

ROTATING BRINKMAN HYBRID NANOFLUID THROUGH AN ACCELERATED DISK

(Bendalir Nano Hibrid Brinkman Berputar Melalui Cakera Dipercepat)

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ABSTRACT

The prospective applications of mono nanofluids in heat transmission are greatly enhanced by their promising thermophysical characteristics. Changes in the ratio or concentration of the nanoparticles can modify the characteristics of mono nanofluids. However, the stable thermophysical properties were disclosed in the restricted range by the mono nanofluids, which only used one type of nanoparticles (metallic or non-metallic). Hybrid nanofluids have been created recently to address this weakness by enhancing the thermophysical features and heat transport capabilities of the basic fluid. The current investigation focuses the Brinkman hybrid nanofluid flow through an accelerated disk with the effect of rotation. Here, the considered hybrid nanofluid are the combination of alumina and copper nanoparticles suspending in water as the base fluid. The closed form solutions for governing momentum and energy equations have been attained with the help of Laplace transformations. Analysis of velocity and temperature distributions affected by the related parameters is carried out via graphical illustration, while the skin friction and Nusselt number are analysed in tables. The results expose that the primary and secondary velocity decelerates when imposing to the higher volume fraction of hybrid nanofluid parameter even though the heat transfer enhances. In addition, the results of both velocity and temperature distributions satisfied the imposed conditions.

Keywords: Brinkman fluid; rotating fluid; hybrid nanofluid; accelerated disk; Laplace transform

ABSTRAK

Aplikasi prospektif bendalir nano mono dalam pemindahan haba dipertingkatkan dengan ketara oleh ciri-ciri termofizik yang menjanjikan. Perubahan dalam nisbah atau kepekatan zarah nano boleh mengubah suai ciri-ciri bendalir nano mono. Walau bagaimanapun, sifat termofizik yang stabil hanya didedahkan dalam julat terhad oleh bendalir nano mono, yang hanya menggunakan satu jenis zarah nano (logam atau bukan logam). Bendalir nano hibrid telah dibangunkan baru-baru ini untuk mengatasi kelemahan ini dengan meningkatkan ciri-ciri termofizik dan keupayaan pemindahan haba bendalir asas. Kajian semasa menumpukan kepada aliran bendalir nano hibrid Brinkman melalui cakera yang dipercepatkan dengan kesan putaran. Di sini, bendalir nano hibrid yang dipertimbangkan adalah gabungan zarah nano alumina dan tembaga yang terampai dalam air sebagai bendalir asas. Penyelesaian berbentuk tertutup untuk persamaan menakluk bagi momentum dan tenaga telah diperolehi dengan bantuan penjelmaan Laplace. Analisis terhadap taburan halaju dan suhu yang dipengaruhi oleh parameter berkaitan dijalankan melalui ilustrasi grafik, manakala geseran kulit dan nombor Nusselt dianalisis dalam bentuk jadual. Hasilnya menunjukkan bahawa halaju primer dan sekunder menurun apabila parameter pecahan isipadu bendalir nano hibrid yang lebih tinggi dikenakan, walaupun pemindahan haba meningkat. Di samping itu, keputusan kedua-dua taburan halaju dan suhu memenuhi syarat-syarat yang dikenakan.

Kata kunci: bendalir Brinkman; bendalir berputar; bendalir nano hibrid; cakera dipercepatkan; transformasi Laplace

1. Introduction

In the 19th century, Darcy (1856) introduced the study of fluid flow through porous media, leading to Darcy's Law. This law, however, fell short for fast-moving fluids in highly porous media. To address this, Brinkman (1949) developed a new model, known as Brinkman fluid. Research into Brinkman fluid flow and heat transfer is crucial for understanding fluid behavior in porous media, with applications in enhanced oil recovery, chemical reactors, filtration systems, and geothermal energy (Ramzan *et al.* 2021). Numerous researchers have conducted this study by considering various effects, such as radiation and ramped wall temperature (Zakaria *et al.* 2013), magnetohydrodynamics (MHD) and accelerated plate (Ali *et al.* 2013), and channel flow (Hashim *et al.* 2023).

Research on Brinkman fluid flow has progressed with the inclusion of nanoparticles, significantly enhancing thermal conductivity and modifying fluid dynamics (Noranuar *et al.* 2023a). This advancement promises improved heat transfer and performance in industrial applications (Noranuar *et al.* 2023b). Ali *et al.* (2016) conducted a comparative analysis of nanofluid flows with water-based nanoparticles of silver (*Ag*), copper (*Cu*), titanium oxide (*TiO₂*), and alumina (*Al₂O₃*). They found that a high thermal diffusivity of *Cu* nanoparticle has boosted fluid motion, resulting in a velocity advantage for *Cu* over *Ag*. This remarkable property of *Cu* nanoparticles was also observed by Zainal *et al.* (2024), who reported that the heat transfer rate of *Cu*-water nanofluid is higher than that of *Al₂O₃*-water nanofluid. Allehiany *et al.* (2023) found that adding clay nanoparticles to water markedly improved heat and mass transfer rates. Kumar *et al.* (2018) explored chemically reactive Brinkman nanofluid flows influenced by radiation, MHD, and porosity, using an exponentially accelerated plate, while Khan *et al.* (2019) studied these effects with an oscillating plate. Both studies found that increasing the nanoparticle volume fraction led to a reduction in fluid velocity due to higher resistance, which resulted from an increase of viscosity. Accelerating the plate results in a more predictable flow pattern compared to an oscillating plate, with fewer fluctuations in velocity. Abbas *et al.* (2023) derived exact solutions for Brinkman nanofluid flow in a channel using Laplace transformations, with inverse Laplace obtained via Stehfest's and Tzou's algorithm. In this study, the increased velocity is attributed to strong buoyancy forces, resulting from high values of both the Grashof number (*Gr*) and the modified Grashof number (*Gm*). Sidin and Bachok (2024) investigated nanofluid flow past a moving plate using carbon nanotube (CNTs) nanoparticles and found that the heat profile increases with the CNTs volume fraction. Ali *et al.* (2019) studied the MHD Casson nanofluid over an accelerated plate, focussing on the application of fractional derivative. The continuous acceleration of the plate promotes a more uniform flow direction, enhancing convective heat transfer.

Hybrid nanofluids, which combine different nanoparticles, emerged as an enhancement over traditional nanofluids (Nadeem *et al.* 2023; Jusoh *et al.* 2023). Sheikh *et al.* (2022) and Saqib *et al.* (2019) highlighted the superior thermal performance of hybrid Brinkman nanofluids in channel flows. Ikram *et al.* (2021) hybridized Brinkman fluid flow between parallel plates using *TiO₂* and *Ag* nanoparticles in water, analyzing heat transfer under heat absorption effects. The authors found that heat transfer is more effectively enhanced with hybrid nanoparticles compared to single nanoparticles. These findings were justified by

Jalaluddin *et al.* (2023), demonstrating an improvement in the thermal performance of hybrid nanofluids. Shafie *et al.* (2019) examined the impact of an inclined magnetic field on hybrid Brinkman nanofluid flow over an oscillating plate. Raza *et al.* (2023) conducted a comparative study of hybrid nanoparticles in water and carboxymethyl cellulose, finding that the thermal profile and velocity of water-based nanofluid were slightly higher than those of carboxymethyl cellulose-based Brinkman hybrid nanofluid. Despite these advances, no studies have focused on the rotating effect in Brinkman hybrid nanofluid flow over an accelerated plate, indicating a potential area for further research.

The study of fluid flow in rotating systems is essential for various engineering and geophysical applications. Introducing rotation to Brinkman fluid flow adds complexity to its dynamics and heat transfer. Mohamad *et al.* (2020) examined the effects of rotation on Brinkman fluid flow constrained by a moving vertical plate. The study stated that the presence of Coriolis force in the rotating system led to the formation of primary and secondary flows, which decelerate with increasing values of Brinkman fluid parameter. Shamshuddin *et al.* (2023) extended this study on rotating Brinkman fluid by introducing the nanoparticle effect. The study found that increasing the dispersion of molybdenum disulfide (MoS_2) nanoparticles in engine oil reduced both primary and secondary velocities. Ali *et al.* (2017) analyzed the same nanofluid under thermal radiation. Numerous researchers further examined the dynamics of rotating nanofluid flow, with Das *et al.* (2015) adopting the viscous fluid model and Zin *et al.* (2017) using the Jeffrey fluid model.

In rotating systems, the combined effects of Coriolis forces and hybrid nanoparticles introduce complex interactions that modify primary and secondary flow patterns, leading to improved energy dissipation and increased fluid velocity. These enhancements benefit applications requiring precise temperature control and efficient thermal management, such as advanced cooling systems, industrial lubricants, and energy solutions. To date, reviews of rotating flow have not emphasized the Brinkman fluid model in hybrid nanofluid studies, although various researchers have explored other fluid models in this context. Suganya *et al.* (2021) explored rotating viscous hybrid nanofluid flow using Al_2O_3 and silicon dioxide (SiO_2) in water, while Krishna and Chamkha (2023) used $Ag - TiO_2$ in water and ethylene glycol. Khan *et al.* (2022a) investigated rotating second grade hybrid nanofluid beyond a moving plate, observing that the hybrid nanoparticles volume fraction is a source to heat transfer enhancement. Khan *et al.* (2022b) extended the study by Khan *et al.* (2022a), investigating the heat and mass transfer over an oscillating plate. The study found that high value of Gr and Gm contributed to the velocity improvement, which is driven by buoyancy force. Despite these studies, the rotating effect on Brinkman hybrid nanofluid flow remains unexplored. This study addresses this gap, investigating rotating flow and heat transfer over an accelerated plate, using the Laplace transform method for exact temperature and velocity solutions.

2. Mathematical Formulation

Consider the unsteady natural convection flow through a vertical accelerating disk using a $Cu - Al_2O_3$ /water hybrid nanofluid as depicted in Figure 1. The x -axis is oriented upward along the disk, and the y -axis is perpendicular to it. Initially, both the disk and fluid are stationary with a uniform temperature T_∞ . At time $t > 0$, the disk accelerates at a constant rate A and the plate temperature increases to T_w . The fluid then begins solid body rotation with an angular velocity Ω along the y -axis, causing the fluid to accelerate in the x -direction. The Boussinesq approximation and hybrid nanofluid model are used for the

momentum and energy equations, which are given as (Ali *et al.* 2013; Saqib *et al.* 2019; Mohamad *et al.* 2020),

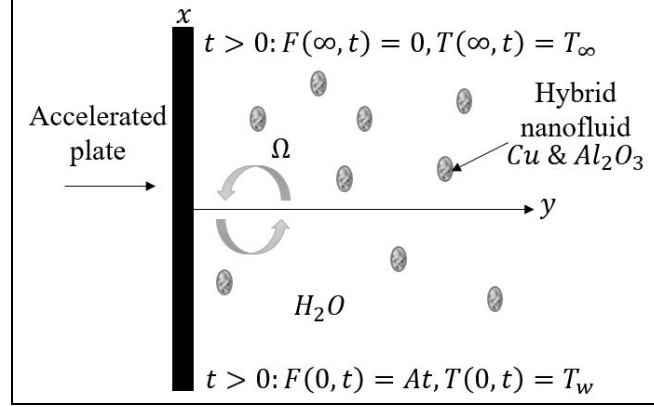


Figure 1: Physical diagram and coordinate system

$$\frac{\partial F}{\partial t} + \beta F + 2i\Omega F = \frac{\mu_{hnf}}{\rho_{hnf}} \frac{\partial^2 F}{\partial y^2} + g(\beta_T)_{hnf}(T - T_\infty), \quad (1)$$

$$(\rho C_p)_{hnf} \frac{\partial T}{\partial t} = k_{hnf} \frac{\partial^2 T}{\partial y^2}, \quad (2)$$

where $F = F(y, t) = u(y, t) + iw(y, t)$ is a complex velocity with $u(y, t)$ and $w(y, t)$ are primary and secondary velocities, respectively. Meanwhile, μ_{hnf} is dynamic viscosity of hybrid nanofluid, ρ_{hnf} is density of hybrid nanofluid, g is the acceleration due to gravity, $(\beta_T)_{hnf}$ is thermal expansion coefficient of hybrid nanofluid, $T = T(y, t)$ is temperature of hybrid nanofluid, $(C_p)_{hnf}$ is the heat capacitance of hybrid nanofluid, and k_{hnf} is thermal conductivity of hybrid nanofluid. Eq. (1) and Eq. (2) are accompanied by the conditions for initial and boundary as (Ali *et al.* 2013)

$$\begin{aligned} F(y, 0) &= 0, \quad T(y, 0) = T_\infty; \quad \forall y > 0, \\ F(0, t) &= At, \quad T(0, t) = T_w; \quad \forall t > 0, \\ F(\infty, t) &= 0, \quad T(\infty, t) = T_\infty; \quad \forall t > 0, \end{aligned} \quad (3)$$

where A is acceleration parameter. Next, the following suitable dimensionless variables (Ali *et al.* 2013)

$$F^* = \frac{F}{(v_f A)^{\frac{1}{3}}}, \quad t^* = \frac{t A^{\frac{2}{3}}}{v_f^{\frac{1}{3}}}, \quad y^* = \frac{y A^{\frac{1}{3}}}{v_f^{\frac{2}{3}}}, \quad T^* = \frac{T - T_\infty}{T_w - T_\infty}, \quad (4)$$

and hybrid nanofluid correlations (Saqib *et al.* 2019; Jalaluddin *et al.* 2023)

$$\begin{aligned}
 \phi_{hnf} &= \phi_1 + \phi_2, \quad \mu_{hnf} = \frac{\mu_f}{(1 - (\phi_1 + \phi_2))^{2.5}}, \\
 (\rho\beta_T)_{hnf} &= (1 - \phi_{hnf})(\rho\beta_T)_f + \phi_1(\rho\beta_T)_1 + \phi_2(\rho\beta_T)_2, \\
 (\rho C_p)_{hnf} &= (1 - \phi_{hnf})(\rho C_p)_f + \phi_1(\rho C_p)_1 + \phi_2(\rho C_p)_2, \\
 \frac{k_{hnf}}{k_f} &= \frac{\frac{\phi_1 k_1 + \phi_2 k_2}{\phi_{hnf}} + 2k_f + 2(\phi_1 k_1 + \phi_2 k_2) - 2k_f \phi_{hnf}}{\frac{\phi_1 k_1 + \phi_2 k_2}{\phi_{hnf}} + 2k_f - 2(\phi_1 k_1 + \phi_2 k_2) - 2k_f \phi_{hnf}}.
 \end{aligned} \tag{5}$$

are substituted into Eq. (1) to Eq. (3), which simplifies to following dimensionless governing equations (the sign * is ignored for simplicity purpose)

$$\frac{\partial F}{\partial t} + B_r F + 2i\omega F = \frac{1}{a_3} \frac{\partial^2 F}{\partial y^2} + a_4 Gr T(y, t), \tag{6}$$

$$\frac{\partial T}{\partial t} = \frac{a_1}{a_2 Pr} \frac{\partial^2 T}{\partial y^2}, \tag{7}$$

where the subscription 1, 2, f , and hnf in Eq. (5) represent nanoparticles of Al_2O_3 , nanoparticles of Cu , base fluid, and hybrid nanofluid, and associated with conditions as

$$\begin{aligned}
 F(y, 0) &= 0, \quad T(y, 0) = T_\infty; \quad \forall y > 0, \\
 F(0, t) &= t, \quad T(0, t) = T_w; \quad \forall t > 0, \\
 F(\infty, t) &= 0, \quad T(\infty, t) = T_\infty; \quad \forall t > 0.
 \end{aligned} \tag{8}$$

In Eq. (6) and Eq. (7), B_r , ω , Gr , and Pr are dimensionless parameters known as Brinkman parameter, rotation parameter, Grashof number, and Prandtl number, which are defined as

$$B_r = \frac{\beta v_f^{\frac{1}{3}}}{A^{\frac{2}{3}}}, \quad \omega = \frac{\Omega v_f^{\frac{1}{3}}}{A^{\frac{2}{3}}}, \quad Gr = \frac{g(\beta_T)_f(T_w - T_\infty)}{A}, \quad Pr = \frac{v_f(\rho C_p)_f}{k_f}. \tag{9}$$

Meanwhile, a_1 , a_2 , a_3 , and a_4 are constants, define as

$$\begin{aligned}
 a_1 &= \frac{k_{hnf}}{k_f}, \quad a_2 = (1 - \phi_{hnf}) + \frac{\phi_1(\rho C_p)_1 + \phi_2(\rho C_p)_2}{(\rho C_p)_f}, \\
 a_3 &= (1 - (\phi_1 + \phi_2))^{2.5} \left((1 - \phi_{hnf}) + \frac{\phi_1 \rho_1 + \phi_2 \rho_2}{\rho_f} \right),
 \end{aligned} \tag{10}$$

$$a_4 = (1 - \phi_{hnf}) + \frac{\phi_1 (\rho\beta_T)_1 + \phi_2 (\rho\beta_T)_2}{(\rho\beta_T)_f},$$

where the correlation of nanofluid is referred to thermophysical properties for water, Al_2O_3 , and Cu as in Table 1 (Saqib *et al.* 2019; Jalaluddin *et al.* 2023).

Table 1: Thermophysical properties for water, Al_2O_3 , and Cu (Saqib *et al.* 2019; Jalaluddin *et al.* 2023)

Materials/Properties	Water	Cu	Al_2O_3
$\rho (kg / m^3)$	997.1	8933	3970
$C_p (J / kgK)$	4179	385	765
$K (W / mK)$	0.613	400	40
$\beta_T \times 10^{-5} (K^{-1})$	21	1.67	0.85

3. Solution of Problem

Eq. (6) to Eq. (8) are employed with the Laplace transform, which results in

$$\frac{d^2 \bar{F}(y, q)}{dy^2} - (a_3 q + a_3 b_2) \bar{F}(y, q) = -a_3 a_4 Gr \frac{1}{q} \exp(-y \sqrt{b_1 q}), \quad (11)$$

$$\frac{d^2 \bar{T}(y, q)}{dy^2} - b_1 \bar{T}(y, q) = 0, \quad (12)$$

$$\bar{F}(0, q) = \frac{1}{q^2}, \quad \bar{F}(\infty, q) = 0, \quad (13)$$

$$\bar{T}(0, q) = 1, \quad \bar{T}(\infty, q) = 0. \quad (14)$$

Eq. (13) and Eq. (14) are used, and several manipulations are applied to solve Eq. (11) and Eq. (12). To obtain the exact solutions for velocity and temperature, the resultant Laplace solutions are tackled by the inverse of Laplace, which obtains

$$F(y, t) = F_1(y, t) - F_2(y, t) + F_3(y, t) + F_4(y, t) - F_5(y, t), \quad (15)$$

$$T(y, t) = \operatorname{erfc}\left(\frac{y}{2} \sqrt{\frac{b_1}{t}}\right), \quad (16)$$

where

$$\begin{aligned}
 F_1(y,t) &= \frac{1}{4\sqrt{b_7}} \left[\begin{aligned} &\left(y\sqrt{a_3} + 2t\sqrt{b_7} \right) \exp\left(y\sqrt{a_3b_7} \right) \operatorname{erfc} \left(\frac{y}{2}\sqrt{\frac{a_3}{t}} + \sqrt{b_7t} \right) + \\ &\left(y\sqrt{a_3} - 2t\sqrt{b_7} \right) \exp\left(-y\sqrt{a_3b_7} \right) \operatorname{erfc} \left(\frac{y}{2}\sqrt{\frac{a_3}{t}} - \sqrt{b_7t} \right) \end{aligned} \right], \\
 F_2(y,t) &= \frac{b_6Gr}{2} \left[\begin{aligned} &\exp\left(y\sqrt{a_3b_7} \right) \operatorname{erfc} \left(\frac{y}{2}\sqrt{\frac{a_3}{t}} + \sqrt{b_7t} \right) + \\ &\exp\left(-y\sqrt{a_3b_7} \right) \operatorname{erfc} \left(\frac{y}{2}\sqrt{\frac{a_3}{t}} - \sqrt{b_7t} \right) \end{aligned} \right], \\
 F_3(y,t) &= \frac{b_6Gr}{2} \left[\begin{aligned} &\exp\left(b_5t + y\sqrt{a_3(b_5+b_7)} \right) \operatorname{erfc} \left(\frac{y}{2}\sqrt{\frac{a_3}{t}} + \sqrt{(b_5+b_7)t} \right) + \\ &\exp\left(b_5t - y\sqrt{a_3(b_5+b_7)} \right) \operatorname{erfc} \left(\frac{y}{2}\sqrt{\frac{a_3}{t}} - \sqrt{(b_5+b_7)t} \right) \end{aligned} \right], \\
 F_4(y,t) &= b_6Gr \operatorname{erfc} \left(\frac{y}{2}\sqrt{\frac{b_1}{t}} \right), \\
 F_5(y,t) &= \frac{b_6Gr}{2} \left[\begin{aligned} &\exp\left(b_5t + y\sqrt{b_1b_5} \right) \operatorname{erfc} \left(\frac{y}{2}\sqrt{\frac{b_1}{t}} + \sqrt{b_5t} \right) + \\ &\exp\left(b_5t - y\sqrt{b_1b_5} \right) \operatorname{erfc} \left(\frac{y}{2}\sqrt{\frac{b_1}{t}} - \sqrt{b_5t} \right) \end{aligned} \right],
 \end{aligned}$$

with $b_1 = \frac{a_2Pr}{a_1}$, $b_2 = B_r + 2i\omega$, $b_3 = b_1 - a_3$, $b_4 = a_3b_2$, $b_5 = \frac{b_4}{b_3}$, $b_6 = \frac{a_2a_4}{b_3b_5}$, and $b_7 = \frac{b_4}{a_3}$ are constants.

4. Results and Discussion

This section focused on the physical behavior of Brinkman fluid flow and heat transfer under the impacts of Brinkman parameter (B_r), rotation parameter (ω), hybrid nanoparticle volume fraction (ϕ_{hnf}), Grashof number (Gr), and time (t). In a rotating fluid flow, the primary and secondary velocities appeared as the consequences of the interplay between rotational forces such as centrifugal force and Coriolis force. The verification process uses the study by Mohamad *et al.* (2020) as a benchmark. For this purpose, the present study is assigned values of $B_r = 2$, $\omega = 0.5$, $t = 1$, $\phi_1 = \phi_2 = 0$, and $Gr = 0$, while the previous study used $B_r = 2$, $\omega = 0.5$, and $t = 1$. The validity of the current solutions is confirmed as demonstrated in Figure 2, as they show good agreement with the results of Mohamad *et al.* (2020).

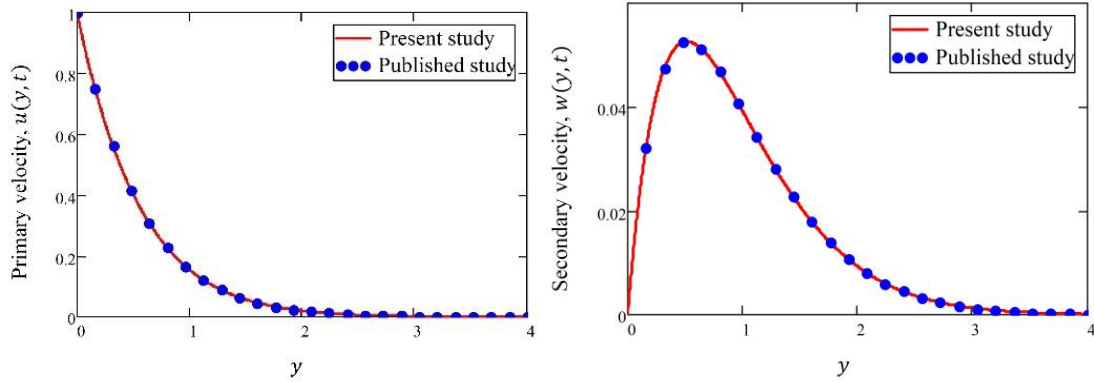


Figure 2: Verification of present study with the published study by Mohamad *et al.* (2020)

Figure 3 illustrates the effects of B_r on velocity profiles. A higher B_r indicates smaller pore sizes, resulting in decreased permeability and increased fluid resistance through the porous medium. Therefore, increase the B_r values result a decrease in primary and secondary velocity profiles. When Brinkman parameter is neglected ($B_r = 0$), it implies that the viscous resistance of the porous medium is not influencing the flow, and the fluid flow behave as Newtonian fluid model. In other words, the porous medium is either absent or, or its resistance to the fluid flow is minimal due to a medium with very high permeability. Therefore, the fluid with $B_r = 0$ can achieve higher velocity compared to those with $B_r > 0$.

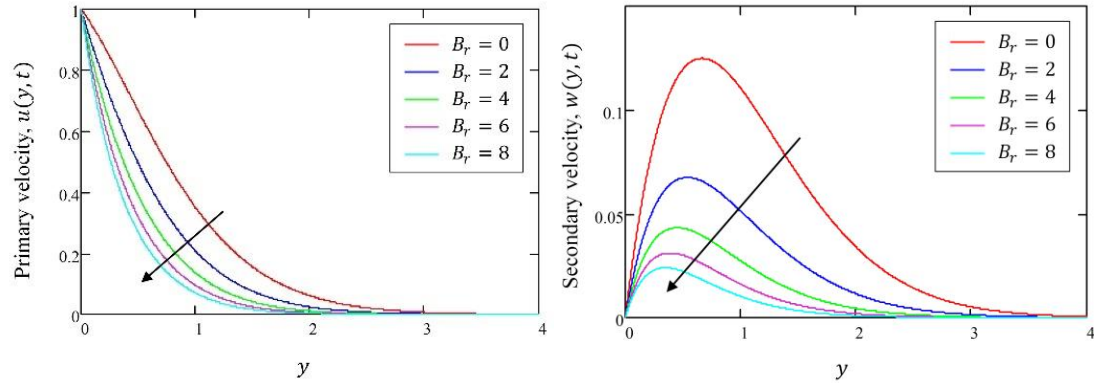
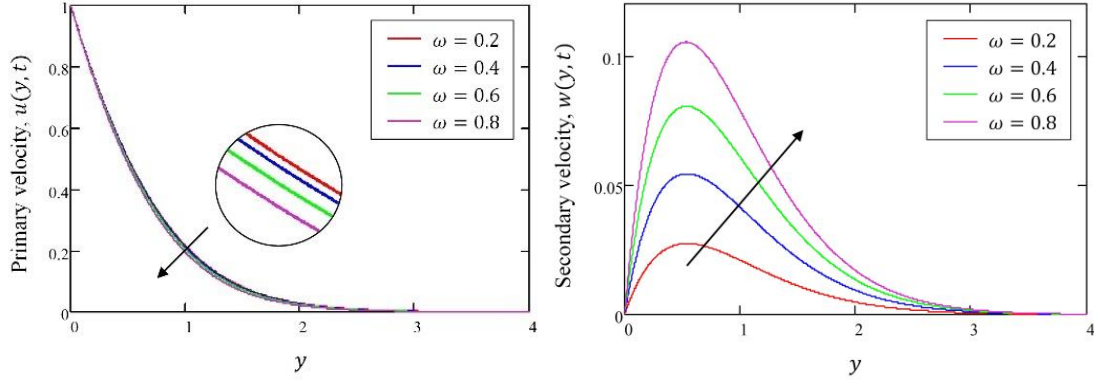
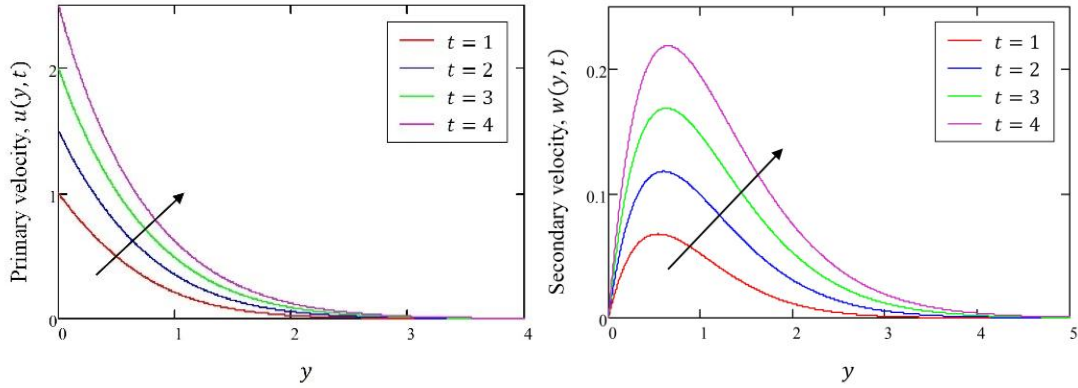


Figure 3: Primary and secondary velocity profiles with the effects of Brinkman parameter (B_r)

Figure 4 shows that an increase in ω decreases the primary velocity while increasing the secondary velocity. Increased rotation strengthens centrifugal forces, reducing primary velocity by diverting fluid into rotational paths, while enhanced rotational motion and Coriolis forces boost secondary velocity by accelerating fluid perpendicular to the primary flow. The effects of increasing t is illustrated in Figure 5, where an elevated profile is exhibited. This situation is due an increase of temperature difference over time, enhancing buoyancy forces. These stronger buoyancy forces drive more vigorous convection, increasing the velocities in both primary and secondary directions.


 Figure 4: Primary and secondary velocity profiles with the effects of rotation parameter (ω)

 Figure 5: Primary and secondary velocity profiles with the effects of time (t)

In Figure 6, increasing ϕ_{hnf} decreases both primary and secondary velocities due to higher viscosity. Adding more nanoparticles to the base fluid increases its overall viscosity. A more viscous fluid experiences greater resistance to flow, which directly reduces both primary and secondary velocities. Meanwhile, an opposite trend is exhibited for temperature profile as shown in Figure 7 under the effects of hybrid nanoparticles. An increased volume fraction of hybrid nanoparticles in a fluid enhances its thermal properties, leading to more efficient heat transfer and an elevated temperature profile.

Figure 8 demonstrates an enhancement profile of primary and secondary velocities due to an increase in Gr . A higher Gr indicates stronger buoyancy forces relative to viscous forces. This leads to more vigorous convection currents, which enhances the movement of the fluid. When the Grashof number in a convective fluid flow is neglected ($Gr = 0$), it implies that the buoyancy forces driving the flow due to temperature differences are absent or insignificant. This condition indicates that the flow is not influenced by natural convection (thermal buoyancy effects), and any fluid motion is instead likely dominated by other forces, such as viscous or external forces like pressure gradients (forced convection) or by the initial velocity of the fluid. Therefore, the velocity profile with $Gr = 0$ is lower compared to the velocity profile with $Gr > 0$.

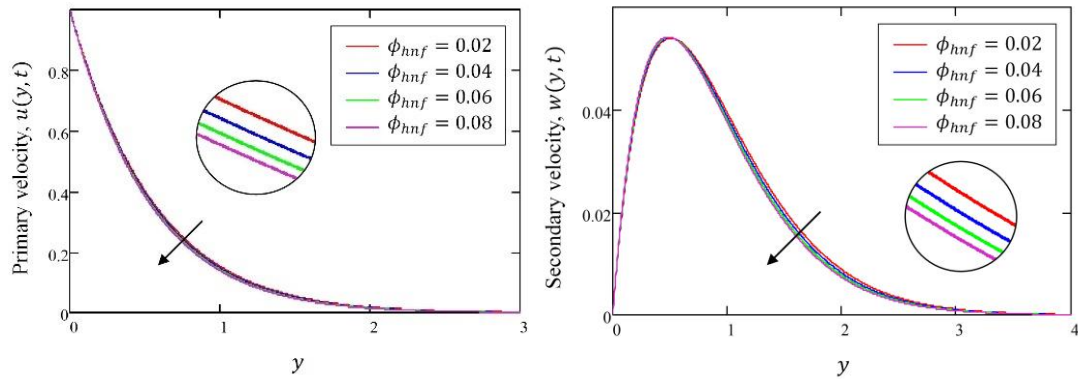


Figure 6: Primary and secondary velocity profiles with the effects of volume fraction of hybrid nanoparticle (ϕ_{hnf})

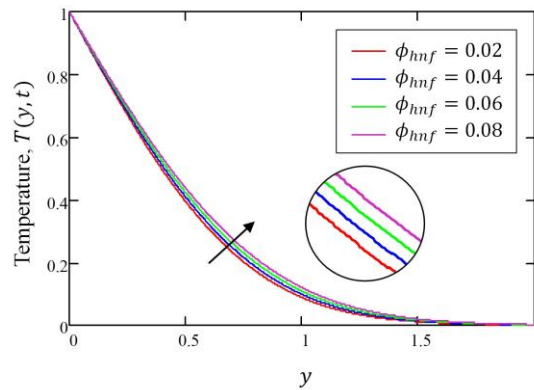


Figure 7: Temperature profile with the effects of volume fraction of hybrid nanoparticle (ϕ_{hnf})

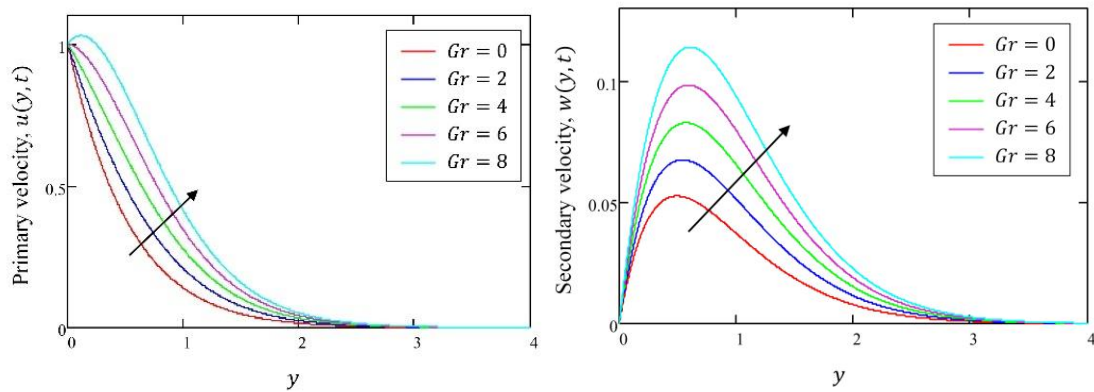


Figure 8: Primary and secondary velocity profiles with the effects of Grashof number (Gr)

The comparison of the velocity and thermal performance of hybrid nanofluid (Water + Cu + Al_2O_3), nanofluids (Water + Cu & Water + Al_2O_3), and conventional fluid (water) is shown in Fig. 9 and Fig. 10. The results reveal that the velocity profile of hybrid nanofluid (Water + Cu + Al_2O_3) is lower compared to velocity profile of nanofluids and conventional fluid. This effect is due to the addition of Cu and Al_2O_3 increases the viscosity of the hybrid nanofluid, generating high resistance and results a low velocity. The density of Cu is higher compared to density of Al_2O_3 , whereby the suspension of Cu in water yields a

high viscosity nanofluid and potentially resulting in lower velocity for nanofluid of Water + Cu compared to nanofluid of Water + Al_2O_3 . However, the viscosity effect in nanofluid is less pronounced than in hybrid nanofluid, so the velocity profile is generally higher than hybrid nanofluid but lower than those of the water. In the case of temperature profile, the presence of Cu and Al_2O_3 , greatly increases the effective thermal conductivity of the hybrid nanofluid. This allows the hybrid nanofluid to dissipate heat more effectively, resulting in a more uniform temperature profile with a steeper gradient near the boundaries. This enhanced heat transfer characteristic results in the temperature of hybrid nanofluid higher than in single component nanofluids (Water + Cu & Water + Al_2O_3) and conventional water. The temperature profile for Water + Cu shows better heat transfer compared to Water + Al_2O_3 due to Cu higher thermal conductivity. Both single component nanofluids offer improved heat transfer over water alone, but they fall short of the hybrid nanofluid's performance due to the lack of combined thermal enhancement from multiple nanoparticles.

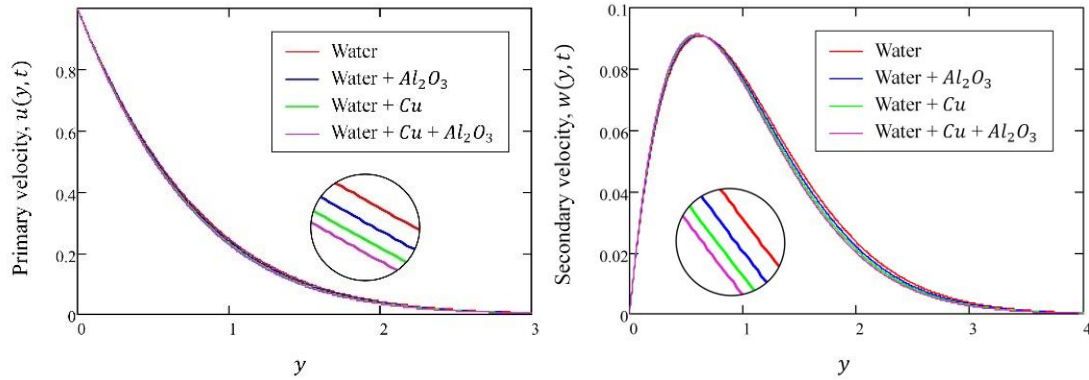


Figure 9: Comparison of hybrid nanofluid, nanofluids, and conventional fluid for primary and secondary velocity profiles

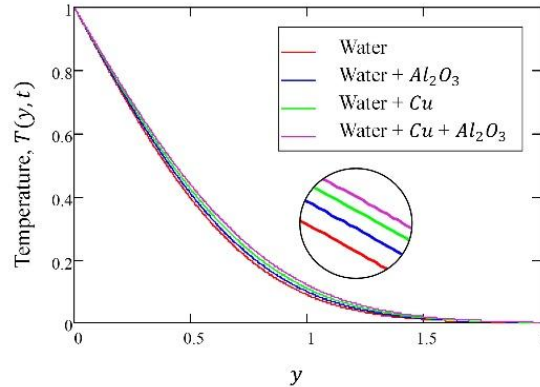


Figure 10: Comparison of hybrid nanofluid, nanofluids, and conventional fluid for temperature profiles

5. Conclusion

This study analytically investigated the unsteady free convection flow of Brinkman hybrid nanofluid over an accelerated plate in the presence of rotation effects, utilizing the Laplace transform method. Key findings reveal that increasing the Brinkman parameter decreases both primary and secondary velocity profiles, indicating higher viscous resistance. Conversely,

higher Grashof numbers and time enhance these velocities due to stronger buoyancy effects and flow development. An increase in the rotation parameter decreases primary but increases secondary velocities due to the Coriolis effect. Additionally, increasing the hybrid nanoparticle volume fraction reduces both primary and secondary velocities but raises the temperature profile due to enhanced thermal conductivity. These insights are significant for applications in cooling systems for rotating machinery, microfluidic devices, and energy systems, where precise heat transfer and fluid flow control are crucial. The study provides a foundation for optimizing such systems for improved performance and efficiency.

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