in river water, including the Water Quality Index (WQI), which was created by DOE Malaysia, the Nemerow pollution index (NPI), and the Fuzzy Complex Index (FCI). In 1985, DOE Malaysia and Universiti Malaya, Malaysia, collaborated to create the WQI method in Malaysia for determining water quality levels (Arsad 2009). The WQI is a straightforward approach for determining a single value representing the overall water quality level based on various water quality parameters (Department of Environment Malaysia 2019). The WQI simplifies these variables' intricate logical data into a single, easily analysed value (Mohammadpour *et al*. 2023). The NPI is another approach to identifying river water pollution. According to Xu *et al*. (2014), the NPI is a weighted multi-factor environmental quality measure considering extreme or remarkable maximum values. The NPI considers a variety of pollutants, including solid waste pollutants, water pollutants, and air pollutants. Zhang *et al*. (2018), Su *et al*. (2022a), and Su *et al*. (2022b) used the NPI in water quality assessment to determine water pollution. Next, the FCI approach introduced by Zhu and Hu (2010) can evaluate the level of water pollution and water quality by categorising the river water to develop a thorough water quality index to evaluate Donghu Lake, China's water quality trend. Based on the idea of fuzzy set theory, fuzzy approaches are already among the most common methods used in evaluating river water. To reduce the risk of river water pollution, however, to effectively and completely clean the river, early problem classification is necessary.

Shafie *et al*. (2023) introduced generalised trapezoidal L-R intuitionistic fuzzy numbers (GTrLRIFNs) with Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method to classify the river water pollution in Johor, Malaysia. Shafie *et al*. (2023) evaluates alternatives based on the Euclidean distance of GTrLRIFNs from an ideal solution and a negative ideal solution. Along with TOPSIS, Shafie *et al*. (2023) also used Criteria Importance Through Intercriteria Correlation (CRITIC) approach to unbiasedly establish the criteria weights according to their interactions.

Hence, this study aims to classify river water pollution using generalised L-R intuitionistic fuzzy number (GLRIFN) with the VIKOR (GLRIFN-VIKOR) method for several rivers in Johor, Malaysia that incorporate a confidence level to quantify the uncertainty associated with each value, offering a more robust representation to cater the problem of ambiguity, vagueness, inconsistency, and imprecise uncertainty with a more reliable evaluation due to consideration of confidence level. The adoption of GTrLRIFNs over other fuzzy models is justified by their enhanced flexibility and expressiveness in representing expert knowledge and environmental uncertainty. Unlike conventional fuzzy numbers, GTrLRIFNs consider the non-linear functions for left and right membership and non-membership functions, along with the confidence level in the evaluation making GTrLRIFNs highly suitable for modeling the vagueness, imprecision, ambiguity, and inconsistency information in environmental decision-making.

By identifying the compromise option that is closest to the ideal, VIKOR is used to resolve choice problems with competing and non-commensurable criteria. A comprehensive examination of data uncertainty is presented by the GLRIFN-VIKOR, which has the advantage of better representing the human evaluation process through membership and non-membership functions. Additionally, incorporating confidence level values will bring a new dimension of data to evaluate the judgment behaviour of the decision-makers.

2. Materials and Methods

In this section, the preliminaries, arithmetic, and aggregation operation of generalised L-R intuitionistic fuzzy number (GLRIFN) with illustrative examples are defined.

2.1. *Preliminaries*

This section provides the following definitions for GTrLRIFNs:

Definition 1 Shafie *et al*. (2023) A generalised trapezoidal L-R intuitionistic fuzzy numbers (GTrLRIFNs) $A=\left(m_a,n_a;m_a',n_a';l_a,r_a;l_a',r_a';h_a;h_a'\right)_{LR}$ defined by a membership and non-membership functions $A=\left\{\left\langle x,\mu_A\left(x\right),\nu_A\left(x\right)\right\rangle \middle| x\in X\right\}$ with the condition $0\le h_a+h_a'\le 1$, where

$$\mu_A\left(x\right)=\begin{cases} h_a\cdot L\left(\dfrac{m_a-x}{l_a}\right) & ;\ -\infty\le x\le m_a \\ h_a & ;\ m_a\le x\le n_a \\ h_a\cdot R\left(\dfrac{x-n_a}{r_a}\right) & ;\ n_a\le x\le +\infty \end{cases}$$

$$\nu_A\left(x\right)=\begin{cases} 1-\left(1-h_a'\right)\cdot L\left(\dfrac{m_a'-x}{l_a'}\right) & ;\ -\infty\le x\le m_a' \\ h_a' & ;\ m_a'\le x\le n_a' \\ 1-\left(1-h_a'\right)\cdot R\left(\dfrac{x-n_a'}{r_a'}\right) & ;\ n_a'\le x\le +\infty \end{cases} \tag{1}$$

such that $m_a,n_a,m_a',n_a',l_a,r_a,l_a',r_a'\in\mathbb{R}$, $m_a\le n_a$, $m_a'\le n_a'$, $h_a\in\left(0,1\right]$, and $h_a'\in\left[0,1\right)$.

The GTrLRIFNs denoted as $A=\left(m_a,n_a;m_a',n_a';l_a,r_a;l_a',r_a';h_a;h_a'\right)_{LR}$ where m_a and n_a are the core of membership degrees, m_a' and n_a' are the core of non-membership degrees, l_a and r_a are the left spread and right spread of the membership functions, μ_A respectively, while l_a' and r_a' are the left spread and right spread of the non-membership functions, ν_A respectively. The functions L and R denote the left and right reference functions of μ_A and

ν_A respectively. The values of h_a and h_a' represent the height of the core for membership and non-membership degrees respectively, such that $h_a : X \rightarrow (0,1]$ and $h_a' : X \rightarrow [0,1)$; $0 \leq h_a + h_a' \leq 1$.

2.2. *Arithmetic operations of generalised trapezoidal L-R intuitionistic fuzzy numbers*

Suppose two generalised trapezoidal L-R intuitionistic fuzzy numbers (GTrLRIFNs) $A = (m_a, n_a; m_a', n_a'; l_a, r_a; l_a', r_a'; h_a; h_a')_{LR}$ and $B = (m_b, n_b; m_b', n_b'; l_b, r_b; l_b', r_b'; h_b; h_b')_{LR}$. The definition of the sum, subtraction, product, inverse, and quotient of two GTrLRIFNs are introduced in Definition 4 to Definition 8 respectively.

Definition 2 Let $C = A \oplus B$ be the sum of two GTrLRIFNs. Then,

$$C = (m_a + m_b, n_a + n_b; m_a' + m_b', n_a' + n_b'; l_a + l_b, r_a + r_b; l_a' + l_b', r_a' + r_b'; \min(h_a, h_b); \max(h_a', h_b'))_{LR} \tag{2}$$

for $A > 0, B > 0$.

The construction of GTrLRIFNs addition is as follows. Note that the L is left reference function and the R is right reference function. The formula for GTrLRIFNs addition is by considering the increasing and decreasing parts of membership and non-membership functions. The increasing part of membership function of GTrLRIFNs $\mu_L^A(x)$ and $\mu_L^B(x)$ are as follows:

$$\mu_L^A(x) = h_a \cdot L\left(\frac{m_a - x_a}{l_a}\right), \quad \mu_L^B(x) = h_b \cdot L\left(\frac{m_b - x_b}{l_b}\right)$$

where $\mu_L^A(x)$ and $\mu_L^B(x)$ are fixed value in $(0,1]$. This is equivalent to

$$x_a = m_a - l_a L^{-1}\left(\frac{\mu_L^A(x)}{h_a}\right), \quad x_b = m_b - l_b L^{-1}\left(\frac{\mu_L^B(x)}{h_b}\right).$$

This implies

$$x_c = x_a + x_b = (m_a + m_b) - (l_a + l_b) L^{-1}\left(\frac{\mu_R^C(x)}{\min(h_a, h_b)}\right),$$

$$\mu_R^C(x) = \min(h_a, h_b) \cdot L\left(\frac{(m_a + m_b) - x_c}{l_a + l_b}\right).$$

The decreasing part of membership function of GTrLRIFNs $\mu_R^A(x)$ and $\mu_R^B(x)$ are as follows:

$$\mu_R^A(x) = h_a \cdot R\left[\frac{x_a - n_a}{r_a}\right], \quad \mu_R^B(x) = h_b \cdot R\left[\frac{x_b - n_b}{r_b}\right]$$

where $\mu_R^A(x)$ and $\mu_R^B(x)$ are fixed value in $(0,1]$. This is equivalent to

$$x_a = n_a + r_a R^{-1}\left(\frac{\mu_R^A(x)}{h_a}\right), \quad x_b = n_b + r_b R^{-1}\left(\frac{\mu_R^B(x)}{h_b}\right).$$

This implies

$$x_c = x_a + x_b = (n_a + n_b) + (r_a + r_b) R^{-1}\left(\frac{\mu_R^C(x)}{\min(h_a, h_b)}\right),$$

$$\mu_R^C(x) = \min(h_a, h_b) \cdot R\left(\frac{x_c - (n_a + n_b)}{r_a + r_b}\right).$$

The decreasing part of non-membership function of GTrLRIFNs $\nu_L^A(x)$ and $\nu_L^B(x)$ are as follows:

$$\nu_L^A(x) = 1 - (1 - h_a') \cdot L\left(\frac{m_a' - x_a}{l_a'}\right), \quad \nu_L^B(x) = 1 - (1 - h_b') \cdot L\left(\frac{m_b' - x_b}{l_b'}\right),$$

where $\nu_L^A(x)$ and $\nu_L^B(x)$ are fixed value in $[0,1)$. This is equivalent to

$$x_a = m_a' - l_a' L^{-1}\left(\frac{1 - \nu_L^A(x)}{1 - h_a'}\right), \quad x_b = m_b' - l_b' L^{-1}\left(\frac{1 - \nu_L^B(x)}{1 - h_b'}\right).$$

This implies

$$x_c = x_a + x_b = (m_a' + m_b') - (l_a' + l_b') L^{-1}\left(\frac{1 - \nu_R^C(x)}{1 - \max(h_a', h_b')}\right),$$

$$\nu_R^C(x) = 1 - (1 - \max(h_a', h_b')) \cdot L\left(\frac{(m_a' + m_b') - x_c}{l_a' + l_b'}\right).$$

The increasing part of non-membership function of GTrLRIFNs $\nu_R^A(x)$ and $\nu_R^B(x)$ are as follows:

$$\nu_R^A(x) = 1 - (1 - h_a') \cdot R\left(\frac{x_a - n_a'}{r_a'}\right), \quad \nu_R^B(x) = 1 - (1 - h_b') \cdot R\left(\frac{x_b - n_b'}{r_b'}\right),$$

where $\nu_R^A(x)$ and $\nu_R^B(x)$ are fixed value in $[0,1)$. This is equivalent to

$$x_a = n_a' + r_a' R^{-1}\left(\frac{1-\nu_R^A(x)}{1-h_a'}\right), \quad x_b = n_b' + r_b' R^{-1}\left(\frac{1-\nu_R^B(x)}{1-h_b'}\right).$$

This implies

$$x_c = x_a + x_b = (n_a' + n_b') + (r_a' + r_b') R^{-1}\left(\frac{1-\nu_R^C(x)}{1-\max(h_a', h_b')}\right),$$

$$\nu_R^C(x) = 1 - (1-\max(h_a', h_b')) \cdot R\left(\frac{x_c - (n_a' + n_b')}{r_a' + r_b'}\right).$$

Example 1 Let $A = (5,7;5,7;1,2;4,3;0.7;0.3)_{LR}$ and $B = (6,7;6,7;3,1;2,3;0.6;0.2)_{LR}$ be two GTrLRIFNs. Therefore, using Definition 2 (Eq. (2)),

$$\begin{aligned} A \oplus B &= (5+6, 7+7; 5+6, 7+7; 1+3, 2+1; 4+2, 3+3; \min(0.7, 0.6); \max(0.3, 0.2))_{LR} \\ &= (11,14;11,14;4,3;6,6;0.6;0.3)_{LR}. \end{aligned}$$

Definition 3 Let $C = A \ominus B$ be the difference between the two GTrLRIFNs. Suppose the formula for the opposite of GTrLRIFNs is

$$-(m,n;m',n';l,r;l',r';h;h')_{LR} = (-m,-n;-m',-n';r,l;r',l';h;h')_{RL}. \tag{3}$$

Then, the subtraction of two GTrLRIFNs is

$$\begin{aligned} C = (&m_a - m_b, n_a - n_b; m_a' - m_b', n_a' - n_b'; l_a + r_b, r_a + l_b; l_a' + r_b', r_a' + l_b'; \\ &\min(h_a, h_b); \max(h_a', h_b'))_{LR}. \end{aligned} \tag{4}$$

for $A > 0, B > 0$.

Example 2 Let $A = (8,10;8,10;5,4;4,3;0.6;0.4)_{LR}$ and $B = (4,5;4,5;3,1;2,3;0.7;0.2)_{LR}$ be two GTrLRIFNs. Therefore, using Definition 3 (Eq. (4)),

$$\begin{aligned} A \ominus B &= (8-4, 10-5; 8-4, 10-5; 5+1, 4+3; 4+3, 3+2; \min(0.6, 0.7); \max(0.4, 0.2))_{LR} \\ &= (4,5;4,5;6,7;7,5;0.6;0.4)_{LR}. \end{aligned}$$

Definition 4 Let $C = A \otimes B$ be the product of two GTrLRIFNs. If $A > 0, B > 0$, then

$$\begin{aligned} C = (&m_a m_b; n_a n_b; m_a' m_b'; n_a' n_b'; m_a l_b + m_b l_a, n_a r_b + n_b r_a; \\ &m_a' l_b' + m_b' l_a', n_a' r_b' + n_b' r_a'; \min(h_a, h_b); \max(h_a', h_b'))_{LR} \end{aligned} \tag{5}$$

The construction of GTrLRIFNs multiplication is as follows. The formula for GTrLRIFNs multiplication is constructed by considering the increasing and decreasing parts of membership and non-membership functions. The increasing part of membership function of GTrLRIFNs $\mu_L^A(x)$ and $\mu_L^B(x)$ are as follows:

$$\mu_L^A(x) = h_a \cdot L\left(\frac{m_a - x_a}{l_a}\right), \quad \mu_L^B(x) = h_b \cdot L\left(\frac{m_b - x_b}{l_b}\right)$$

where $\mu_L^A(x)$ and $\mu_L^B(x)$ are fixed value in $(0,1]$. This is equivalent to

$$x_a = m_a - l_a L^{-1}\left(\frac{\mu_L^A(x)}{h_a}\right), \quad x_b = m_b - l_b L^{-1}\left(\frac{\mu_L^B(x)}{h_b}\right).$$

This implies

$$\begin{aligned} x_c &= x_a \cdot x_b \\ &= m_a m_b - (m_a l_b + m_b l_a) L^{-1}\left(\frac{\mu_R^C(x)}{\min(h_a, h_b)}\right) + l_a l_b \left[L^{-1}\left(\frac{\mu_R^C(x)}{\min(h_a, h_b)}\right)\right]^2. \end{aligned}$$

The decreasing parts of membership function of GTrLRIFNs $\mu_R^A(x)$ and $\mu_R^B(x)$ are as follows:

$$\mu_R^A(x) = h_a \cdot R\left[\frac{x_a - n_a}{r_a}\right], \quad \mu_R^B(x) = h_b \cdot R\left[\frac{x_b - n_b}{r_b}\right]$$

where $\mu_R^A(x)$ and $\mu_R^B(x)$ are fixed value in $(0,1]$. This is equivalent to

$$x_a = n_a + r_a R^{-1}\left(\frac{\mu_R^A(x)}{h_a}\right), \quad x_b = n_b + r_b R^{-1}\left(\frac{\mu_R^B(x)}{h_b}\right).$$

This implies

$$\begin{aligned} x_c &= x_a \cdot x_b \\ &= n_a n_b + (n_a r_b + n_b r_a) R^{-1}\left(\frac{\mu_R^C(x)}{\min(h_a, h_b)}\right) + r_a r_b \left[R^{-1}\left(\frac{\mu_R^C(x)}{\min(h_a, h_b)}\right)\right]^2. \end{aligned}$$

The decreasing part of non-membership function of GTrLRIFNs $\nu_L^A(x)$ and $\nu_L^B(x)$ are as follows:

$$\nu_L^A(x) = 1-(1-h_a')\cdot L\left(\frac{m_a'-x_a}{l_a'}\right), \quad \nu_L^B(x) = 1-(1-h_b')\cdot L\left(\frac{m_b'-x_b}{l_b'}\right)$$

where $\nu_L^A(x)$ and $\nu_L^B(x)$ are fixed value in $[0,1)$. This is equivalent to

$$x_a = m_a'-l_a'L^{-1}\left(\frac{1-\nu_L^A(x)}{1-h_a'}\right), \quad x_b = m_b'-l_b'L^{-1}\left(\frac{1-\nu_L^B(x)}{1-h_b'}\right).$$

This implies

$$\begin{aligned} x_c &= x_a \cdot x_b \\ &= m_a'm_b' - \begin{pmatrix} m_a'l_b'+ \\ m_b'l_a' \end{pmatrix} L^{-1}\left(\frac{1-\nu_R^C(x)}{1-\max(h_a',h_b')}\right) + l_a'l_b'\left[L^{-1}\left(\frac{1-\nu_R^C(x)}{1-\min(h_a',h_b')}\right)\right]^2. \end{aligned}$$

The increasing part of non-membership function of GTrLRIFNs $\nu_R^A(x)$ and $\nu_R^B(x)$ are as follows:

$$\nu_R^A(x) = 1-(1-h_a')\cdot R\left(\frac{x_a-n_a'}{r_a'}\right), \quad \nu_R^B(x) = 1-(1-h_b')\cdot R\left(\frac{x_b-n_b'}{r_b'}\right)$$

where $\nu_R^A(x)$ and $\nu_R^B(x)$ are fixed value in $[0,1)$. This is equivalent to

$$x_a = n_a'+r_a'R^{-1}\left(\frac{1-\nu_R^A(x)}{1-h_a'}\right), \quad x_b = n_b'+r_b'R^{-1}\left(\frac{1-\nu_R^B(x)}{1-h_b'}\right).$$

This implies

$$\begin{aligned} x_c &= x_a \cdot x_b \\ &= n_a'n_b' + \begin{pmatrix} n_a'r_b'+ \\ n_b'r_a' \end{pmatrix} R^{-1}\left(\frac{1-\nu_R^C(x)}{1-\max(h_a',h_b')}\right) + r_a'r_b'\left[R^{-1}\left(\frac{1-\nu_R^C(x)}{1-\max(h_a',h_b')}\right)\right]^2. \end{aligned}$$

In short, if the term $l_a l_b\left[L^{-1}\left(\frac{\mu_R^C(x)}{\min(h_a,h_b)}\right)\right]^2$ and $l_a'l_b'\left[L^{-1}\left(\frac{1-\nu_R^C(x)}{1-\min(h_a',h_b')}\right)\right]^2$ are neglected, provided the l_a, l_b, l_a', and l_b' are smaller compared with m_a, m_b, m_a', and m_b', and if $r_a r_b\left[R^{-1}\left(\frac{\mu_R^C(x)}{\min(h_a,h_b)}\right)\right]^2$ and $r_a'r_b'\left[R^{-1}\left(\frac{1-\nu_R^C(x)}{1-\max(h_a',h_b')}\right)\right]^2$ are neglected, provided that $r_a, r_b, r_a', $ and r_b' are small compared with $n_a, n_b, n_a',$ and n_b', and/or $\frac{\mu_R^C(x)}{\min(h_a, h_b)}$ and $\frac{1-\nu_R^C(x)}{1-\max(h_a', h_b')}$ are in the neighbourhood of 1, the equation will become simpler.

Example 3 If $A>0, B>0,$ then let $A=(5,7;5,7;1,2;4,3;0.7;0.3)_{LR}$ and $B=(12,15;12,15;3,1;2,3;0.6;0.2)_{LR}$ be two generalised trapezoidal L-R intuitionistic fuzzy numbers. Therefore, using Definition 4 (Eq. (5)),

$$\begin{aligned}A\otimes B &= (5\cdot 12, 7\cdot 15; 5\cdot 12, 7\cdot 15; (5\cdot 3)+(12\cdot 1), (7\cdot 1)+(15\cdot 2);\\ &\quad (5\cdot 2)+(12\cdot 4), (7\cdot 3)+(15\cdot 3); \min(0.7, 0.6); \max(0.3, 0.2))_{LR}\\ &= (60,105;60,105;27,37;34,66;0.6;0.3)_{LR}.\end{aligned}$$

Definition 5 Let A^{-1} be the inverse of GTrLRIFN. Then,

$$\begin{aligned}A^{-1} &= (m_a, n_a; m_a', n_a'; l_a, r_a; l_a', r_a'; h_a; h_a')_{LR}^{-1}\\ &= \left(\frac{1}{m_a}, \frac{1}{n_a}; \frac{1}{m_a'}, \frac{1}{n_a'}; \frac{r_a}{n_a^2}, \frac{l_a}{m_a^2}; \frac{r_a'}{n_a'^2}, \frac{l_a'}{m_a'^2}; h_a; h_a'\right)_{RL}; m_a, n_a, m_a', n_a' > 0.\end{aligned} \tag{6}$$

A similar formula holds when $A<0$ since $-(A^{-1}) = (-A)^{-1}$.

The construction of GTrLRIFN inverse is as follows. Obviously, $\mu_{A^{-1}}(x) = \mu_A\left(\frac{1}{x}\right), \forall x \neq 0, \forall A \in \mathbb{R}$. Let A be a positive GTrLRIFN. If

$$\mu_L^A(x) = h \cdot L_-\left(\frac{m-x}{l}\right); x \leq m,$$

then, when $A=(m,n;m',n';l,r;l',r';h;h')_{LR}$,

$$\mu_{A^{-1}}(x) = \mu_A\left(\frac{1}{x}\right) = \frac{1}{h}\cdot L_+\left(\frac{mx-1}{lx}\right); x \geq \frac{1}{m}.$$

Noted that the $\mu_L^A(x)$ is built from the left curve of A. Moreover, A^{-1} is not a L-R type intuitionistic fuzzy number. However, by considering the neighbourhood of $\frac{1}{m}$ and approximation formula can be used as follows:

$$\mu_{A^{-1}}(x)=\mu_A\left(\frac{1}{x}\right)=\frac{1}{h}\cdot L_+\left(\frac{x-\frac{1}{m}}{\frac{l}{m^2}}\right) \quad ; \quad x\geq\frac{1}{m}.$$

Similarly with right curve of A, $\mu_R^A(x)$.

Example 4 Let $A=(5,7;5,7;1,2;4,3;0.7;0.3)_{LR}$ be the generalised trapezoidal L-R intuitionistic fuzzy number. Therefore, using Definition 5 (Eq. (6)),

$$\begin{aligned}A^{-1}&=\left(\frac{1}{5},\frac{1}{7};\frac{1}{5},\frac{1}{7};\frac{2}{7^2},\frac{1}{5^2};\frac{3}{7^2},\frac{4}{5^2};0.7;0.3\right)_{RL}\\&=\left(\frac{1}{5},\frac{1}{7};\frac{1}{5},\frac{1}{7};\frac{2}{49},\frac{1}{25};\frac{3}{49},\frac{4}{25};0.7;0.3\right)_{RL}.\end{aligned}$$

Definition 6 Let $C=A\oslash B$ be the quotient of two GTrLRIFNs using the identity $A\oslash B=A\otimes B^{-1}$, the division of GTLRIFNs can be obtained as follows. If $A>0,B>0$, then

$$\begin{aligned}C=(&\frac{m_a}{m_b};\frac{n_a}{n_b};\frac{m_a'}{m_b'};\frac{n_a'}{n_b'};\frac{m_a l_b+m_b l_a}{m_b^{2}},\frac{n_a r_b+n_b r_a}{n_b^{2}};\\&\frac{m_a' l_b'+m_b' l_a'}{m_b'^2},\frac{n_a' r_b'+n_b' r_a'}{n_b'^2};\min(h_a,h_b);\max(h_a',h_b'))_{LR}\end{aligned} \tag{7}$$

Example 5 If $A>0,B>0,$ then let $A=(5,7;5,7;1,2;4,3;0.7;0.3)_{LR}$ and $B=(12,15;12,15;3,1;2,3;0.6;0.2)_{LR}$ be two GTrLRIFNs. Therefore, using Definition 6 (Eq. (7)),

$$\begin{aligned}A\oslash B=(&\frac{5}{12},\frac{7}{15};\frac{5}{12},\frac{7}{15};\frac{(5\cdot 3)+(12\cdot 1)}{12^2},\frac{(7\cdot 1)+(15\cdot 2)}{15^2};\\&\frac{(5\cdot 2)+(12\cdot 4)}{12^2},\frac{(7\cdot 3)+(15\cdot 3)}{15^2};\min(0.7,0.6);\max(0.3,0.2))_{LR}\\=&\left(\frac{5}{12},\frac{7}{15};\frac{5}{12},\frac{7}{15};\frac{3}{16},\frac{37}{225};\frac{29}{72},\frac{22}{75};0.6;0.3\right)_{LR}.\end{aligned}$$

2.3. *Generalised trapezoidal L-R intuitionistic fuzzy weighted average*

Generalised L-R intuitionistic fuzzy weighted average (GLRIF-WA) is a process to aggregate generalised L-R intuitionistic fuzzy numbers (GLRIFNs) using weighted average. In the evaluation of weighted averaging for GLRIF-WA, it is considered the scoring criteria, x_i and the relative weight, w_i are both GLRIFNs, and thus it forms a GLRIF-WA problem.

Definition 7 The aggregated value of GTrLRIF-WA $(m_i,n_i;m_i',n_i';l_i,r_i;l_i',r_i';h_i;h_i')_{LR}$ for

$i = 1, 2, ..., n$ by using GTrLRIF-WA operator is also a GTrLRIFNs, and denoted as

$$y = f\begin{pmatrix} x_1, x_2, ..., x_n; \\ w_1, w_2, ..., w_n \end{pmatrix} = \begin{pmatrix} \frac{\sum_{i=1}^{n} m_i^{w_i} m_i^x}{\sum_{i=1}^{n} m_i^w}, \frac{\sum_{i=1}^{n} n_i^{w_i} n_i^x}{\sum_{i=1}^{n} n_i^w}, \frac{\sum_{i=1}^{n} m_i^{w_i}{}' m_i^x{}'}{\sum_{i=1}^{n} m_i^w{}'}, \frac{\sum_{i=1}^{n} n_i^{w_i}{}' n_i^x{}'}{\sum_{i=1}^{n} n_i^w{}'}, \\ \frac{\sum_{i=1}^{n} \left(m_i^{w_i} m_i^x\right) l_i^w + m_i^w \left(m_i^{w_i} l_i^x + m_i^x l_i^{w_i}\right)}{\sum_{i=1}^{n} m_i^{w2}}, \\ \frac{\sum_{i=1}^{n} \left(n_i^{w_i} n_i^x\right) r_i^w + n_i^w \left(n_i^{w_i} r_i^x + n_i^x r_i^{w_i}\right)}{\sum_{i=1}^{n} n_i^{w2}}, \\ \frac{\sum_{i=1}^{n} \left(m_i^{w_i}{}' m_i^x{}'\right) l_i^w{}' + m_i^w{}' \left(m_i^{w_i}{}' l_i^x{}' + m_i^x{}' l_i^{w_i}{}'\right)}{\sum_{i=1}^{n} m_i^{w}{}'^2}, \\ \frac{\sum_{i=1}^{n} \left(n_i^{w_i}{}' n_i^x{}'\right) r_i^w{}' + n_i^w{}' \left(n_i^{w_i}{}' r_i^x{}' + n_i^x{}' r_i^{w_i}{}'\right)}{\sum_{i=1}^{n} n_i^{w}{}'^2}, \\ \wedge_{i=1}^{n} \left(h_i^{w_i}, h_i^x, h_i^w\right), \vee_{i=1}^{n} \min\left(h_i^{w_i}{}', h_i^x{}', h_i^w{}'\right) \end{pmatrix}_{LR} \tag{8}$$

where $\left(m_i^w, n_i^w; m_i^w{}', n_i^w{}'; l_i^w, r_i^w; l_i^w{}', r_i^w{}'; h_i^w; h_i^w{}'\right)_{LR}$ is the relative weight of GTrLRIFNs while $\left(m_i^x, n_i^x; m_i^x{}', n_i^x{}'; l_i^x, r_i^x; l_i^x{}', r_i^x{}'; h_i^x; h_i^x{}'\right)_{LR}$ is the scoring criteria of GTrLRIFNs.

The proposition of monotonicity, commutativity, boundedness, and idempotence for GTrLRIF-WA operator are shown as Proposition 1 to Proposition 4 respectively.

Proposition 1 ***Monotonicity*** Let $A = \left(m_{a_i}, n_{a_i}; m_{a_i}{}', n_{a_i}{}'; l_{a_i}, r_{a_i}; l_{a_i}{}', r_{a_i}{}'; h_{a_i}; h_{a_i}{}'\right)_{LR}$ and $B = \left(m_{b_i}, n_{b_i}; m_{b_i}{}', n_{b_i}{}'; l_{b_i}, r_{b_i}; l_{b_i}{}', r_{b_i}{}'; h_{b_i}; h_{b_i}{}'\right)_{LR}$ for $i = 1, 2, ..., n$ be a collection of GTrLRIFNs. If $m_{a_i} \le m_{b_i}$, $n_{a_i} \le n_{b_i}$, $m_{a_i}{}' \le m_{b_i}{}'$, $n_{a_i}{}' \le n_{b_i}{}'$, $l_{a_i} \le l_{b_i}$, $r_{a_i} \le r_{b_i}$, $l_{a_i}{}' \le l_{b_i}{}'$, $r_{a_i}{}' \le r_{b_i}{}'$, $h_{a_i} \le h_{b_i}$, and $h_{a_i}{}' \le h_{b_i}{}'$, then GTrLRIF-WA

$$f\left(x_{a_1}, x_{a_2}, ..., x_{a_n}; w_{a_1}, w_{a_2}, ..., w_{a_n}\right) \le f\left(x_{b_1}, x_{b_2}, ..., x_{b_n}; w_{b_1}, w_{b_2}, ..., w_{b_n}\right)$$

for all $i = 1, 2, ..., n$.

Proposition 2 ***Commutativity*** Let $A = \left(m_{a_i}, n_{a_i}; m_{a_i}{}', n_{a_i}{}'; l_{a_i}, r_{a_i}; l_{a_i}{}', r_{a_i}{}'; h_{a_i}; h_{a_i}{}'\right)_{LR}$ for $i = 1, 2, ..., n$ be a collection of GTrLRIFNs. $B = \left(m_{b_i}, n_{b_i}; m_{b_i}{}', n_{b_i}{}'; l_{b_i}, r_{b_i}; l_{b_i}{}', r_{b_i}{}'; h_{b_i}; h_{b_i}{}'\right)_{LR}$ is any permutation of the $\left(m_i, n_i; m_i{}', n_i{}'; l_i, r_i; l_i{}', r_i{}'; h_i; h_i{}'\right)_{LR}$ for $i = 1, 2, ..., n$. If $m_{a_i} = m_{b_i}$, $n_{a_i} = n_{b_i}$, $m_{a_i}{}' = m_{b_i}{}'$, $n_{a_i}{}' = n_{b_i}{}'$, $l_{a_i} = l_{b_i}$, $r_{a_i} = r_{b_i}$, $l_{a_i}{}' = l_{b_i}{}'$, $r_{a_i}{}' = r_{b_i}{}'$, $h_{a_i} = h_{b_i}$, and $h_{a_i}{}' = h_{b_i}{}'$ then GTrLRIF-WA

$$f\left(x_{a_1}, x_{a_2}, ..., x_{a_n}; w_{a_1}, w_{a_2}, ..., w_{a_n}\right) = f\left(x_{b_1}, x_{b_2}, ..., x_{b_n}; w_{b_1}, w_{b_2}, ..., w_{b_n}\right)$$

for all $i=1,2,...,n$.

Proposition 3 ***Boundedness*** Let $A=\left(m_{a_i},n_{a_i};m_{a_i}',n_{a_i}';l_{a_i},r_{a_i};l_{a_i}',r_{a_i}';h_{a_i};h_{a_i}'\right)_{LR}$ for $i=1,2,...,n$ be a group of GTrLRIFNs. If

$$\begin{aligned}A^{-}&=(\min_i m_{a_i},\min_i n_{a_i};\min_i m_{a_i}',\min_i n_{a_i}';\\&\quad\min_i l_{a_i},\min_i r_{a_i};\min_i l_{a_i}',\min_i r_{a_i}';\min_i h_{a_i};\max_i h_{a_i}')_{LR}\\A^{+}&=(\max_i m_{a_i},\max_i n_{a_i};\max_i m_{a_i}',\max_i n_{a_i}';\\&\quad\max_i l_{a_i},\max_i r_{a_i};\max_i l_{a_i}',\max_i r_{a_i}';\max_i h_{a_i};\min_i h_{a_i}')_{LR},\end{aligned}$$

then, $A^{-}\le f\left(x_1,x_2,...,x_n;w_1,w_2,...,w_n\right)\le A^{+}$.

Proposition 4 ***Idempotence*** Let $A_i=\left(m_{a_i},n_{a_i};m_{a_i}',n_{a_i}';l_{a_i},r_{a_i};l_{a_i}',r_{a_i}';h_{a_i};h_{a_i}'\right)_{LR}$ for $i=1,2,...,n$ be a group of GTrLRIFNs. If all a_i for $i=1,2,...,n$ are equal such that $A_1=A_2=...=A_n=A$, then GTrLRIF-WA $f\left(x_1,x_2,...,x_n;w_1,w_2,...,w_n\right)=\left(x;w\right)$.

Example 6 Assume that the parameter $x_1=\left(5,7;5,7;1,2;4,3;0.6;0.2\right)_{LR}$, $x_2=\left(6,7;6,7;3,1;2,3;0.6;0.3\right)_{LR}$, and $x_3=\left(8,10;8,10;5,2;3,4;0.9;0.1\right)_{LR}$ be the scoring criteria. Let $w_{x_1}=\left(5,7;5,7;2,3;3,4;0.7;0.1\right)_{LR}$, $w_{x_2}=\left(4,6;4,6;1,2;3,2;0.8;0.1\right)_{LR}$, and $w_{x_3}=\left(6,7;6,7;3,2;1,2;0.7;0.2\right)_{LR}$ be the relative weight. Then, the sum of w_{x_1},w_{x_2},w_{x_3} were obtained $w=\left(15,20;15,20;6,7;7,8;0.7;0.2\right)_{LR}$.

$$w_1=w_{x_1}\oslash w=\left(\frac{1}{3},\frac{7}{20};\frac{1}{3},\frac{7}{20};\frac{4}{15},\frac{109}{400};\frac{16}{45},\frac{17}{50};0.7;0.2\right)_{LR}$$

$$w_2=w_{x_2}\oslash w=\left(\frac{4}{15},\frac{3}{10};\frac{4}{15},\frac{3}{10};\frac{13}{75},\frac{41}{200};\frac{73}{225},\frac{11}{50};0.7;0.2\right)_{LR}$$

$$w_3=w_{x_3}\oslash w=\left(\frac{2}{5},\frac{7}{20};\frac{2}{5},\frac{7}{20};\frac{9}{25},\frac{89}{400};\frac{19}{75},\frac{6}{25};0.7;0.2\right)_{LR}$$

$$y=\frac{w_1x_1+w_2x_2+w_3x_3}{w_1+w_2+w_3}=\left(\frac{97}{225},\frac{161}{400};\frac{97}{225},\frac{161}{400};\frac{823}{1125},\frac{2017}{4000};\frac{2663}{3375},\frac{1289}{2000};0.6;0.3\right)_{LR}.$$

2.4. *VIKOR using GTrLRIFNs with weighted average aggregation operator*

This study classified the water pollution of multiple rivers in Johor, Malaysia, using the generalised trapezoidal L-R intuitionistic fuzzy VIKOR (GTrLRIF VIKOR) approach with a weighted average aggregation operator. The GTrLRIF VIKOR flowchart with a weighted average aggregation operator is displayed in Figure 1.

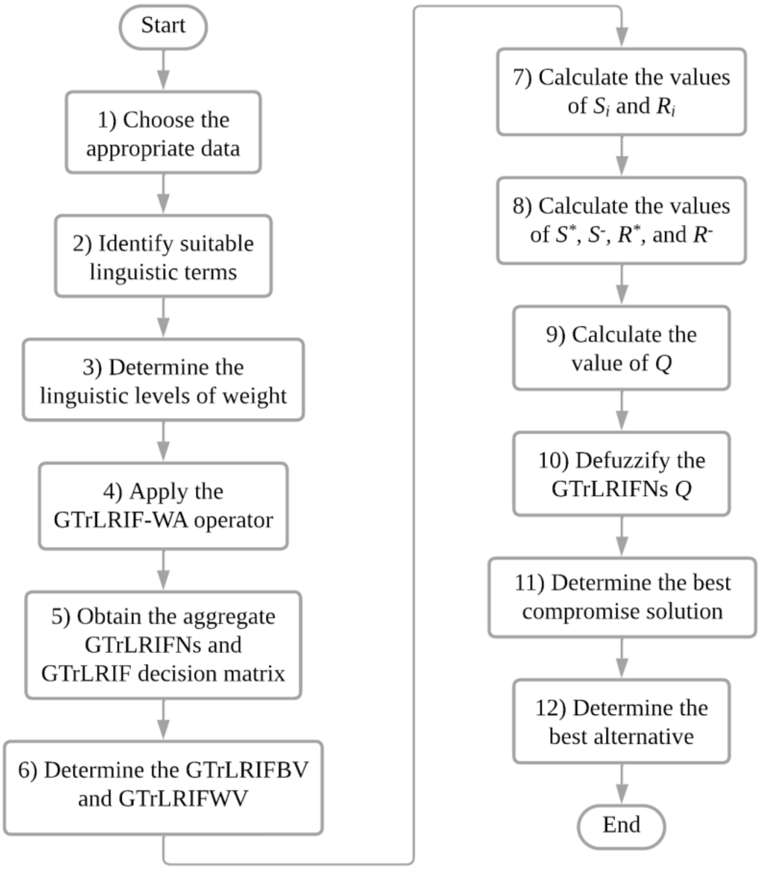

Figure 1: Flowchart of generalised trapezoidal L-R intuitionistic fuzzy VIKOR with weighted average aggregation operator

The following procedures are used to classify river water pollution:

Step 1: Select the relevant information in addition to the evaluation criteria (C) and alternatives (A).

Step 2: Identify relevant linguistic terms for the linguistic variables with respect to each criterion and importance weight of alternative.

Step 3: Determine the linguistic levels of the weight w_i for each $i = 1, 2, ..., n$.

Step 4: Apply the GTrLRIF-WA aggregation operator using Eq. (8) and get the aggregated GTrLRIFNs of the GTrLRIF decision matrix.

Step 5: After obtaining the aggregate weight of alternatives and rating of criterion, the GTrLRIF decision matrix is obtained by constructing a generalised L-R intuitionistic fuzzy decision matrix D.

$$D=\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}, \qquad i=1,2,...,m; \qquad j=1,2,...,n, \tag{9}$$

where x_{ij} is the rating of alternative A_i with respect to criterion C_j.

Step 6: Determine the generalised trapezoidal L-R intuitionistic fuzzy best value (GTrLRIFBV) denoted as f_j^* and generalised trapezoidal L-R intuitionistic fuzzy worst value (GTrLRIFWV) denoted as f_j^-.

$$f_j^* = \begin{cases} \max_i x_{ij} & ;\text{for benefit criteria} \\ \min_i x_{ij} & ;\text{for cost criteria} \end{cases} \tag{10}$$

$$f_j^- = \begin{cases} \min_i x_{ij} & ;\text{for benefit criteria} \\ \max_i x_{ij} & ;\text{for cost criteria} \end{cases} \tag{11}$$

where

$$\max_i x_{ij} = \left\langle \begin{array}{c} \max\left(m_i - l_i, m_j - l_j\right), \max\left(m_i, m_j\right), \max\left(n_i, n_j\right), \\ \max\left(n_i + r_i, n_j + r_j\right); \max\left(m_i' - l_i', m_j' - l_j'\right), \\ \max\left(m_i', m_j'\right), \max\left(n_i', n_j'\right), \max\left(n_i' + r_i', n_j' + r_j'\right); \\ \min\left(h_i, h_j\right); \max\left(h_i', h_j'\right) \end{array} \right\rangle_{LR},$$

$$\min_i x_{ij} = \left\langle \begin{array}{c} \min\left(m_i - l_i, m_j - l_j\right), \min\left(m_i, m_j\right), \min\left(n_i, n_j\right), \\ \min\left(n_i + r_i, n_j + r_j\right); \min\left(m_i' - l_i', m_j' - l_j'\right), \\ \min\left(m_i', m_j'\right), \min\left(n_i', n_j'\right), \min\left(n_i' + r_i', n_j' + r_j'\right); \\ \max\left(h_i, h_j\right); \min\left(h_i', h_j'\right) \end{array} \right\rangle_{LR}$$

for $i=1,2,3,...,k; j=1,2,3,...,k$.

Step 7: Calculate the values of S_i and R_i for $i=1,2,3,...,k$.

$$S_i = \frac{\sum_{j=1}^{n} w_j \left(f_j^* - x_{ij}\right)}{\left(f_j^* - f_j^-\right)}$$

$$= \sum_{j=1}^{n} w_j \times \frac{\left\langle \begin{matrix} f_{mj}^* - x_{mj}, f_{nj}^* - x_{nj}; f_{m'j}^* - x_{m'j}, f_{n'j}^* - x_{n'j}; f_{lj}^* + x_{rj}, f_{rj}^* + x_{lj}; \\ f_{l'j}^* + x_{r'j}, f_{r'j}^* + x_{l'j}; \min\left(h_{f_j^*}, h_{x_{ij}}\right); \max\left(h'_{f_j^*}, h'_{x_{ij}}\right) \end{matrix} \right\rangle_{LR}}{\left\langle \begin{matrix} f_{mj}^* - f_{mj}^-, f_{nj}^* - f_{nj}^-; f_{m'j}^* - f_{m'j}^-, f_{n'j}^* - f_{n'j}^-; f_{lj}^* + f_{rj}^-, f_{rj}^* + f_{lj}^-; \\ f_{l'j}^* + f_{r'j}^-, f_{r'j}^* + f_{l'j}^-; \min\left(h_{f_j^*}, h_{f_j^-}\right); \max\left(h'_{f_j^*}, h'_{f_j^-}\right) \end{matrix} \right\rangle_{LR}} \tag{12}$$

$$R_i = \max_j \left[\frac{w_j \left(f_j^* - x_{ij} \right)}{\left(f_j^* - f_j^- \right)} \right]$$

$$= \max_j \left(w_j \times \frac{\left\langle \begin{matrix} f_{mj}^* - x_{mj}, f_{nj}^* - x_{nj}; f_{m'j}^* - x_{m'j}, f_{n'j}^* - x_{n'j}; f_{lj}^* + x_{rj}, f_{rj}^* + x_{lj}; \\ f_{l'j}^* + x_{r'j}, f_{r'j}^* + x_{l'j}; \min\left(h_{f_j^*}, h_{x_{ij}}\right); \max\left(h'_{f_j^*}, h'_{x_{ij}}\right) \end{matrix} \right\rangle_{LR}}{\left\langle \begin{matrix} f_{mj}^* - f_{mj}^-, f_{nj}^* - f_{nj}^-; f_{m'j}^* - f_{m'j}^-, f_{n'j}^* - f_{n'j}^-; f_{lj}^* + f_{rj}^-, f_{rj}^* + f_{lj}^-; \\ f_{l'j}^* + f_{r'j}^-, f_{r'j}^* + f_{l'j}^-; \min\left(h_{f_j^*}, h_{f_j^-}\right); \max\left(h'_{f_j^*}, h'_{f_j^-}\right) \end{matrix} \right\rangle_{LR}} \right) \tag{13}$$

where $S_i = A_i$ with respect to all criteria calculated by the sum of the distance for GLRIFBV while $R_i = A_i$ with respect to the j^{th} criterion, calculated by maximum distance of GLRIFWV.

Step 8: State the values of S^*, S^-, R^*, and R^-.

$$S^* = \min_i S_i, \qquad S^- = \max_i S_i, \qquad R^* = \min_i R_i, \qquad R^- = \max_i R_i \tag{14}$$

Step 9: Calculate and rate the alternatives by the index Q_i for $i = 1,2,3,...,k$ by using Eq. (15).

$$\begin{aligned} Q_i = {} & \vartheta \left[\frac{\left\langle \begin{matrix} S_{mi} - S_m^*, S_{ni} - S_n^*; S_{m'i} - S_{m'}^*, S_{n'i} - S_{n'}^*; S_{li} + S_r^*, S_{ri} + S_l^*; \\ S_{l'i} + S_{r'}^*, S_{r'i} + S_{l'}^*; \min\left(h_{S_i}, h_{S^*}\right); \max\left(h'_{S_i}, h'_{S^*}\right) \end{matrix} \right\rangle_{LR}}{\left\langle \begin{matrix} S_m^- - S_m^*, S_n^- - S_n^*; S_{m'}^- - S_{m'}^*, S_{n'}^- - S_{n'}^*; S_l^- + S_r^*; S_r^- + S_l^*; \\ S_{l'}^- + S_{r'}^*; S_{r'}^- + S_{l'}^*; \min\left(h_{S^-}, h_{S^*}\right); \max\left(h_{S^-}, h_{S^*}\right) \end{matrix} \right\rangle_{LR}} \right] \\ & + \left(1 - \vartheta\right) \left[\frac{\left\langle \begin{matrix} R_{mi} - R_m^*, R_{ni} - R_n^*; R_{m'i} - R_{m'}^*, R_{n'i} - R_{n'}^*; R_{li} + R_r^*, R_{ri} + R_l^*; \\ R_{l'i} + R_{r'}^*, R_{r'i} + R_{l'}^*; \min\left(h_{R_i}, h_{R^*}\right); \max\left(h'_{R_i}, h'_{R^*}\right) \end{matrix} \right\rangle_{LR}}{\left\langle \begin{matrix} R_m^- - R_m^*, R_n^- - R_n^*; R_{m'}^- - R_{m'}^*, R_{n'}^- - R_{n'}^*; R_l^- + R_r^*; R_r^- + R_l^*; \\ R_{l'}^- + R_{r'}^*; R_{r'}^- + R_{l'}^*; \min\left(h_{R^-}, h_{R^*}\right); \max\left(h_{R^-}, h_{R^*}\right) \end{matrix} \right\rangle_{LR}} \right] \end{aligned} \tag{15}$$

where ϑ is introduced as weight of the strategy of "the maximum group utility" and often set to be 0.5 (Muhamad *et al*. 2018).

Step 10: Defuzzify the value of Q_i. The defuzzification procedure makes use of the graded mean integration (Devi 2011). A GTrLRIFNs in a notation $Q_i = \left(Q_{(m-l)i}, Q_{(m)i}, Q_{(n)i}, Q_{(n+r)i}; Q_{(m'-l')i}, Q_{(m')i}, Q_{(n')i}, Q_{(n'+r')i}; Q_{(h)i}; Q_{(h')i}\right)_{LR}$ is converted into an exact number by utilising Eq. (16).

$$Q = \frac{\left(Q_{(m-l)i} + Q_{(m)i} + Q_{(n)i} + Q_{(n+r)i}\right)Q_{(h)i} + \left(Q_{(m'-l')i} + Q_{(m')i} + Q_{(n')i} + Q_{(n'+r')i}\right)Q_{(h')i}}{8} \tag{16}$$

The right and left utilities can be combined to obtain the crisp value. The list Q_i implicit the detachment estimates of A_i from the top alternative. Therefore, the lower the VIKOR index Q, the preferable the alternative (Sunarsih *et al*. 2020).

Step 11: Determine the best compromise solution. The alternative A' is the best ranked by the measure Q if the following two conditions are satisfied:
[*Condition 1*]: Reasonable advantage.

$$Q(A'') - Q(A') \geq DQ; \qquad DQ = \frac{1}{m-1} \tag{17}$$

where A' and A'' are the first and second position in the ranking list by Q; m is the number of alternatives.

[*Condition 2*]: Acceptable stability.
The alternative A' must be ranked first with the minimum of $S(A')$ or/and $R(A')$. The compromise solution can be found if one of these requirements is not satisfied:

- Alternative A' and A'' if condition 2 is not satisfies, or
- Alternative $A', A'', ... A^m$ if condition 1 is not satisfied: A^m can be calculated by the equation $Q(A^m) - Q(A') < DQ$ for maximum m.

Step 12: Determine the best alternative by sorting the VIKOR index Q in ascending order.

2.5. *Determination of generalised trapezoidal L-R intuitionistic fuzzy numbers*

This study replicates the work of Shafie *et al*. (2023) on the generalised trapezoidal L-R intuitionistic fuzzy number (GTrLRIFN). The trapezoidal L-R intuitionistic fuzzy numbers (TrLRIFNs) for linguistic variables are modified based on the method used by Lee and Wang (2010) that utilises the data's minimum, maximum, mean, and standard deviation. This study has used 100 river data from the bootstrap method to determine the minimum (min), maximum (max), mean, and standard deviation (SD) used in Lee and Wang's (2010) method. Table 1 shows the sample formulation of the membership function for DO. All elements of the river, which are DO, BOD, COD, SS, pH, and AN, have used the same formulation for the evaluation. In Shafie *et al*. (2023), decision-makers determined the level of confidence, which depended on their degree of certainty regarding the reliability of the dataset for each parameter associated with the rivers. The confidence level is important because it acknowledges that humans have significance for determining the height of membership and non-membership functions. Due to the fact that decision-makers frequently come from various backgrounds with differing levels of experience, education, and other traits that may impact the evaluation process, it is imperative to include the confidence level. Next, this study also used the same L and R functions used by Shafie *et al*. (2023), which is $y = \frac{1}{1+x^2}$.

Table 1: Formulation of membership function for DO

Linguistic Variable	Linguistic Term	TrLRIFNs
DO	VC	$(Min, Mean-SD; Min, Mean-SD; 0, SD; 0, SD)_{LR}$
	C	$\left(\frac{Min+Mean}{2}, \frac{2Mean-SD}{2}; \frac{Min+Mean}{2}, \frac{2Mean-SD}{2}; \frac{SD}{2}, SD; \frac{SD}{2}, SD\right)_{LR}$
	SP	$(Mean, Mean; Mean, Mean; SD, SD; SD, SD)_{LR}$
	P	$\left(\frac{2Mean+SD}{2}, \frac{Mean+Max}{2}; \frac{2Mean+SD}{2}, \frac{Mean+Max}{2}; SD, \frac{SD}{2}; SD, \frac{SD}{2}\right)_{LR}$
	VP	$(Mean+SD, Max; Mean+SD, Max; SD, 0; SD, 0)_{LR}$

where VC = Very Clean, C = Clean, SP = Slightly Polluted, P = Polluted, VP = Very Polluted.

3. Implementation of the Proposed Methodology in the Classification of River Water Pollution

The data for five rivers have been selected for further evaluation which has been provided by DOE Malaysia: the Kim Kim River (A_1), Sayong River (A_2), Telor River (A_3), Pelepah River (A_4), and Bantang River (A_5) from 2019 to 2021. The six criteria are DO (C_1), BOD (C_2), COD (C_3), SS (C_4), pH (C_5), and AN (C_6) were used for evaluating the five possible alternatives. While DO and pH were categorised as cost criteria in this study, BOD, COD, SS, and AN were classed as benefit criteria. The suggested method is currently being used to address this issue, and the procedure is summed up as follows:

Step 1: This study averaged river data (2019 until 2021) for each of the five alternatives.

Step 2: The relevant linguistic terms for the linguistic variables with respect to each criterion and the importance weight of alternative were determined based on Table 2, are shown in Table 3 to Table 5. Table 2 is obtained by modifying the water quality index classification from Department of Environment (2019), aiming to derive alternative weights as GTrLRIFNs.

Step 3: Linguistic levels of weight have been determined for each alternative from 2019 to 2021. It can be shown in Table 3 to Table 5 which indicates the class of WQI. The GTrLRIFNs for the class of WQI can be determined by Table 1, which indicates the linguistic terms for alternative weights.

Table 2: Linguistic terms for alternative weights

Alternative Weight	GTrLRIFNs
Class I (VC)	$(1.00,4.15; 1.00,4.15; 0.00,8.55; 0.00,8.55; 1; 0)_{LR}$
Class II (C)	$(12.70,18.10; 12.70,18.10; 8.55,13.60; 8.55,13.60; 1; 0)_{LR}$
Class III (SP)	$(31.70,39.90; 31.70,39.90; 13.60,15.17; 13.60,15.17; 1; 0)_{LR}$
Class IV (P)	$(55.07,62.04; 55.07,62.04; 15.17,22.46; 15.17,22.46; 1; 0)_{LR}$
Class V (VP)	$(84.50,100.00; 84.50,100.00; 22.46,0.00; 22.46,0.00; 1; 0)_{LR}$

Table 3: Data ratings for every alternative in 2019

Alternative (River)	WQI	C_1 (DO)	C_2 (BOD)	C_3 (COD)	C_4 (SS)	C_5 (pH)	C_6 (AN)
A_1 (Kim Kim)	IV	P	VP	P	C	C	VP
A_2 (Sayong)	II	C	SP	C	VC	SP	C
A_3 (Telor)	III	VC	SP	C	SP	SP	SP
A_4 (Pelepah)	II	VC	SP	C	C	C	C
A_5 (Bantang)	II	VC	SP	C	VC	VC	SP

Table 4: Data ratings for every alternative in 2020

Alternative (River)	WQI	C_1 (DO)	C_2 (BOD)	C_3 (COD)	C_4 (SS)	C_5 (pH)	C_6 (AN)
A_1 (Kim Kim)	IV	P	SP	SP	C	C	VP
A_2 (Sayong)	II	VC	C	C	C	P	VC
A_3 (Telor)	II	VC	C	C	SP	C	C
A_4 (Pelepah)	II	C	C	C	C	C	SP
A_5 (Bantang)	I	VC	C	VC	VC	VC	VC

Table 5: Data ratings for every alternative in 2021

Alternative (River)	WQI	C_1 (DO)	C_2 (BOD)	C_3 (COD)	C_4 (SS)	C_5 (pH)	C_6 (AN)
A_1 (Kim Kim)	III	SP	P	SP	VC	C	VP
A_2 (Sayong)	II	VC	VC	VC	C	P	C
A_3 (Telor)	II	VC	C	VC	C	SP	VC
A_4 (Pelepah)	II	C	C	C	C	C	SP
A_5 (Bantang)	I	VC	VC	VC	VC	C	VC

Step 4: Apply the GTrLRIF-WA aggregation operator using Eq. (8) and obtained the aggregated values of GTrLRIFNs.

Step 5: The GTrLRIF decision matrix is obtained using Eq. (9) from the aggregated values of GTrLRIFNs, as shown in Table 6.

Step 6: Calculate the generalised trapezoidal L-R intuitionistic fuzzy best value (GTrLRIFBV), f_j^* and generalised trapezoidal L-R intuitionistic fuzzy worst value (GTrLRIFWV), f_j^- using Eqs. (10) and (11) respectively, as shown in Table 7.

Table 6: Generalised Trapezoidal L-R Intuitionistic Fuzzy Decision Matrix

Alternative (River)	C_1 (DO)	C_2 (BOD)	C_3 (COD)	C_4 (SS)	C_5 (pH)	C_6 (AN)
A_1 (Kim Kim)	(2.20,2.52; 2.20,2.52; 0.56,1.52; 0.56,1.52; 0.91; 0.09)$_{LR}$	(11.89,12.98; 11.89,12.98; 2.37,7.70; 2.37,7.70; 0.89; 0.11)$_{LR}$	(44.48,45.16; 44.48,45.16; 6.75,27.20; 6.75,27.20; 0.91; 0.09)$_{LR}$	(28.13,31.52; 28.13,31.52; 5.19,25.70; 5.19,25.70; 0.91; 0.09)$_{LR}$	(6.70,6.79; 6.70,6.79; 0.54,3.68; 0.54,3.68; 0.93; 0.07)$_{LR}$	(5.53,6.41; 5.53,6.41; 1.19,3.48; 1.19,3.48; 0.91; 0.09)$_{LR}$
A_2 (Sayong)	(6.86,7.01; 6.86,7.01; 0.72,11.92; 0.72,11.92; 0.85; 0.15)$_{LR}$	(2.69,2.77; 2.69,2.77; 4.90,0.52; 4.90,0.52; 0.83; 0.17)$_{LR}$	(9.70,10.31; 9.70,10.31; 17.03,1.48; 17.03,1.48; 0.83; 0.17)$_{LR}$	(21.33,28.17; 21.33,28.17; 39.61,9.25; 39.61,9.25; 0.76; 0.24)$_{LR}$	(4.80,4.82; 4.80,4.82; 8.38,0.42; 8.38,0.42; 0.86; 0.14)$_{LR}$	(0.09,0.11; 0.09,0.11; 0.17,0.04; 0.17,0.04; 0.85; 0.15)$_{LR}$
A_3 (Telor)	(6.92,7.10; 6.92,7.10; 6.35,0.67; 6.35,0.67; 0.88; 0.12)$_{LR}$	(3.49,3.52; 3.49,3.52; 2.77,0.73; 2.77,0.73; 0.88; 0.12)$_{LR}$	(11.35,12.22; 11.35,12.22; 9.19,1.88; 9.19,1.88; 0.88; 0.12)$_{LR}$	(71.52,71.99; 71.52,71.99; 66.14,21.16; 66.14,21.16; 0.85; 0.15)$_{LR}$	(5.86,5.88; 5.86,5.88; 5.35,0.58; 5.35,0.58; 0.86; 0.14)$_{LR}$	(0.18,0.18; 0.18,0.18; 0.19,0.10; 0.19,0.10; 0.88; 0.12)$_{LR}$
A_4 (Pelepah)	(6.25,6.46; 6.25,6.46; 10.91,0.72; 10.91,0.72; 0.93; 0.07)$_{LR}$	(2.50,2.59; 2.50,2.59; 4.46,0.40; 4.46,0.40; 0.93; 0.07)$_{LR}$	(11.38,11.80; 11.38,11.80; 20.02,1.58; 20.02,1.58; 0.93; 0.07)$_{LR}$	(28.41,33.94; 28.41,33.94; 54.52,13.33; 54.52,13.33; 0.93; 0.07)$_{LR}$	(6.60,6.66; 6.60,6.66; 11.45,0.60; 11.45,0.60; 0.94; 0.06)$_{LR}$	(0.44,0.45; 0.44,0.45; 0.84,0.13; 0.84,0.13; 0.93; 0.07)$_{LR}$
A_5 (Bantang)	(8.17,8.24; 8.17,8.24; 12.36,13.51; 12.36,13.51; 0.93; 0.07)$_{LR}$	(3.07,3.09; 3.07,3.09; 5.01,3.80; 5.01,3.80; 0.93; 0.07)$_{LR}$	(9.62,9.90; 9.62,9.90; 15.27,12.86; 15.27,12.86; 0.93; 0.07)$_{LR}$	(3.49,4.87; 3.49,4.87; 5.51,7.17; 5.51,7.17; 0.91; 0.09)$_{LR}$	(7.17,7.27; 7.17,7.27; 10.88,11.69; 10.88,11.69; 0.94; 0.06)$_{LR}$	(0.40,0.40; 0.40,0.40; 0.83,0.55; 0.83,0.55; 0.93; 0.07)$_{LR}$

Table 7: GTrLRIFBV and GTrLRIFWV

	C_1 (DO)	C_2 (BOD)	C_3 (COD)	C_4 (SS)	C_5 (pH)	C_6 (AN)
f_j^*	(2.20,2.52; 2.20,2.52; 7.27,1.52; 7.27,1.52; 0.93; 0.07)$_{LR}$	(11.89,12.98; 11.89,12.98; 2.37,7.70; 2.37,7.70; 0.83; 0.17)$_{LR}$	(44.48,45.16; 44.48,45.16; 6.75,27.20; 6.75,27.20; 0.83; 0.17)$_{LR}$	(71.52,71.99; 71.52,71.99; 48.58,21.16; 48.58,21.16; 0.76; 0.24)$_{LR}$	(4.80,4.82; 4.80,4.82; 9.66,0.42; 9.66,0.42; 0.94; 0.06)$_{LR}$	(5.53,6.41; 5.53,6.41; 1.19,3.48; 1.19,3.48; 0.85; 0.15)$_{LR}$
f_j^-	(8.17,8.24; 8.17,8.24; 6.53,13.51; 6.53,13.51; 0.85; 0.15)$_{LR}$	(2.50,2.59; 2.50,2.59; 4.71,0.40; 4.71,0.40; 0.93; 0.07)$_{LR}$	(9.62,9.90; 9.62,9.90; 18.26,1.88; 18.26,1.88; 0.93; 0.07)$_{LR}$	(3.49,4.87; 3.49,4.87; 29.60,7.17; 29.60,7.17; 0.93; 0.07)$_{LR}$	(7.17,7.27; 7.17,7.27; 1.01,11.69; 1.01,11.69; 0.86; 0.14)$_{LR}$	(0.09,0.11; 0.09,0.11; 0.52,0.04; 0.52,0.04; 0.93; 0.07)$_{LR}$

Step 7: Calculate the values of S_i and R_i using Eqs. (12) and (13) respectively, as shown in Table 8.

Step 8: The values of S^*, S^-, R^*, and R^- are determined using Eq. (14), as shown in Table 9.

Table 8: List of S_i and R_i

Alternative (River)	A_1 (Kim Kim)	A_2 (Sayong)	A_3 (Telor)	A_4 (Pelepah)	A_5 (Bantang)
S_i	(0.20,0.19; 0.20,0.19; -1.12,0.40; -1.12,0.40; 0.76; 0.24)$_{LR}$	(0.79,0.77; 0.79,0.77; -0.91,0.07; -0.91,0.07; 0.76; 0.24)$_{LR}$	(0.70,0.70; 0.70,0.70; -1.47,0.28; -1.47,0.28; 0.76; 0.24)$_{LR}$	(0.82,0.82; 0.82,0.82; -1.67,-0.02; -1.67,-0.02; 0.76; 0.24)$_{LR}$	(0.98,0.98; 0.98,0.98; -3.04,-0.24; -3.04,-0.24; 0.76; 0.24)$_{LR}$
R_i	(0.10,0.10; 0.10,0.10; -1.54,0.14; -1.54,0.14; 0.76; 0.24)$_{LR}$	(0.19,0.19; 0.19,0.19; -0.88,0.45; -0.88,0.45; 0.76; 0.24)$_{LR}$	(0.17,0.18; 0.17,0.18; -0.90,0.39; -0.90,0.39; 0.76; 0.24)$_{LR}$	(0.19,0.19; 0.19,0.19; -1.24,0.45; -1.24,0.45; 0.76; 0.24)$_{LR}$	(0.22,0.22; 0.22,0.22; -2.06,0.41; -2.06,0.41; 0.76; 0.24)$_{LR}$

Table 9: Value of S^*, S^-, R^*, and R^-

	Value of S^*, S^-, R^*, and R^-
S^*	(0.20,0.19; 0.20,0.19; -1.12,0.40; -1.12,0.40; 0.76; 0.24)$_{LR}$
S^*	(0.98,0.98; 0.98,0.98; -3.04,0.00; -3.04,0.00; 0.76; 0.24)$_{LR}$
R^*	(0.10,0.10; 0.10,0.10; -0.96,0.14; -0.96,0.14; 0.76; 0.24)$_{LR}$
R^-	(0.22,0.22; 0.22,0.22; -2.06,0.42; -2.06,0.42; 0.76; 0.24)$_{LR}$

Step 9: Calculate the Q_i and rate the alternatives by the index Q_i by using Eq. (15), as shown in Table 10.

Step 10: Defuzzify the value of Q_i to determine the ranking of preferable alternative using Eq. (16), as shown in Table 11.

Table 10: List of Q_i

Alternative (River)	A_1 (Kim Kim)	A_2 (Sayong)	A_3 (Telor)	A_4 (Pelepah)	A_5 (Bantang)
Q_i	(0.00,0.00; 0.00,0.00; -6.40,-3.79; -6.40,-3.79; 0.76; 0.24)$_{LR}$	(0.73,0.73; 0.73,0.73; -10.56,-4.83; -10.56,-4.83; 0.76; 0.24)$_{LR}$	(0.62,0.64; 0.62,0.64; -9.98,-4.68; -9.98,-4.68; 0.76; 0.24)$_{LR}$	(0.77,0.77; 0.77,0.77; -12.93,-5.00; -12.93,-5.00; 0.76; 0.24)$_{LR}$	(1.00,1.00; 1.00,1.00; -19.73,-5.99; -19.73,-5.99; 0.76; 0.24)$_{LR}$

Table 11: List of Q_i and rank for alternatives

Alternative (River)	A_1 (Kim Kim)	A_2 (Sayong)	A_3 (Telor)	A_4 (Pelepah)	A_5 (Bantang)
Q_i	1.61	5.33	4.82	6.79	10.93
Rank	1	3	2	4	5

Step 11: Using the double condition, compromised solution (A') by the index Q are determined.
[*Condition 1*]: Acceptable advantage.
By using Eq. (17),

$$Q(A'') - Q(A') = 4.82 - 1.61 = 3.21 \geq 0.25,$$
$$Q(A''') - Q(A') = 5.33 - 1.61 = 3.72 \geq 0.25.$$

[*Condition 2*]: Reasonable range in decision making as shown in Table 12.

Table 12: Reasonable range in decision making

Reasonable Range	Rank
Rank by Q_i	$A_1 \succ A_3 \succ A_2 \succ A_4 \succ A_5$
Rank by S_i	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$
Rank by R_i	$A_3 \succ A_1 \succ A_2 \succ A_4 \succ A_5$

The result shows that Condition 1 and Condition 2 are satisfied. Therefore, A_1 is the solution.

Step 12: The minimum value of $Q(A')$ indicates the best alternative. It is suggested that the compromise solution is A_1 owing to its closeness to the best alternative.

4. Discussion of Results and Comparative Analysis

The procedure of classifying river water pollution was challenging and complex due to the numerous aspects that needed to be considered simultaneously, as well as the classification's subjectivity and ambiguity. The five rivers used in this study have all been evaluated from 2019 to 2021. DO, BOD, COD, SS, pH, and AN are the criteria taken into account to classify river water pollution. The preferable solution was ranked using a GTrLRIF VIKOR method that incorporates the generalised trapezoidal L-R intuitionistic weighted average (GTrLRIF-WA) aggregation operator.

Table 13: Comparison of proposed method with Water Quality Index (WQI)

Alternative (River)	GLRIF VIKOR		Water Quality Index		
	Q_i	Rank	WQI	Rank	Class
A_1 (Kim Kim)	1.61	1	49.77	1	IV (P)
A_2 (Sayong)	5.33	3	84.23	3	II (C)
A_3 (Telor)	4.82	2	84.18	2	II (C)
A_4 (Pelepah)	6.79	4	84.44	4	II (C)
A_5 (Bantang)	10.93	5	93.69	5	I (VC)
Order	$A_1 \succ A_3 \succ A_2 \succ A_4 \succ A_5$		$A_1 \succ A_3 \succ A_2 \succ A_4 \succ A_5$		

According to the values of Q_i, the polluted river water in ranked is Kim Kim River (A_1), Telor River (A_3), Sayong River (A_2), Pelepah River (A_4), and Bantang River (A_5). The result of the proposed method has been compared with the Water Quality Index (WQI) method used by Malaysia in Table 13. It shows that for GTrLRIF VIKOR and WQI methods, the Kim Kim

River is the most polluted river, while the Bantang River is the cleanest river. The result also shows the same rank of river water pollution using the proposed method and classical WQI. However, unlike the classical WQI method, the proposed GTrLRIF VIKOR method incorporates fuzzy theory to better handle uncertainty and imprecision in river water pollution assessment. It utilises linguistic terms and considers both membership and non-membership functions to represent the degree of truth and falsity, enhancing the depth of the evaluation. Additionally, the GTrLRIF VIKOR method integrates a confidence level, which makes the evaluation more reliable. The classical WQI method is a single-valued method that does not take into account the confidence level in evaluating the data. This can lead to inaccurate results, especially when the data is uncertain. This makes the proposed GTrLRIF VIKOR method more flexible, adaptive, and practical, especially when dealing with uncertain environmental data.

Table 14: Result of river water pollution using fuzzy complex index method

Alternative (River)	Class I (Very Clean)	Class II (Clean)	Class III (Slightly Polluted)	Class IV (Polluted)	Class V (Very Polluted)
A_1 (Kim Kim)	0.0632	0	0.2194	**0.4358**	0.2816
A_2 (Sayong)	**0.7795**	0.2019	0	0.0186	0
A_3 (Telor)	**0.9247**	0.0319	0	0.0434	0
A_4 (Pelepah)	0.3645	**0.5807**	0.0548	0	0
A_5 (Bantang)	**1**	0	0	0	0
Order	$A_1 \succ A_4 \succ A_2 \succ A_3 \succ A_5$				

In addition to the comparison with the WQI method, the GTrLRIF VIKOR approach was also evaluated against another fuzzy-based method, the Fuzzy Complex Index (FCI). This comparison provides deeper insights into the relative performance of fuzzy decision-making frameworks under similar conditions. Table 14 shows the result of river water pollution using FCI method. For FCI method, each alternative is assigned to a pollution class based on the highest membership value. Accordingly, the order of river water pollution is Kim Kim River (A_1), Pelepah River (A_4), Sayong River (A_2), Telor River (A_3), and Bantang River (A_5). The results also show that the Kim Kim River is the most polluted river, while the Bantang River is the cleanest river, which is consistent with the findings obtained using the proposed method of GTrLRIF VIKOR. This alignment further supports the robustness and reliability of the fuzzy-based approaches in assessing river water quality under the given criteria. However, the proposed method of GTrLRIF VIKOR offers an additional advantage by incorporating the confidence level to explicitly quantify the uncertainty, providing a more robust representation to cater the problem of uncertainty with a more reliable evaluation. In contrast, the FCI method integrates multiple dimensions of information to form a comprehensive decision-making framework based on the principles of fuzzy relation, that consider the inherent uncertainty in real-world data. The findings of this study highlight the critical need for immediate attention and targeted intervention in managing the water quality of the Kim Kim River, as it emerged as the most polluted among the rivers evaluated.

The Kim Kim River's polluted water is caused by a variety of factors. Chemicals like methane, hydrogen chloride, acrylonitrile, acrolein, benzoene, xylene, and methyl mercaptan were exposed because Kim Kim River was located in an industrial area with heavy industries like shipbuilding, petrochemical manufacturing, logistics and transportation, and oil palm storage and distribution (Ismail *et al*. 2020; Yap *et al*. 2019). The Kim Kim River became famous in 2019 due to the water pollution incident that affected over 1000 people, and 209 people were hospitalised while 111 schools near the river were subsequently closed (Shamsuddin *et al*. 2019). This is due to the 2.43 tons of chemical waste dumped into the Kim

Kim River that caused river pollution and led to serious water pollution, which consisted of Ethyl Benzene, Toluene, Xylene, D-Limonene, and Benzene (Department of Chemistry 2019; Hussein 2019). Bantang River became the cleanest river since it is located away from the industrial area; hence the river is used for Forest Eco Park. Based on the aquatic insect composition and WQI value, the Bantang River provided an appropriate physical and chemical variable (Zakaria & Mohamed 2019). The good water quality of the Bantang River is a testament to the importance of protecting Malaysia's rivers from pollution.

Based on the aggregated river data (Table 6), the DO at Kim Kim River is too low due to the industrial wastewater discharge, especially in 2019 and 2020. DO in rivers reflects the breathing of aquatic life (Zhi *et al*. 2021). The changes in DO concentration in the river can affect the BOD and COD in the river water, as shown in the Kim Kim River dataset. The high concentration of BOD and COD at Kim Kim River is also due to the industrial wastewater discharge containing organic materials. The relative presence of organic contaminants in water is determined by the COD, which is a frequently used measure for environmental monitoring and impact assessment (Parsimehr *et al*. 2018). Besides that, Kim Kim River also has a high concentration of AN due to the discharge of wastewater from the fertiliser industry near Kim Kim River. Abdullah *et al*. (2023) stated that animal waste and sewage are the main causes of AN, whereas garbage is the main cause of high BOD. Contradicting to the Bantang River, the cleanest river in this study, the concentration of SS is the lowest, and the pH value is the highest due to the strategic location of the Bantang River, which is located away from human activities, leading to higher biodiversity and natural decomposition. These findings underscore the critical importance of understanding and managing the various sources of pollutants in rivers to safeguard water quality and ecological balance.

In 2018, The pH value of tap water samples from Pasir Gudang, which is near Kim Kim River was analysed by Nurani Zulkifli *et al*. (2018). Comparisons were made between the pH value and the standard range of 6.5 to 8.5 for tap and drinking water as recommended by DOE Malaysia. The measured values exhibit significant variation within the pH range of 6.00 to 8.65. The industrial activities in the vicinity of Kim Kim River have considerable ramifications not only on its water quality but also on the surrounding areas, highlighting the urgency for stricter regulatory measures and sustainable practices to mitigate these environmental impacts.

Therefore, a key policy priority should be to plan for the long-term care of river water for future generations. To guarantee the sustainability of Malaysian river water today and in the future, several issues must be addressed. DOE Malaysia should invest in green technology for remote sensing and monitoring to help identify real-time pollution sources, track changes in water quality, and enable prompt responses to pollution incidents. The government also needs to formulate stringent pollution control regulations and spread awareness campaigns to ensure that industries and communities adhere to strict waste disposal and wastewater treatment standards. Hence, prioritising the comprehensive and sustainable management of Malaysia's river water resources is critical for the well-being of present and future generations.

5. Conclusion

Classifying river water pollution is important to environment and human health because it allows us to identify and mitigate potential risks, implement targeted pollution control measures, and safeguard both aquatic ecosystems and the well-being of communities that depend on these rivers for various purposes. The GTrLRIF-WA aggregation operator was therefore incorporated into this study's new GTrLRIF VIKOR method to classify river water pollution for a number of rivers in Johor, Malaysia, including the Kim Kim, Sayong, Telor, Pelepah, and Bantang River, from 2019 to 2021.

The evaluation process to classify river water pollution consider DO, BOD, COD, SS, pH, and AN in it. The findings show that, in comparison to the other rivers, the Kim Kim River is the most polluted. According to the values of Q, the polluted river water ranked is Kim Kim River, Telor River, Sayong River, Pelepah River, and Bantang River. By implementing this approach to a broader range of rivers in Johor, Malaysia, this study has not only identified the most polluted river but also created a valuable approach for a comprehensive and effective means to assess and categorise river water pollution.

The results were also compared with those obtained using the classical WQI method by DOE Malaysia and the Fuzzy Composite Index (FCI) method. Across all methods, the findings consistently indicate that the Bantang River has the cleanest water, while the Kim Kim River is the most polluted. Additionally, the results of the proposed method and traditional WQI methods have the same rank of polluted river water. The level of confidence is taken into account in the proposed GTrLRIF VIKOR method. Given that, it has the advantage of offering a comprehensive analysis of data uncertainty, with a better representation of the human evaluation process attributable to the usage of membership and non-membership functions. Furthermore, adding confidence level values will add another layer of data to assess the decision-makers' judgemental behaviour. The study's findings demonstrated how well GTrLRIF VIKOR classified river water pollution.

The primary contribution of this research lies in the development of the arithmetic operation and the aggregation operator of GTrLRIFNs along with the successful application of the GTrLRIF VIKOR method as a comprehensive decision-support tool for water quality assessment. Its ability to handle uncertainty and linguistic information makes it especially useful for environmental decision-making where expert judgment plays a critical role. Furthermore, this approach can be extended to other environmental or decision-making domains beyond water quality assessment.

Despite its strengths, the study has several limitations. The data were limited to five rivers in one Malaysian state and only covered a three-year period. Additionally, the method depends heavily on the decision-maker, which can introduce subjective bias. Lastly, the current framework does not incorporate real-time or continuous monitoring data, which could further enhance its applicability in dynamic environments.

Future research could focus on expanding the geographical scope of the study, integrating real-time sensor data, and automating parts of the decision-making process through AI-driven techniques. Moreover, comparative studies with other advanced multi-criteria decision-making (MCDM) methods could further validate and refine the proposed framework.

The findings of this study can be used as a basis for future resource allocation and targeted pollution management strategies to enhance water quality, preserve aquatic ecosystem health, and enhance the welfare of communities that depend on these rivers. It emphasises the importance of proactive environmental management to ensure the long-term sustainability of these vital water resources. Therefore, GTrLRIF VIKOR is an appropriate method to address the problem of river water pollution and can be applied in any other field due to their extensive benefits.

Acknowledgments

The authors would like to thank the anonymous referees for providing constructive comments and suggestions.

References

Abdullah M.H., Halim H., Rahman R., Rameli N. & Saleh N.S. 2023. Incidents of poisonous gas spreading in Pasir Gudang, Johor, Malaysia: Responsibilities, roles and social impacts on communities. *RES MILITARIS* **13**(1): 3660-3671.

Afroz R., Masud M.M., Akhtar R. & Duasa J.B. 2014. Water pollution: Challenges and future direction for water resource management policies in Malaysia. *Environment and Urbanization ASIA* **5**(1): 63-81.

Arsad A. 2009. Development of river restoration plan for upstream tributary of Sungai Pulai based on water quality and land used activities. Master Thesis. Universiti Teknologi Malaysia.

Chowdhury M.S.U., Othman F., Jaafar W.Z.W., Mood N.C. & Adham M.I. 2018. Assessment of pollution and improvement measure of water quality parameters using scenarios modeling for Sungai Selangor basin. *Sains Malaysiana* **47**(3): 457-469.

Deng W., Li X., Guo Y., Huang J. & Zhang L. 2024. Ecological assessment of water environment in Huizhou Region of China based on DPSIR theory and entropy weight TOPSIS model. *Water* **16**(18): 2579.

Department of Chemistry Malaysia. 2019. Pencemaran Sungai Kim Kim: Sejauh mana industri kimia peka terhadap tanggungjawab kepada alam sekitar dan penduduk sekitarnya? Buletin OSH, Bil. 1 (2019). https://www.kimia.gov.my/wp-content/uploads/2020/Buletin%20OSH/Buletin%20OSH%20Bil%201%202019.pdf (1 January 2025).

Department of Environment Malaysia. 2019. Interim national water quality standards for Malaysia. https://www.doe.gov.my/wp-content/uploads/2021/10/ii-Standard-Kualiti-Air-Kebangsaan.pdf (17 January 2025).

Devi K. 2011. Extension of VIKOR method in intuitionistic fuzzy environment for robot selection. *Expert Systems with Applications* **38**(11): 14163-14168.

Gaćina R., Bajić S., Dimitrijević B., Šubaranović T., Beljić Č. & Bajić D. 2024. Application of the VIKOR method for selecting the purpose of recultivated terrain after the end of coal mining. *Proceedings of the Bulgarian Academy of Sciences* **77**(7): 1031-1041

Garai T. 2024. λ-possibility-center based MCDM technique on the control of Ganga river pollution under non-linear pentagonal fuzzy environment. *Journal of Ambient Intelligence and Humanized Computing* **15**(8): 3243-3253.

Hussein I.N.A. 2019. 2.43 tan sisa bahan kimia Sungai Kim Kim. Harian Metro. https://www.hmetro.com.my/utama/2019/03/433504/243-tan-sisa-bahan-kimia-dari-sungai-kim-kim-metrotv

Ismail S.N.S., Abidin E.Z. & Rasdi I. 2020. A case study of Pasir Gudang chemical toxic pollution: A review on health symptoms, psychological manifestation and biomarker assessment. *Malaysian Journal of Medicine and Health Sciences* **16**(SUPP11): 175-184.

Karbasi Ahvazi A., Ebadi T., Zarghami M. & Hashemi S.H. 2024. Application of multi-criteria group decision-making for water quality management. *Environmental Monitoring and Assessment* **196**(8): 683.

Khan A. 2017. Fuzzy logic approach to quantify water pollution. *International Journal of Engineering Science and Computing* **7**(5): 12227-12233.

Lee C.-S. & Wang M.-H. 2010. A fuzzy expert system for diabetes decision support application. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)* **41**(1): 139-153.

Liu J., Li Y.P. & Huang G.H. 2013. Mathematical modeling for water quality management under interval and fuzzy uncertainties. *Journal of Applied Mathematics* **2013**: 731568.

Mohammadpour A., Samaei M.R., Baghapour M.A., Sartaj M., Isazadeh S., Azhdarpoor A., Alipour H. & Khaneghah A.M. 2023. Modeling, quality assessment, and Sobol sensitivity of water resources and distribution system in Shiraz: A probabilistic human health risk assessment. *Chemosphere* **341**: 139987.

Muhamad S.N.N., Abd Halim R., Wan Shahidan W.N. & Sarkam S.F. 2018. Ranking academic performance using fuzzy vikor: A case of secondary schools at Perlis. *Journal of Computing Research and Innovation* **3**(4): 31-38.

Nurani Zulkifli S., Abdul Rahim H. & Adilla Subha N. 2018. Analysis of bacterial contaminant in Pasir Gudang, Johor tap water supply–varies pH value observation. *International Journal of Engineering, Transactions B: Applications* **31**(8): 1455-1463.

Ostad-Ali-Askari K. & Kianmehr P. 2024. Evaluation of water quality by Fuzzy DEMATEL and statistical analysis. *SSRN*. https://ssrn.com/abstract=5073295

Pak H.Y., Chuah C.J., Yong E.L. & Snyder S.A. 2021. Effects of land use configuration, seasonality and point source on water quality in a tropical watershed: A case study of the Johor River Basin. *Science of the Total Environment* **780**: 146661.

Parsimehr M., Shayesteh K., Godini K. & Varkeshi M.B. 2018. Using multilayer perceptron artificial neural network for predicting and modeling the chemical oxygen demand of the Gamasiab River. *Avicenna Journal of Environmental Health Engineering* **5**(1): 15-20.

Sahoo S.K. & Goswami S.S. 2023. A comprehensive review of multiple criteria decision-making (MCDM) Methods: Advancements, applications, and future directions. *Decision Making Advances* **1**(1): 25-48.

Samsudin M.S., Azid A., Khalit S.I., Saudi A.S.M. & Zaudi M.A. 2017. River water quality assessment using APCS-MLR and statistical process control in Johor River Basin, Malaysia. *International Journal of Advanced and Applied Sciences* **4**(8): 84-97.
Shafie M.A., Mohamad D. & Awang Kechil S. 2023. A multi-criteria generalised L-R intuitionistic Fuzzy TOPSIS with CRITIC for river water pollution classification. *Malaysian Journal of Fundamental and Applied Sciences* **19**(6): 1152-1175.
Shamsuddin M.S., Rohaizad A.W., Ramzi A.H., Masruddin M.F. & Jazam M.F.F. 2019. Isu pencemaran air di Sungai Kim Kim berdasarkan saranan Al-Quran. *Proceedings of the International Conference on Islamic Civilization and Technology Management*,
Shinde S.P., Barai V.N., Gavit B.K., Kadam S.A., Atre A.A., Pande C.B., Pal S.C., Radwan N., Tolche A.D. & Elkhrachy I. 2024. Assessment of groundwater potential zone mapping for semi-arid environment areas using AHP and MIF techniques. *Environmental Sciences Europe* **36**(1): 87.
Su K., Wang Q., Li L., Cao R. & Xi Y. 2022a. Water quality assessment of Lugu Lake based on Nemerow pollution index method. *Scientific Reports* **12**(1): 13613.
Su K., Wang Q., Li L., Cao R., Xi Y. & Li G. 2022b. Water quality assessment based on Nemerow pollution index method: A case study of Heilongtan reservoir in central Sichuan province, China. *PloS ONE* **17**(8): e0273305.
Sunarsih S., Pamurti R.D., Khabibah S. & Hadiyanto H. 2020. Analysis of priority scale for watershed reforestation using trapezoidal fuzzy VIKOR method: A case study in Semarang, Central Java Indonesia. *Symmetry* **12**(4): 507.
Xu B., Lin C.Y. & Mao X.W. 2014. Analysis of applicability of Nemerow pollution index to evaluation of water quality of Taihu Lake. *Water Resources Protection (Shui Ziyuan Baohu)* **30**(2): 38–40.
Yap C.K., Peng S.H.T. & Leow C.S. 2019. Contamination in Pasir Gudang area, Peninsular Malaysia: What can we learn from Kim Kim River chemical waste contamination? *Journal of Humanities and Education Development* **1**(2): 82-87.
Zakaria M.Z. & Mohamed M. 2019. Comparative analysis of biotic indices in water quality assessment: Case study at Sg. Bantang, Johor. *IOP Conference Series: Earth and Environmental Science* **269**(1): 012047.
Zhang Q., Feng M. & Hao X. 2018. Application of Nemerow index method and integrated water quality index method in water quality assessment of Zhangze Reservoir. *IOP Conference Series: Earth and Environmental Science* **128**: 012160.
Zhi W., Feng D., Tsai W.-P., Sterle G., Harpold A., Shen C. & Li L. 2021. From hydrometeorology to river water quality: Can a deep learning model predict dissolved oxygen at the continental scale? *Environmental Science & Technology* **55**(4): 2357-2368.
Zhu L. & Hu H. 2010. Fuzzy complex index in water quality assessment of municipalities. *Journal of Water Resource and Protection* **2**(9): 809-813.

Faculty of Computer and Mathematical Sciences
Universiti Teknologi MARA (UiTM)
40450 Shah Alam
Selangor, MALAYSIA
E-mail: asyranshafie@uitm.edu.my, norhanimah@tmsk.uitm.edu.my[*]*,*
daud@tmsk.uitm.edu.my, seripah@tmsk.uitm.edu.my

Received: 29 April 2025
Accepted: 4 August 2025

[*]Corresponding author