

INTEGRATING HYBRID NANOFLUID ON BIOCONVECTION FLOW FOR A BRINKMAN TYPE FLUID IN CHANNEL

(Mengintegrasikan Bendalir Nano Hibrid pada Aliran Biokonveksi bagi Bendalir Jenis Brinkman dalam Saluran)

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ABSTRACT

With the escalating global water crisis, carbon membrane technology presents a promising solution for purifying both water and wastewater. Titanium oxide nanoparticles (TiO_2) are commonly incorporated to enhance membrane performance, offering significant advantages in water treatment systems due to their multifunctional properties, such as improved adsorption, catalytic activity, and antimicrobial effects. By combining TiO_2 with carbon in a hybrid membrane, the limitations of using metal or carbon catalysts alone are effectively addressed. This article mathematically simulates the performance of a hybrid membrane in water treatment systems by deriving the governing equations for the bioconvection flow of a Brinkman-type fluid, taking into account the effects of hybrid nanoparticles. The simulation is conducted within a channel under appropriate initial and boundary conditions, which are then transformed into dimensionless form using dimensionless variables. Analytical solutions are obtained with the help of the Laplace transform method. The results indicate that as the volume fraction of hybrid nanoparticles increases, both fluid velocity and microorganism profiles decrease, while temperature profiles exhibit the opposite effect. These findings align with experimental evidence supporting the integration of hybrid nanomaterials in water treatment technologies, ultimately enhancing the efficiency of the treatment process.

Keywords: bioconvection flow; brinkman type fluid; hybrid nanoparticles; channel; Laplace transform method; water treatment

ABSTRAK

Dengan krisis air global yang semakin meningkat, teknologi membran karbon memberikan penyelesaian yang menjanjikan untuk membersihkan kedua-dua air dan air sisa. Zarah nano titanium oksida (TiO_2) biasanya digabungkan untuk meningkatkan prestasi membran, menawarkan kelebihan ketara dalam sistem rawatan air kerana sifat pelbagai fungsinya, seperti penyerapan yang lebih baik, aktiviti pemangkin dan kesan antimikrob. Dengan menggabungkan TiO_2 dengan karbon dalam membran hibrid, had penggunaan logam atau pemangkin karbon sahaja ditangani dengan berkesan. Artikel ini secara matematik mensimulasikan prestasi membran hibrid dalam sistem rawatan air dengan memperoleh persamaan menakluk untuk aliran biokonveksi bendalir jenis Brinkman, dengan mengambil kira kesan zarah nano hibrid. Simulasi dijalankan dalam saluran di bawah syarat awal dan sempadan yang sesuai, yang kemudiannya diubah menjadi bentuk tidak berdimensi menggunakan pembolehubah tidak berdimensi. Penyelesaian analisis diperoleh dengan bantuan kaedah penjelmaan Laplace. Keputusan menunjukkan bahawa apabila pecahan isipadu zarah nano hibrid meningkat, kedua-dua profil halaju bendalir dan mikroorganisma berkurangan, manakala profil suhu menunjukkan kesan yang bertentangan. Penemuan ini sejajar dengan bukti eksperimen yang menyokong penyepaduan bahan nano hibrid dalam teknologi rawatan air, akhirnya meningkatkan kecekapan proses rawatan.

Kata kunci: aliran biokonveksi; bendalir jenis Brinkman; zarah nano hibrid; saluran; kaedah penjelmaan Laplace; rawatan air

1. Introduction

The Brinkman fluid model is an extension of Darcy's law and has been explored by Hallack *et al.* (2019) and Hashim *et al.* (2023) for flow in porous media, incorporating viscous shear effects within the fluid. It becomes particularly useful when considering fluids that exhibit significant viscosity relative to the permeability of the medium, such as in wastewater treatment systems. In wastewater treatment, Brinkman fluids represent the behavior of water that contains suspended particles or sludge, where the fluid's viscosity plays a crucial role in determining flow characteristics (Hong *et al.* 2018). Baird *et al.* (2021) employed the Navier-Stokes-Brinkman equations coupled with a convection-diffusion equation to model contaminant transport in water. By incorporating the Brinkman equation, researchers can more accurately simulate viscous shear and permeability changes, improving the precision of complex groundwater flow models (Yao and Zhang 2023). As the Brinkman parameter increases, it indicates a higher dominance of viscous forces, leading to more resistance as the wastewater moves through the porous medium. This is relevant because in wastewater treatment, porous media like membranes, sand filters, or gravel beds are used to filter out contaminants. A higher Brinkman parameter suggests that the fluid (wastewater) experiences greater difficulty flowing through these media, which can affect the efficiency of the filtration process (Alrabaiah 2021). To further explain, in porous media where permeability is low, the viscous forces dominate, slowing down the fluid. The Brinkman parameter quantifies this balance between viscosity and permeability, which is important in designing and optimizing wastewater filtration systems to ensure adequate flow while still capturing contaminants effectively. By understanding this parameter, engineers can adjust the characteristics of the porous medium (e.g., its permeability) or the operational conditions to improve treatment efficiency. Studies by Ali *et al.* (2016; 2017), Saqib *et al.* (2021), Abbas *et al.* (2023), and Allehiany *et al.* (2023) have delved into the analytical aspects of Brinkman fluid flow with nanoparticle effects.

Bioconvection occurs when microorganisms, like bacteria or algae, move in a fluid due to concentration, temperature, or light gradients, causing density variations (Alsaedi *et al.* 2023). In polluted water, these microorganisms can create density gradients by forming dense colonies or producing metabolic by-products, leading to observable patterns and currents (Habibishandiz *et al.* 2023; Ramamonjy *et al.* 2023). Studies have modeled bioconvection with various microorganisms. Ahmad *et al.* (2020) examined gyrotactic microbes in nanofluid flow within a porous medium, finding that porosity and permeability do not impact the gyrotactic mechanism. Rana *et al.* (2021) explored magnetically influenced Casson nanofluid bioconvection, emphasizing magnetic control of velocity. Abbasi *et al.* (2021) studied peristaltic viscoelastic flow in asymmetric channels, while Hayat *et al.* (2021) focused on Prandtl Eyring nanofluid with gyrotactic microorganisms, noting a decrease in microorganism profiles with a higher bioconvective Lewis number. Integrating the Brinkman fluid model with bioconvection enhances water treatment simulations by improving mixing, aeration, and microbial degradation, while the Brinkman model captures complex flow dynamics in free-flowing and porous regions. This combination optimizes water treatment system design and pollutant removal. Numerical studies on bioconvective Brinkman nanofluid flow have also

considered factors such as Darcy number (Zhang *et al.* 2020), chemical reactions (Geetha *et al.* 2023), rotational effects (Mishra *et al.* 2024), and magnetic fields (Shahid *et al.* 2021).

Traditional water treatment methods often struggle to remove metals and microorganisms effectively, sometimes leading to harmful disinfection byproducts. Nanomaterials have emerged as a promising alternative, characterized by their nanoscale size (less than 100 nm) and unique mechanical, electrical, optical, and magnetic properties (Boukhessaim *et al.* 2022; Noranuar *et al.* 2023). Many studies on bioconvection have incorporated nanomaterials like copper (Cu), silver (Ag), aluminum oxide (Al_2O_3), and titanium oxide (TiO_2). Among carbon nanomaterials (CNMs), carbon nanotubes (CNTs) stand out due to their excellent adsorption properties (Noranuar *et al.* 2021; Saba *et al.* 2019). Composed of graphene sheets rolled into cylinders, CNTs offer high specific surface areas and efficient adsorption of various contaminants, making them highly effective for water and wastewater treatment. CNTs are often embedded in membranes, a type of porous media used to separate contaminants in polluted water, including Brinkman fluids (Piai *et al.* 2020). These membranes, with pores as small as a few nanometers, block contaminants while allowing water molecules to pass through (Piai *et al.* 2020). In bioconvection studies, Ramzan *et al.* (2019) found that increasing CNTs volume fraction enhances fluid velocity. Shah *et al.* (2020) expanded this by considering thermal radiation, rotating microorganisms, Joule heating, and chemical reactions. Hussain *et al.* (2024a; 2024b) explored CNTs nanofluids with motile microorganisms and inclined magnetic fields, also considering chemical reactions. These studies (Ramzan *et al.* 2019; Shah *et al.* 2020; Hussain *et al.* 2024a; Hussain *et al.* 2024b) utilized MATLAB's bcp4c package without implementing the Brinkman fluid model.

Metal and carbon filters each offer unique benefits, but their limitations often prevent complete water purification. To overcome this, hybrid filtration systems combining metal and carbon materials are increasingly favored in modern water treatment. TiO_2 -carbon hybrid nanomaterials are particularly effective, offering enhanced adsorption, catalytic activity, and antimicrobial properties (Basso Peressut *et al.* 2023). Research by Diniz *et al.* (2023) shows that integrating activated carbon with TiO_2 increases photocatalytic efficiency, resulting in up to 10 times greater pollutant degradation compared to commercial TiO_2 alone. Similarly, Abdullah *et al.* (2023) found that this hybridization significantly improves adsorption and photocatalytic degradation, achieving nearly complete pollutant removal, including up to 99.2% Chemical Oxygen Demand (COD) reduction. In bioconvection studies, Asjad *et al.* (2022) explored hybrid Ag- TiO_2 nanomaterials in fluids with motile microorganisms, while Hussain *et al.* (2022), Khan *et al.* (2022), and Raza *et al.* (2023) examined the numerical impact of hybrid nanoparticles on bioconvection flow using various fluid models.

In summary, most studies have relied on numerical methods, with none specifically exploring the impact of hybrid TiO_2 -CNTs nanomaterials on bioconvection flow in a Brinkman fluid from an analytical perspective. This study aims to address this gap by analytically examining bioconvection flow in a Brinkman hybrid nanofluid containing TiO_2 and CNTs nanomaterials within a channel, utilizing the Laplace transform method. The hybrid nanomaterials of TiO_2 and CNTs are employed as hybrid membrane filtration due to their exceptional contaminant adsorption capabilities and photocatalytic properties, significantly improving the effectiveness of wastewater treatment.

2. Mathematical Formulation

Consider a hybrid Brinkman-type nanofluid with a combination of single-wall (SWCNTs) and Titanium oxide (TiO_2), and motile microorganisms moving unsteadily within a channel

as illustrated in Figure 1. The channel lies along the x -axis and is perpendicular to the y -axis, with a width d and infinite length. The flow is between two parallel plates at $y=0$ and $y=d$. At $t=0$, both the fluid and plates are stationary, with system temperature and microorganism concentration at T_d and N_d , respectively. For $t>0$, the plate at $y=0$ moves with velocity u_0 , and the temperature and microorganism concentration rise to T_w and N_d . This motion and the temperature difference accelerate the fluid in the x -direction. Due to the infinite length of the plates, physical quantities depend only on the y -coordinate and time t . The momentum, energy, and bioconvection equations incorporate Brinkman-type fluid with hybrid nanofluid models are given as (Hashim *et al.* 2023; Asjad *et al.* 2022)

$$\rho_{hnf} \left(\frac{\partial u}{\partial t} + \beta^* u \right) = \mu_{hnf} \frac{\partial^2 u}{\partial y^2} + g \left[(\rho \beta_T)_{hnf} (T - T_d) - \gamma (\rho_m - \rho) (N - N_d) \right], \quad (1)$$

$$(\rho C_p)_{hnf} \frac{\partial T}{\partial t} = k_{hnf} \frac{\partial^2 T}{\partial y^2}, \quad (2)$$

$$\frac{\partial N}{\partial t} = D_{hnf} \frac{\partial^2 N}{\partial y^2}. \quad (3)$$

Here, $u = u(y, t)$, $T = T(y, t)$, and $N = N(y, t)$ represent the profiles for velocity, temperature, and bioconvection, respectively. The relationship for the nanofluid used in the governing equations is outlined in Table 1, along with the corresponding equations. The subscript 1 and $CNTs$ represent TiO_2 and single wall carbon nanotubes ($SWCNTs$) nanoparticle, and hnf , f , and bf represent hybrid nanofluid, base fluid (water), and TiO_2 nanofluid. For spherical shaped nanocomposites, the value of m should be equal to 3 (Saba *et al.* 2019). This correlation is determined based on the thermophysical properties listed in Table 2.

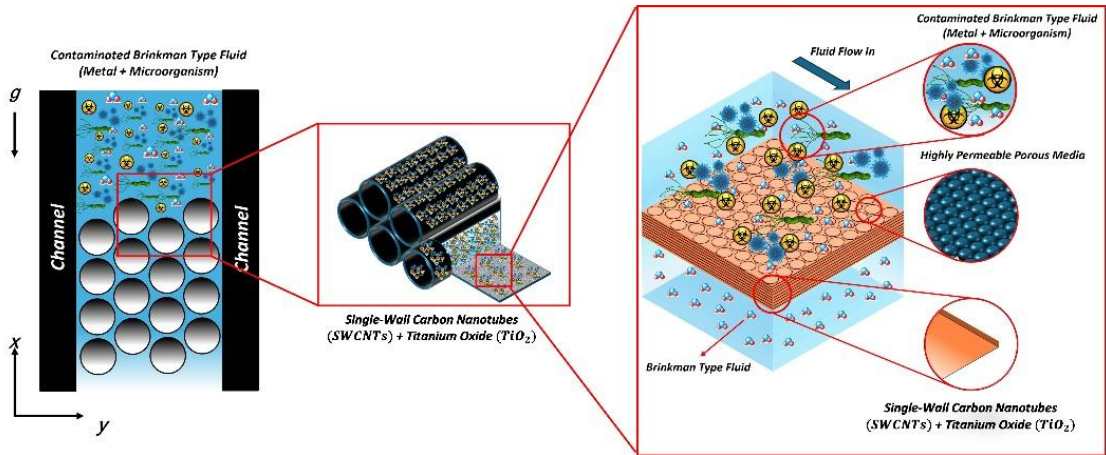


Figure 1: Physical model of the problem

Table 1: Hybrid nanofluid correlation (Saba *et al.* 2019; Raza *et al.* 2023)

Properties	Correlation
Dynamic viscosity, μ	$\mu_{hnf} = \frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_{CNTs})^{2.5}},$
Density, ρ	$\frac{\rho_{hnf}}{\rho_f} = (1-\phi_{CNTs}) \left(1 - \left(1 - \frac{\rho_l}{\rho_f} \right) \phi_1 \right) + \phi_{CNTs} \frac{\rho_{CNTs}}{\rho_f},$
Thermal expansion coefficient, $(\rho\beta_T)$	$\frac{(\rho\beta_T)_{hnf}}{(\rho\beta_T)_f} = (1-\phi_{CNTs}) \left(1 - \left(1 - \frac{(\rho\beta_T)_l}{(\rho\beta_T)_f} \right) \phi_1 \right) + \phi_{CNTs} \frac{(\rho\beta_T)_{CNTs}}{(\rho\beta_T)_f},$
Specific heat capacitance, (ρC_p)	$\frac{(\rho C_p)_{hnf}}{(\rho C_p)_f} = (1-\phi_{CNTs}) \left(1 - \left(1 - \frac{(\rho C_p)_l}{(\rho C_p)_f} \right) \phi_1 \right) + \phi_{CNTs} \frac{(\rho C_p)_{CNTs}}{(\rho C_p)_f},$
Thermal conductivity, k	$\frac{k_{hnf}}{k_{bf}} = \frac{1-\phi_{CNTs} + 2\phi_{CNTs} \frac{k_{CNTs}}{(k_{CNTs} - k_{bf})} \ln \left(\frac{k_{CNTs} + k_{bf}}{2k_{bf}} \right)}{1-\phi_{CNTs} + 2\phi_{CNTs} \frac{k_{bf}}{(k_{CNTs} - k_{bf})} \ln \left(\frac{k_{CNTs} + k_{bf}}{2k_{bf}} \right)},$ where $\frac{k_{bf}}{k_f} = \frac{k_1 + (m-1)k_f - (m-1)\phi_1(k_f - k_1)}{k_1 + (m-1)k_f + \phi_1(k_f - k_1)}$
Mass diffusivity, D	$D_{hnf} = (1-\phi_1)(1-\phi_{CNTs})D_f$

 Table 2: Thermophysical properties for water, $SWCNTs$, and TiO_2 (Saba *et al.* 2019; Raza *et al.* 2023)

Properties/ Materials	Water	$SWCNTs$	TiO_2
$\rho (Kg m^{-3})$	997.1	2600	425
$C_p (JKg^{-1}K^{-1})$	4179	425	686.2
$k (Wm^{-1}K^{-1})$	0.613	6600	8.9538
$\beta (K^{-1})$	21×10^{-5}	27×10^{-5}	0.9×10^{-5}

The governing equations in Eq. (1) to Eq. (3) are subjected to the conditions (Noranuar *et al.* 2021; Asjad *et al.* 2022)

$$u(y,0)=0, \quad T(y,0)=T_d, \quad N(y,0)=N_d; \quad \forall 0 \leq y \leq d, \quad (4)$$

$$u(0,t)=u_0, \quad T(0,t)=T_w, \quad N(0,t)=N_w; \quad \forall t > 0, \quad (5)$$

$$u(d,t)=0, \quad T(d,t)=T_d, \quad N(d,t)=N_d; \quad \forall t > 0. \quad (6)$$

The related dimensionless variables [36]

$$u^* = \frac{u}{u_0}, \quad y^* = \frac{y}{d}, \quad t^* = \frac{tv_f}{d^2}, \quad T^* = \frac{T - T_d}{T_w - T_d}, \quad N^* = \frac{N - N_d}{N_w - N_d} \quad (7)$$

are substituted into Eq. (1) to Eq. (6) and the following dimensionless governing equations are formed as

$$A_2 \frac{\partial u}{\partial t} + A_2 B_r u = A_1 \frac{\partial^2 u}{\partial y^2} + Gr(A_3 T - RaN), \quad (8)$$

$$\frac{\partial T}{\partial t} = \frac{1}{A_6} \frac{\partial^2 T}{\partial y^2}, \quad (9)$$

$$\frac{\partial N}{\partial t} = \frac{1}{A_7} \frac{\partial^2 N}{\partial y^2}, \quad (10)$$

with their conditions

$$u(y, 0) = 0, \quad T(y, 0) = 0, \quad N(y, 0) = 0; \quad \forall 0 \leq y \leq 1, \quad (11)$$

$$u(0, t) = 1, \quad T(0, t) = 1, \quad N(0, t) = 1; \quad \forall t > 0, \quad (12)$$

$$u(1, t) = 0, \quad T(1, t) = 0, \quad N(1, t) = 0; \quad \forall t > 0. \quad (13)$$

In the above equations,

$$\begin{aligned} A_1 &= \frac{1}{(1 - \phi_1)^{2.5} (1 - \phi_{CNTs})^{2.5}}, \quad A_2 = (1 - \phi_{CNTs}) \left(1 - \frac{\rho_l}{\rho_f} \phi_1 \right) + \phi_{CNTs} \frac{\rho_{CNTs}}{\rho_f}, \\ A_3 &= (1 - \phi_{CNTs}) \left(1 - \left(1 - \frac{(\rho\beta_T)_l}{(\rho\beta_T)_f} \right) \phi_1 \right) + \phi_{CNTs} \frac{(\rho\beta_T)_{CNTs}}{(\rho\beta_T)_f}, \\ A_4 &= (1 - \phi_{CNTs}) \left(1 - \left(1 - \frac{(\rho C_p)_l}{(\rho C_p)_f} \right) \phi_1 \right) + \phi_{CNTs} \frac{(\rho C_p)_{CNTs}}{(\rho C_p)_f}, \\ A_5 &= \frac{k_{hmf}}{k_f}, \quad A_6 = \frac{A_4 \text{Pr}}{A_5}, \quad A_7 = \frac{Lb}{(1 - \phi_1)(1 - \phi_{CNTs})}, \end{aligned} \quad (14)$$

are constants with dimensionless variables as Brinkman parameter (B_r), Grashof number (Gr), bioconvection Rayleigh number (Ra), Prandtl number (Pr), and bioconvection Lewis number (Lb), which defined as

$$\begin{aligned} B_r &= \frac{d^2 \beta^*}{\nu_f}, \quad Gr = \frac{gd^2 \beta_{Tf} (T_w - T_d)}{\nu_f u_0}, \quad Ra = \frac{\gamma(\rho_m - \rho)(N_w - N_d)}{(\rho\beta_T)_{nf} (T_w - T_d)}, \\ Pr &= \frac{\nu_f (\rho C_p)_f}{k_f}, \quad Lb = \frac{\nu_f}{D_f}. \end{aligned} \quad (15)$$

3. Solution of Problem

Employing the Laplace transform on Eq. (8) to Eq. (13) gives

$$\frac{d^2 \bar{u}(y, s)}{dy^2} - (D_1 s + B_{r1}) \bar{u}(y, s) = -D_2 A_3 \bar{T}(y, s) + D_2 Ra_1 \bar{N}(y, s), \quad (16)$$

$$\frac{d^2 \bar{T}(y, s)}{dy^2} - A_6 s \bar{T}(y, s) = 0, \quad (17)$$

$$\frac{d^2 \bar{N}(y, s)}{dy^2} - A_7 s \bar{N}(y, s) = 0, \quad (18)$$

$$\bar{u}(0, s) = \bar{T}(0, s) = \bar{N}(0, s) = \frac{1}{s}, \quad (19)$$

$$\bar{u}(1, s) = \bar{T}(1, s) = \bar{N}(1, s) = 0. \quad (20)$$

Eq. (16) to Eq. (18) are solved using Eq. (19) and Eq. (20) along with several manipulations. The exact solutions for velocity, temperature, and bioconvection are derived by applying the inverse of Laplace to the resultant Laplace solutions, which gives

$$u(y, t) = -u_1(y, t) + u_2(y, t) - u_3(y, t) - u_4(y, t) + u_5(y, t) + u_6(y, t) - u_7(y, t) + u_8(y, t) + u_9(y, t) - u_{10}(y, t) - u_{11}(y, t) + u_{12}(y, t) + u_{13}(y, t) - u_{14}(y, t) + u_{15}(y, t) - u_{16}(y, t) - u_{17}(y, t) + u_{18}(y, t), \quad (21)$$

$$T(y, t) = \sum_{n=0}^{\infty} \left[\operatorname{erfc} \left(\frac{(2n+y)\sqrt{A_6}}{2\sqrt{t}} \right) - \operatorname{erfc} \left(\frac{(2n+2-y)\sqrt{A_6}}{2\sqrt{t}} \right) \right], \quad (22)$$

$$N(y, t) = \sum_{n=0}^{\infty} \left[\operatorname{erfc} \left(\frac{(2n+y)\sqrt{A_7}}{2\sqrt{t}} \right) - \operatorname{erfc} \left(\frac{(2n+2-y)\sqrt{A_7}}{2\sqrt{t}} \right) \right], \quad (23)$$

where

$$\begin{aligned} u_1(y, t) &= \frac{1}{2} \sum_{n=0}^{\infty} \left[\exp(\xi_1) \operatorname{erfc} \left(\frac{\xi_2}{2\sqrt{t}} + \sqrt{D_7 t} \right) + \exp(-\xi_1) \operatorname{erfc} \left(\frac{\xi_2}{2\sqrt{t}} - \sqrt{D_7 t} \right) \right], \\ u_2(y, t) &= \frac{Gr_1}{2} \sum_{n=0}^{\infty} \left[\exp(\xi_1) \operatorname{erfc} \left(\frac{\xi_2}{2\sqrt{t}} + \sqrt{D_7 t} \right) + \exp(-\xi_1) \operatorname{erfc} \left(\frac{\xi_2}{2\sqrt{t}} - \sqrt{D_7 t} \right) \right], \\ u_3(y, t) &= \frac{Gr_1}{2} \times \sum_{n=0}^{\infty} \left[\exp(\xi_4) \operatorname{erfc} \left(\frac{\xi_2}{2\sqrt{t}} + \sqrt{\eta_1} \right) + \exp(\xi_5) \operatorname{erfc} \left(\frac{\xi_2}{2\sqrt{t}} - \sqrt{\eta_1} \right) \right], \\ u_4(y, t) &= \frac{Ra_1}{2} \sum_{n=0}^{\infty} \left[\exp(\xi_1) \operatorname{erfc} \left(\frac{\xi_2}{2\sqrt{t}} + \sqrt{D_7 t} \right) + \exp(-\xi_1) \operatorname{erfc} \left(\frac{\xi_2}{2\sqrt{t}} - \sqrt{D_7 t} \right) \right], \\ u_5(y, t) &= \frac{Ra_1}{2} \sum_{n=0}^{\infty} \left[\exp(\xi_7) \operatorname{erfc} \left(\frac{\xi_2}{2\sqrt{t}} + \sqrt{\eta_2} \right) + \exp(\xi_8) \operatorname{erfc} \left(\frac{\xi_2}{2\sqrt{t}} - \sqrt{\eta_2} \right) \right], \end{aligned}$$

$$\begin{aligned}
 u_6(y, t) &= \frac{1}{2} \sum_{n=0}^{\infty} \left[\exp(\xi_9) \operatorname{erfc} \left(\frac{\xi_{10}}{2\sqrt{t}} + \sqrt{D_7 t} \right) + \exp(-\xi_9) \operatorname{erfc} \left(\frac{\xi_{10}}{2\sqrt{t}} - \sqrt{D_7 t} \right) \right], \\
 u_7(y, t) &= \frac{Gr_1}{2} \sum_{n=0}^{\infty} \left[\exp(\xi_9) \operatorname{erfc} \left(\frac{\xi_{10}}{2\sqrt{t}} + \sqrt{D_7 t} \right) + \exp(-\xi_9) \operatorname{erfc} \left(\frac{\xi_{10}}{2\sqrt{t}} - \sqrt{D_7 t} \right) \right], \\
 u_8(y, t) &= \frac{Gr_1}{2} \sum_{n=0}^{\infty} \left[\exp(\xi_{12}) \operatorname{erfc} \left(\frac{\xi_{10}}{2\sqrt{t}} + \sqrt{\eta_1} \right) + \exp(\xi_{13}) \operatorname{erfc} \left(\frac{\xi_{10}}{2\sqrt{t}} - \sqrt{\eta_1} \right) \right], \\
 u_9(y, t) &= \frac{Ra_1}{2} \sum_{n=0}^{\infty} \left[\exp(\xi_9) \operatorname{erfc} \left(\frac{\xi_{10}}{2\sqrt{t}} + \sqrt{D_7 t} \right) + \exp(-\xi_9) \operatorname{erfc} \left(\frac{\xi_{10}}{2\sqrt{t}} - \sqrt{D_7 t} \right) \right], \\
 u_{10}(y, t) &= \frac{Ra_1}{2} \sum_{n=0}^{\infty} \left[\exp(\xi_{15}) \operatorname{erfc} \left(\frac{\xi_{10}}{2\sqrt{t}} + \sqrt{\eta_2} \right) + \exp(\xi_{16}) \operatorname{erfc} \left(\frac{\xi_{10}}{2\sqrt{t}} - \sqrt{\eta_2} \right) \right], \\
 u_{11}(y, t) &= Gr_1 \sum_{n=0}^{\infty} \left[\operatorname{erfc} \left(\frac{\xi_{17}}{2\sqrt{t}} \right) \right], u_{13}(y, t) = Gr_1 \sum_{n=0}^{\infty} \left[\operatorname{erfc} \left(\frac{\xi_{21}}{2\sqrt{t}} \right) \right], \\
 u_{12}(y, t) &= \frac{Gr_1}{2} \sum_{n=0}^{\infty} \left[\exp(\xi_{19}) \operatorname{erfc} \left(\frac{\xi_{17}}{2\sqrt{t}} + \sqrt{D_5 t} \right) + \exp(\xi_{20}) \operatorname{erfc} \left(\frac{\xi_{17}}{2\sqrt{t}} - \sqrt{D_5 t} \right) \right], \\
 u_{14}(y, t) &= \frac{Gr_1}{2} \sum_{n=0}^{\infty} \left[\exp(\xi_{23}) \operatorname{erfc} \left(\frac{\xi_{21}}{2\sqrt{t}} + \sqrt{D_5 t} \right) + \exp(\xi_{24}) \operatorname{erfc} \left(\frac{\xi_{21}}{2\sqrt{t}} - \sqrt{D_5 t} \right) \right], \\
 u_{15}(y, t) &= Ra_1 \sum_{n=0}^{\infty} \left[\operatorname{erfc} \left(\frac{\xi_{25}}{2\sqrt{t}} \right) \right], u_{17}(y, t) = Ra_1 \sum_{n=0}^{\infty} \left[\operatorname{erfc} \left(\frac{\xi_{29}}{2\sqrt{t}} \right) \right], \\
 u_{16}(y, t) &= \frac{Ra_1}{2} \sum_{n=0}^{\infty} \left[\exp(\xi_{27}) \operatorname{erfc} \left(\frac{\xi_{25}}{2\sqrt{t}} + \sqrt{D_6 t} \right) + \exp(\xi_{28}) \operatorname{erfc} \left(\frac{\xi_{25}}{2\sqrt{t}} - \sqrt{D_6 t} \right) \right], \\
 u_{18}(y, t) &= \frac{Ra_1}{2} \sum_{n=0}^{\infty} \left[\exp(\xi_{31}) \operatorname{erfc} \left(\frac{\xi_{29}}{2\sqrt{t}} + \sqrt{D_6 t} \right) + \exp(\xi_{32}) \operatorname{erfc} \left(\frac{\xi_{29}}{2\sqrt{t}} - \sqrt{D_6 t} \right) \right],
 \end{aligned}$$

with

$$\begin{aligned}
 D_1 &= \frac{A_2}{A_1}, D_2 = \frac{Gr}{A_1}, B_{r1} = D_1 B_r, D_3 = A_6 - D_1, D_4 = A_7 - D_1, \\
 D_5 &= \frac{B_{r1}}{D_3}, D_6 = \frac{B_{r1}}{D_4}, D_7 = \frac{B_{r1}}{D_1}, Gr_1 = \frac{D_2 A_3}{D_3 D_5}, Ra_1 = \frac{D_2 Ra}{D_4 D_6}, \\
 \xi_1 &= (2n + 2 - y) \sqrt{D_1 D_7}, \xi_2 = (2n + 2 - y) \sqrt{D_1}, \\
 \xi_3 &= (2n + 2 - y) \sqrt{D_1 (D_5 + D_7)}, \xi_4 = D_5 t + \xi_3, \xi_5 = D_5 t - \xi_3, \\
 \eta_1 &= (D_5 + D_7) t, \xi_6 = (2n + 2 - y) \sqrt{D_1 (D_6 + D_7)}, \xi_7 = D_6 t + \xi_6, \\
 \xi_8 &= D_6 t - \xi_6, \eta_2 = (D_6 + D_7) t, \xi_9 = (2n + y) \sqrt{D_1 D_7}, \\
 \xi_{10} &= (2n + y) \sqrt{D_1}, \xi_{11} = (2n + y) \sqrt{D_1 (D_5 + D_7)}, \xi_{12} = D_5 t + \xi_{11}, \\
 \xi_{13} &= D_5 t - \xi_{11}, \xi_{14} = (2n + y) \sqrt{D_1 (D_6 + D_7)}, \xi_{15} = D_6 t + \xi_{14}, \\
 \xi_{16} &= D_6 t - \xi_{14}, \xi_{17} = (2n + 2 - y) \sqrt{A_6}, \xi_{18} = (2n + 2 - y) \sqrt{A_6 D_5},
 \end{aligned}$$

$$\begin{aligned}\xi_{19} &= D_5 t + \xi_{18}, \xi_{20} = D_5 t - \xi_{18}, \xi_{21} = (2n + y)\sqrt{A_6}, \\ \xi_{22} &= (2n + y)\sqrt{A_6 D_5}, \xi_{23} = D_5 t + \xi_{22}, \xi_{24} = D_5 t - \xi_{22}, \\ \xi_{25} &= (2n + 2 - y)\sqrt{A_7}, \xi_{26} = (2n + 2 - y)\sqrt{A_7 D_6}, \xi_{27} = D_6 t + \xi_{26}, \\ \xi_{28} &= D_6 t - \xi_{26}, \xi_{29} = (2n + y)\sqrt{A_7}, \xi_{30} = (2n + y)\sqrt{A_7 D_6}, \\ \xi_{31} &= D_6 t + \xi_{30}, \xi_{32} = D_6 t - \xi_{30}\end{aligned}$$

are constants.

4. Physical quantities

In fluid dynamics and heat transfer, skin friction (τ), Nusselt number (Nu), and microorganism density number (Nm) are key physical quantities used to assess flow, heat, and mass transfer characteristics.

Skin friction refers to the resistance that a fluid flow exerts along the surface it contacts, stemming from viscous shear forces in the fluid. Nusselt number is a dimensionless measure that represents the rate of heat transfer due to convection relative to conduction across a fluid boundary layer. Microorganism density number often described in terms of concentration profiles, represents the density or concentration of microorganisms in a fluid system. It is typically expressed in terms of mass or cell concentration per unit volume.

The dimensionless forms of τ , Nu , and Nm are derived as follows.

$$\tau = \frac{1}{(1 - \phi_1)^{2.5} (1 - \phi_{CNTs})^{2.5}} \left(\frac{\partial u}{\partial y} \right) \bigg|_{y=0,1}, \quad (24)$$

$$Nu = - \frac{k_{hnf}}{k_f} \left(\frac{\partial T}{\partial y} \right) \bigg|_{y=0,1}, \quad (25)$$

$$Nm = - (1 - \phi_1) (1 - \phi_{CNTs}) \left(\frac{\partial N}{\partial y} \right) \bigg|_{y=0,1}. \quad (26)$$

These forms are obtained by substituting the dimensional expressions of τ , Nu , and Nm with dimensionless variables and nanofluid constants. Equations (21) to (23) are then differentiated with respect to y , and the resulting expressions are incorporated into Equations (24) to (26) to solve for τ , Nu , and Nm . Since the problem involves two plates, all the physical quantities are evaluated at $y=0$ and $y=1$, where the fluid flow contacts each plate, generating friction.

5. Results and Discussion

In this section, the solutions derived from our analytical computations are displayed in graphs with the help of Mathcad software and further analyzed considering the effects of several significant parameters. The following results emphasize the key discoveries of our study and offer a thorough discussion of their implications. The validity of the obtained solution has been confirmed through a comparison with the published study by Hashim *et al.* (2023) as shown in Figure 2 (Velocity profile) and Figure 3 (Temperature profile). By setting the

parameters $B_r = Gr = 2$, $Pr = 6.2$, $t = 1$, and $Ra = \phi_1 = \phi_{CNTs} = 0$ for the present study of velocity profile, and $B_r = Gr = 2$, $Pr = 6.2$, $t = R = 1$, $M = 0$, $K \rightarrow \infty$ for the published study, a direct comparison was made. The same procedure was also applied in the present study to the temperature profile, where the volume fraction parameters were set to zero to ensure that the present solution aligns with the value obtained by Hashim *et al.* (2023). Thus, by selecting the appropriate parameter values for both the current and published studies of velocity and temperature profiles, the results demonstrated strong agreement, thereby validating the accuracy of the solutions in the present works.

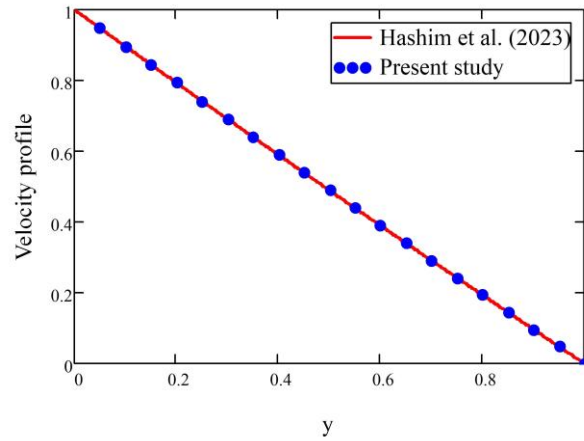


Figure 2: Verification of the present study for velocity profile with the published study by Hashim *et al.* (2023)

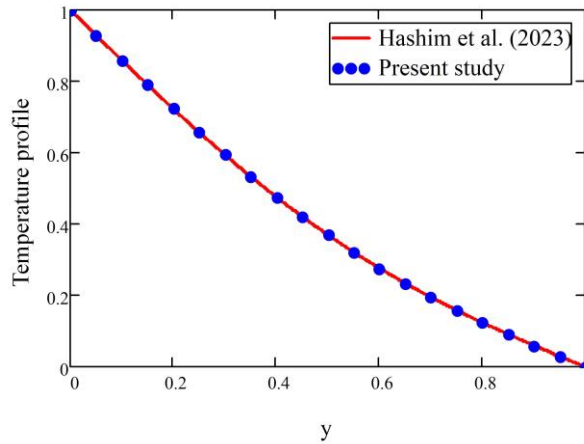


Figure 3: Verification of the present study for temperature profile with the published study by Hashim *et al.* (2023)

Figure 4 to Figure 6 demonstrate the influences of hybrid nanoparticles ($TiO_2 - CNTs$) volume fraction (ϕ_1 and ϕ_{CNTs}) on temperature, microorganisms, and velocity profiles, respectively. Hybrid nanoparticles, especially TiO_2 and $CNTs$, exhibit high thermal conductivity. When their volume fraction in the fluid increases, they significantly enhance the thermal transport properties of the fluid. Nanoparticles absorb heat more efficiently and distribute it across the fluid, raising the overall temperature as shown in Figure 4. In contrast, increasing their volume fraction causes reduced microorganisms profile as depicted in Figure 5. This situation can be attributed to both TiO_2 and $CNTs$ possess inherent antimicrobial properties. Increasing their volume fraction enhances the interaction between these

nanoparticles and the microorganisms, either physically damaging them through contact or chemically deactivating them. The elevated temperature may also contribute to the destruction of some microorganisms. Moreover, the hybridized nanoparticles, particularly $TiO_2 - CNTs$, are known for their high thermal conductivity and unique ability to facilitate fluid transport. This enhances the momentum transfer in the fluid, effectively increasing its velocity. The nanoparticles also act as micro-stirrers, promoting more uniform distribution and reducing drag forces within the flow, which contributes to the increase in velocity as seen in Figure 6. In water filtration, the increase in fluid velocity ensures a more efficient and rapid transport of water through the membrane, reducing the time required for filtration and improving overall throughput. The improved temperature also led to the degradation of more organic pollutants and microorganisms, making the water cleaner. The reduction in microorganism concentration minimizes biofouling, which is one of the common challenges in membrane filtration. This extends the operational life of the membrane and reduces the need for frequent cleaning or replacement, making the filtration system more cost-effective.

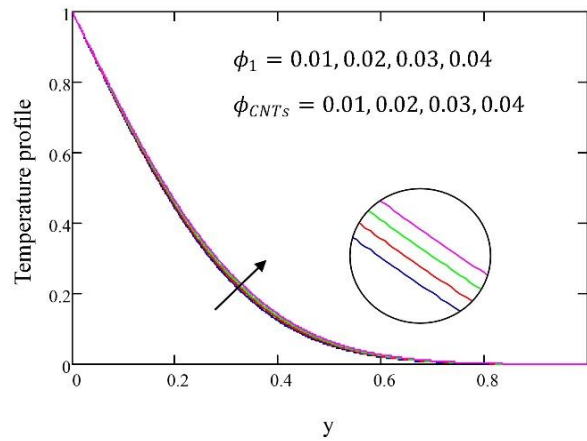


Figure 4: Temperature profile versus hybrid nanoparticles volume fraction

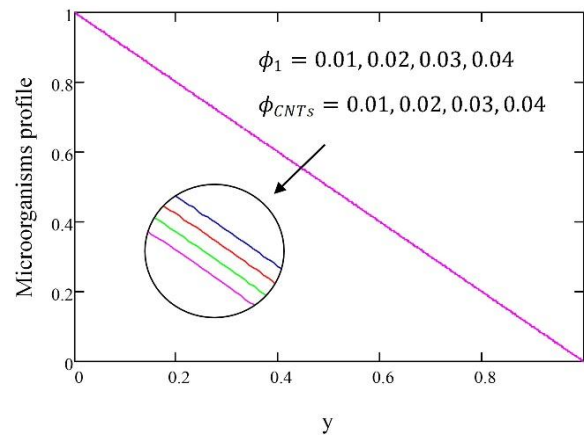


Figure 5: Microorganisms profile versus hybrid nanoparticles volume fraction

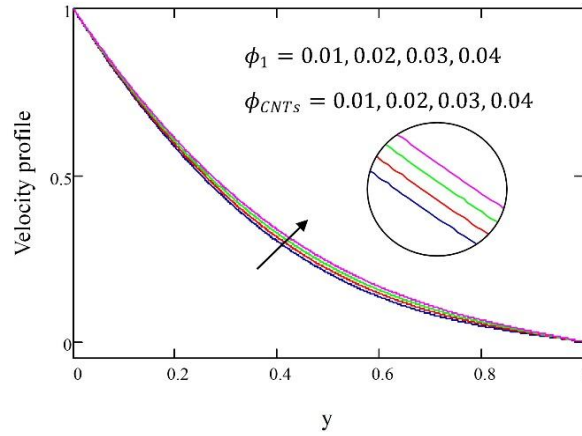


Figure 6: Velocity profile versus hybrid nanoparticles volume fraction

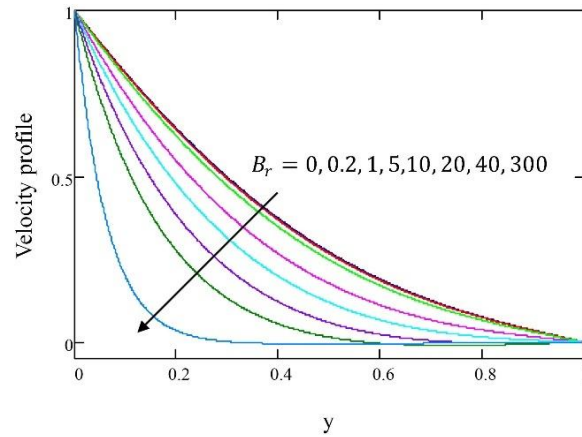


Figure 7: Velocity profile versus Brinkman parameter

Figure 7 shows the impact of the Brinkman parameter (B_r) on the velocity profile, emphasizing the balance between viscous forces and permeability in porous medium. Physically, the Brinkman parameter is a ratio that compares the fluid's viscosity to the permeability of the porous medium. Minimal velocity changes occur as B_r increases from 0 to 1, with the highest velocity observed at $B_r = 0$. At this point, the Brinkman model reduces to Newtonian behavior due to absent porous resistance. Conversely, a high B_r suggests that viscous effects are more pronounced compared to the medium's permeability. As a result, the fluid encounters greater resistance due to viscosity, which slows down its flow through the porous medium. This physical finding of B_r is in accordance with the findings reported in Alrabaiah *et al.* (2021). In wastewater treatment, higher Brinkman parameters could indicate that the fluid is experiencing a high degree of resistance due to viscosity, which is typical in fluids with suspended solids, sludges, or other contaminants, specifically satisfying the statement in the study of Hong *et al.* (2018) regarding physical factor of sludge behavior. A higher resistance slows down the flow, allowing for more extended contact time with the filter media, thereby enhancing the filtration of particles and improving water purity.

The influence of bioconvection Rayleigh number (Ra) on the velocity profile is depicted in Figure 8. As Ra increases, the density stratification due to microorganisms becomes more significant. This stratification can create stable layers or zones where fluid movement is limited because of the higher concentration of microorganisms. In these stratified areas, the vertical movement of both the microorganisms and the fluid can be impeded, leading to a decrease in the overall velocity of the bioconvection currents. In wastewater treatment plants, zones with limited or no fluid movement (dead zones) can reduce efficiency. These dead zones can occur in areas of high microorganism concentration. By controlling Ra , wastewater engineers can minimize these areas, ensuring that bioconvection currents remain strong enough to prevent the formation of stagnant layers, which would otherwise slow down the treatment process.

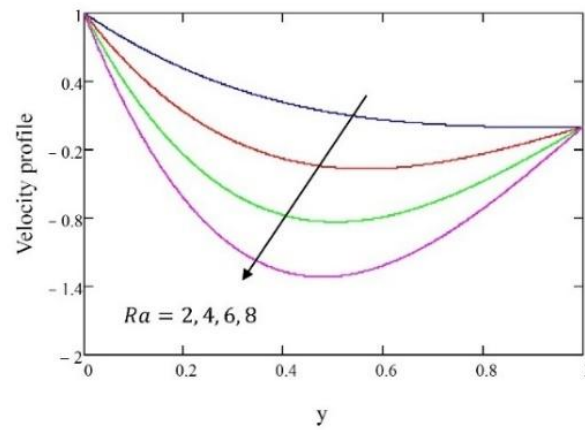


Figure 8: Velocity profile versus bioconvection Rayleigh number

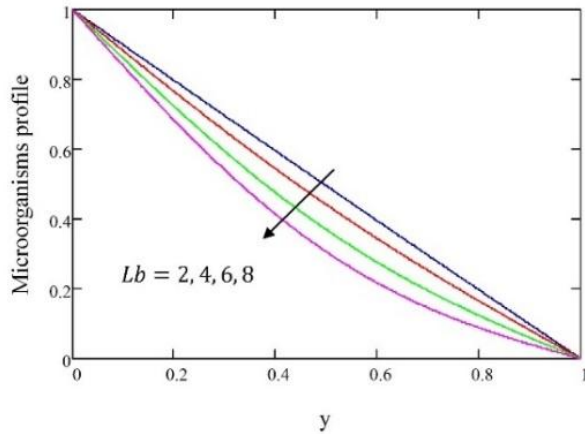


Figure 9: Microorganisms profile versus bioconvection Lewis number

Figure 9 illustrates the microorganism concentration profile with the effect of bioconvection Lewis number (Lb). An increased Lb signifies that microorganisms spread more swiftly through the fluid. This faster diffusion promotes a more uniform distribution of microorganisms, which in turn minimizes the formation of dense clusters or bands. The insights from Lb can be highly beneficial in wastewater treatments, such as biofilters or

bioreactors, which can experience problems when microorganisms cluster in specific areas, leading to localized clogging or reduced efficiency. A higher Lb indicates that microorganisms diffuse more evenly, minimizing the risk of dense clusters that could clog filters or pipes. This improves the system's operational stability and reduces the need for maintenance or system downtime. In summary, these findings offer a framework for better control over the fluid dynamics within water filtration systems. By understanding and manipulating the B_r , Ra , and Lb , filtration systems can be designed or adjusted to maximize efficiency, minimize energy consumption, and improve contaminant removal.

Table 3: Results of Skin friction (τ)

ϕ_1	ϕ_{CNTs}	B_r	Gr	Ra	Lb	τ	
						τ_0	τ_1
0.02	0.02	2	2	2	2	-2.1632	0.3306
0.03	0.03	2	2	2	2	-2.2025	0.3256
0.02	0.02	4	2	2	2	-2.7146	0.2791
0.02	0.02	2	4	2	2	-2.8086	0.6612
0.02	0.02	2	2	4	2	-3.3470	0.8674
0.02	0.02	2	2	2	4	-2.0942	0.2616

Table 4: Results of Nusselt number (Nu) and microorganism density number (Nm)

ϕ_1	ϕ_{CNTs}	Lb	Nu		Nm	
			Nu_0	Nu_1	Nm_0	Nm_1
0.02	0.02	2	1.4324	0.6613	0.9772	0.9436
0.03	0.03	2	1.4435	0.6947	0.9590	0.9228
0.02	0.02	4	-	-	1.1402	0.7809

Table 3 and Table 4 show the variations in skin friction (τ), Nusselt number (Nu), and microorganism density number (Nm). The results reveal that increasing the volume fraction of hybrid nanoparticles, ϕ_1 and ϕ_{CNTs} , reduces the skin friction at both plates (τ_0 and τ_1) and the microorganism density number at both plates (Nm_0 and Nm_1). However, the Nusselt number (Nu) shows an opposite trend, increasing with higher values of ϕ_1 and ϕ_{CNTs} .

Physically, the reduction in τ values indicate less resistance between the fluid flow and the surface, which can be beneficial in filtration and water treatment systems. Lower skin friction helps minimize water flow resistance through porous media or channels, enhancing contaminant transport and filtration efficiency. Meanwhile, the increase in Nu suggests a higher rate of heat transfer. In temperature-sensitive processes like bioconvection, a higher Nu signifies more efficient thermal management, supporting pollutant degradation or contaminant removal. Effective heat transfer is often necessary in water treatment to facilitate microbial activity, chemical reactions, or thermally activated processes. Additionally, the decreased values of Nm indicate a reduction in microorganism density in certain regions. This outcome is advantageous as it can prevent stagnant layers, promoting efficient mixing and uniform distribution of nutrients and oxygen, which are critical factors for microbial activity in water treatment systems. In summary, integrating hybrid nanoparticles can enhance contaminant removal, prevent clogging, and reduce operational costs, allowing engineers to optimize design and operational parameters, such as channel geometry, flow rates, and temperature.

On the other hand, increasing Gr and Ra reduces τ at plate $y=0$ (τ_0), while it increases τ at plate $y=1$ (τ_1). Additionally, τ_0 shows an increasing trend and τ_1 shows a decreasing trend with increasing Lb . For microorganism density number, an increasing trend in Nm_0 and a decreasing trend in Nm_1 is observed with rising Lb . Evaluating these properties under specific treatment conditions can lead to more sustainable and effective treatment systems.

6. Conclusion

This study analytically investigated the bioconvection flow of Brinkman hybrid nanofluid within a channel using the Laplace transform method. The key findings are:

- Increasing the volume fraction of $TiO_2 - CNTs$ enhances thermal conductivity, raises the fluid temperature, and increases the fluid velocity, leading to improved contaminant degradation and reduced biofouling.
- A higher B_r increases viscous resistance, slowing down fluid flow through porous media.
- Higher Ra intensifies density stratification, creating stable layers that can reduce fluid velocity.
- An increased Lb promotes faster and more uniform diffusion of microorganisms, reducing the formation of dense clusters.
- Increasing nanoparticle volume fractions $TiO_2 - CNTs$ lowers τ values at both plates, reducing fluid resistance.
- Higher $TiO_2 - CNTs$ nanoparticle volume fractions boost Nu values, indicating improved heat transfer rates.
- Decreased Nm values due to increase of $TiO_2 - CNTs$ nanoparticle volume fraction suggests reduced microorganism buildup, preventing stagnant layers.

These findings are beneficial for optimizing water filtration systems by improving efficiency, extending membrane life, and enhancing contaminant removal. Understanding these effects allows for better design and operation of filtration systems in various water treatment scenarios.

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