

ENERGY AND LAPLACIAN ENERGY OF THE RELATIVE CO-PRIME GRAPH FOR SYMMETRIC GROUP OF ORDER SIX

(Tenaga dan Tenaga Laplacian bagi Graf Ko-Perdana secara Relatif bagi Kumpulan Simetrik Peringkat Enam)

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ABSTRACT

The energy of a graph is calculated as the sum of the modulus of eigenvalues of an adjacency matrix for a graph. The Laplacian energy refers to the eigenvalues of the graph's Laplacian matrix. In graph theory, the co-prime graph of a group is a graph where the vertices represent the elements of a group, and two distinct vertices are connected if and only if their orders are relatively prime. This study is then extended to the relative co-prime graph of a group, where the vertices are the elements of the group, and two distinct vertices are adjacent if and only if their orders are co-prime and any of them is in the subgroup. By using the definition of the relative co-prime graph of a group with respect to a subgroup, some graphs of symmetric group of order six are constructed. Some energies and Laplacian energies of the relative co-prime graph related to the symmetric group of order six are also obtained.

Keywords: adjacency matrices; energy; Laplacian energy; relative co-prime graph; symmetric group

ABSTRAK

Tenaga graf dikira sebagai jumlah modulus nilai eigen bagi matriks bersebelahan untuk graf. Tenaga Laplacian merujuk kepada nilai eigen bagi graf matriks Laplacian. Dalam teori graf, graf ko-perdana bagi suatu kumpulan ialah graf di mana bucu-bucu mewakili unsur-unsur bagi suatu kumpulan, dan dua bucu yang berbeza adalah bersambung jika dan hanya jika peringkat bagi unsur-unsur berkenaan adalah perdana secara relatif. Kajian ini kemudiannya diperluaskan selanjutnya kepada graf ko-perdana secara relatif bagi suatu kumpulan, di mana bucu-bucu adalah unsur-unsur bagi suatu kumpulan, dan dua bucu yang berbeza adalah bersebelahan jika dan hanya jika peringkat bagi unsur-unsur berkenaan adalah ko-perdana dan mana-mana antara kedua-dua bucu ini ada di dalam subkumpulan. Dengan menggunakan takrifan bagi graf ko-perdana secara relatif bagi suatu kumpulan, beberapa graf kumpulan simetrik peringkat enam dibina. Beberapa tenaga dan tenaga Laplacian bagi graf ko-perdana secara relatif yang berkaitan dengan kumpulan simetrik peringkat enam juga diperoleh.

Kata kunci: nilai eigen bagi matriks; tenaga; tenaga Laplacian; graf ko-perdana secara relatif; kumpulan simetrik

1. Introduction

Gutman (1978) established the framework for the concept of energy in graph theory, sparking extensive research into this fundamental idea. Alimon *et al.* (2018) computed the energy of commuting graph, non-commuting graph, conjugate graph, and conjugacy class graph of certain metacyclic 2-groups. In the same year, Mahmoud *et al.* (2017b) obtained the energy of the non-commuting graph of dihedral groups. The authors subsequently derived a general formula for the energy of the corresponding graph.

Gutman and Furtula (2017) provided the basic facts on graph energies, for historical and bibliographic data. Estrada and Benzi (2017) highlighted an explanation and proved that the energy of any graph, whether bipartite or not, is expressed as a weighted sum of the traces of even powers of its adjacency matrix.

Ahmad Fadzil *et al.* (2018) determined the energy of the conjugacy class graphs for all nonabelian metabelian groups of order 24. The results indicated that if the energy of the graphs in the study is rational, it must be an even integer.

Moreover, Ahmad Fadzil *et al.* (2020a) determined the energy of the Cayley graphs of alternating groups with respect to the subset S of valency one and two. The authors also computed the energy of the same graph for the symmetric group of order 24 (Ahmad Fadzil *et al.* 2020b). Furthermore, Akbari and Hosseinzadeh (2020) established that for any graph Γ , its energy is at least twice the minimum degree of Γ , with equality occurring if and only if Γ is a complete multipartite graph with equal-sized parts.

The Laplacian energy of a graph is defined as the sum of the absolute differences between the Laplacian eigenvalues and the ratio of twice the number of edges to the number of vertices (Gutman & Zhou 2006). Mahmoud *et al.* (2017a) presented a general formula for the Laplacian energy of the conjugacy class graph of the dihedral group D_{2n} , specifically for cases where n is an odd integer greater than 3. In year 2020, Das *et al.* established a lower bound on the Laplacian energy of chain graphs in terms of order n and characterized the extremal graphs. A year later, Mei *et al.* (2021), derived upper and lower bounds for the Laplacian energy of chain graphs and identified the graph with the maximum Laplacian energy among all connected bicyclic chain graphs by analyzing algebraic connectivity.

The prime graph of a group G , denoted by $\Gamma(G)$ is one of the examples of graph related to a group. The researchers have been exploring the theory and generalizations of the prime graph since 1981. According to Williams (1981), the prime graph of G is a graph whose vertices correspond to the prime numbers dividing the order of G . Two vertices u and v are connected by an edge if and only if G contains an element of order uv . This concept played a fundamental role in understanding the structure and properties of finite groups.

Further developing the field, Sattanathan and Kala (2009) introduced the concept of order prime graph which is associated to G . In this graph, vertices correspond to elements of G and are connected if their ordering is co-prime. The authors characterized certain classes of order prime graph.

Moreover, Ma *et al.* (2014) introduced the co-prime graph of G . The coprime graph was studied extensively by Dorbidi (2016) and Selvakumar and Subajini (2017). Mutharasu *et al.* (2014) is then introduced the generalized co-prime graph where in this article, the authors found that the chromatic number and the clique number of certain generalized co-prime graphs are equal. A year later, Rajkumar and Devi (2015) explored the relationships between the algebraic properties of a group and the graph-theoretic properties of the co-prime graph.

Afterward, Abd Rhani *et al.* (2017) initiated the concept of the relative co-prime graph of a group. By using the definition of this graph, some graph properties such as the dominating number, the degree of the identity element and the independence number are obtained. Recently, Muthulakshmi *et al.* (2024) characterized the structure of order prime graphs of some groups and obtained the independence number and clique number for this graph.

This paper is divided into three sections. The first section provides an introduction, followed by the second section, which discusses the preliminaries, including fundamental concepts, definitions, and relevant previous findings. The third section focuses on the main results, specifically the computation of the energy and Laplacian energy of three graphs associated to the relative co-prime graph of S_3 with respect to a subgroup H .

2. Preliminaries

Some preliminaries involved in this study can be found as is the following.

Definition 2.1. (Gutman 1978) The adjacency matrix of a graph $\Gamma (G)$ associated with a group G , consisting of n vertices and no parallel edges, is defined as follows:

$$E (\Gamma (G)) = \begin{cases} 1, \text{ if } v_i \text{ and } v_j \text{ are adjacent,} \\ 0, \text{ otherwise.} \end{cases} \quad (1)$$

Definition 2.2. (Friedberg *et al.* 2013) The equation $\det (E - \lambda I) = 0$ is known as the characteristic equation of E where E denotes the adjacency matrix, λ is a scalar and I represents the identity matrix. The function $f (\lambda) = \det (\lambda I - E)$ is called the characteristic polynomial of E .

Definition 2.3. (Gutman 1978) Let Γ be a graph, the energy of Γ , denoted by $\varepsilon (\Gamma)$, is the total of the absolute values of its eigenvalues, i.e. if $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigenvalues of Γ , then

$$\varepsilon (\Gamma) = \sum_{i=1}^n |\lambda_i| \quad (2)$$

Definition 2.4. (Bondy & Murty 2008) The degree of a vertex v , denoted as d_v , is the number of edges connected to v , with each loop being counted as two edges.

Definition 2.5. (Gutman & Zhou 2006) Let Γ be a graph. The Laplacian matrix $L (\Gamma)$ of Γ is defined as $n \times n$ matrix,

$$L (\Gamma (G)) = \begin{cases} -1, \text{ if } v_i \text{ and } v_j \text{ are adjacent where } i \neq j, \\ 0, \text{ if } v_i \text{ and } v_j \text{ are not adjacent where } i \neq j, \\ d_i, \text{ if } i = j, \end{cases} \quad (3)$$

where d_i is the degree of vertex $i, i = 1, 2, 3, \dots, n$.

Definition 2.6. (Bondy & Murty 2008) The Laplacian characteristic polynomial of a graph Γ is given by $f (\mu) = \det (\mu I - L (\Gamma))$ where μ is a real number, I represents the identity matrix, and $L (\Gamma)$ is the Laplacian matrix corresponding to the graph Γ .

Definition 2.7. (Chattopadhyay & Panigrahi 2015) Let M be an $n \times n$ square matrix, $\vec{y} \neq \vec{0}$ be a vector in $\mathbb{C}^n$, and λ be a scalar in $\mathbb{C}$. Then, $\vec{y}$ is called an eigenvector of M corresponding to the eigenvalue λ if

$$M \vec{y} = \lambda \vec{y}. \quad (4)$$

Definition 2.8. (Gutman & Zhou 2006) Let Γ be a graph. The Laplacian energy of Γ , denoted by $L\varepsilon (\Gamma)$, is defined as the sum of the absolute deviations of the eigenvalues of its Laplacian matrix from the mean, denoted by

$$L\varepsilon (\Gamma) = \sum_{i=1}^n \left| \mu_i - \frac{2m}{n} \right| \quad (5)$$

where $\mu_1, \mu_2, \dots, \mu_n$ are the Laplacian eigenvalues of graph Γ , n is the number of vertices of the graph Γ , and m is the number of its edges.

Definition 2.9. (Abd Rhani *et al.* 2017) The relative co-prime graph of a group G with respect to a subgroup H , denoted as $\Gamma_{copr} (H, G)$, is a graph whose vertices are elements of G . Two distinct vertices u and v are adjacent if and only if $\gcd (|u|, |v|) = 1$ and any of element u or v is in H .

Definition 2.10. (Fraleigh 2003) Let A be the finite set $\{1, 2, \dots, m\}$. The group consisting of all permutations of A is called the symmetric group on m elements and is denoted by S_m . The order of S_m is given by $m!$.

3. Results and Discussion

This section presents the adjacency matrices of the relative co-prime graph of the symmetric group of order six. Subsequently, the energy and Laplacian energy of this graph are computed.

3.1. The relative co-prime graph of symmetric group of order six

The relative co-prime graphs of the symmetric group of order six are given as in the following.

Note: Only the graph of $H_2 = \{(1), (12)\}$ is provided in Figure 1 for the subgroup of order two, because $H_4 = \{(1), (13)\}$ and $H_5 = \{(1), (23)\}$ has a structure similar to H_2 due to the identical order. In addition, only the graph of $H_3 = \{(1), (123), (132)\}$ is provided, since $H_6 = S_3 = \{(1), (12), (13), (23), (123), (132)\}$ has a graph structure similar to H_3 . The main difference between them is in their vertices, which vary depending on the elements that make up each subgroup.

Proposition 3.1. Let $\Gamma_{copr}(H, S_3)$, be the relative co-prime graph of symmetric group with respect to a subgroup H and S_3 is the symmetric group of order six. Then, the relative co-prime graph of S_3 with respect to subgroups of $H_1 = \{(1)\}$, $H_2 = \{(1), (12)\}$ and $H_3 = \{(1), (123), (132)\}$ are found as in the following.

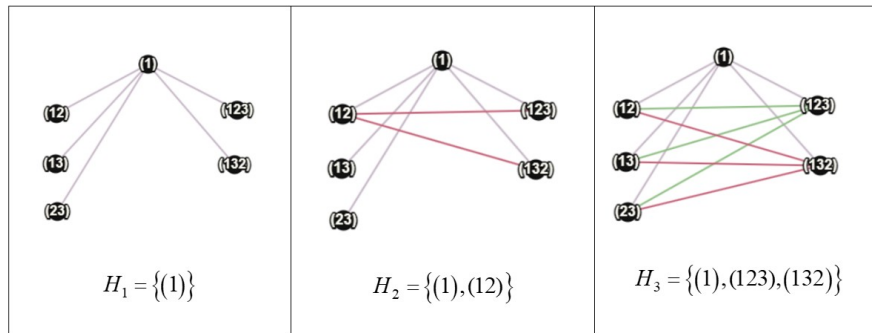


Figure 1: The relative co-prime graphs of S_3 with respect to subgroups H_1 , H_2 and H_3 .

Proof. By Definition 2.9, the relative co-prime graph of a group of S_3 consists of six vertices, and their connections follow from the adjacency rule in Definition 2.9. $\square$

3.2. The energy of the relative co-prime graph of S_3

In this subsection, the energies of the relative co-prime graph of symmetric group of order six are determined by using the eigenvalues of its adjacency matrix. The adjacency matrix represents the connections between the vertices, where each vertex corresponds to an element of the symmetric group S_3 . A connection is formed between two vertices if their corresponding elements have relatively prime orders.

The adjacency matrix of $\Gamma_{copr}(H_1, S_3)$ is given in the following.

$$E(\Gamma_{copr}(H_1, S_3)) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Proposition 3.2. Let G be the symmetric group of order six, S_3 and $\Gamma_{copr}(H_1, G)$ is the relative co-prime graph of G with respect to a subgroup $H_1 = \{(1)\}$. Then, the energy of the relative co-prime graph of G ,

$$\varepsilon(\Gamma_{copr}(H_1, G)) = 4.472.$$

Proof. By Definition 2.1, if a connection exists between vertex i and vertex j , the element $a_{ij} = 1$. Or else, it is 0. From $\Gamma_{copr}(H_1, G)$ of Figure 1, the central vertex connected to all other vertices and no other edges between peripheral vertices. For the adjacency matrix, the eigenvalue polynomial of $E(\Gamma_{copr}(H_1, G))$ is as follows:

$$f(\lambda) = \lambda^4(\lambda^2 - 5).$$

As a result, the eigenvalues of $\Gamma_{copr}(H_1, S_3)$ are $\lambda = 0$ with four repetitions and $\lambda = \sqrt{5}$ and $\lambda = -\sqrt{5}$. Therefore, by Eq. (2), $\varepsilon(\Gamma_{copr}(H_1, G)) = 4|0| + |\sqrt{5}| + |-\sqrt{5}| = 4.472$. $\square$

The adjacency matrix of $\Gamma_{copr}(H_2, S_3)$ is presented below along with the energy of the graph. This matrix represents the connections between elements of the symmetric group, S_3 based on their relative co-primality with respect to the subgroup H_2 .

$$E(\Gamma_{copr}(H_2, S_3)) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Proposition 3.3. Let G be the symmetric group of order six, S_3 and $\Gamma_{copr}(H_2, G)$ is the relative co-prime graph of G with respect to a subgroup $H_2 = \{(1), (12)\}$. Then, the energy of the relative co-prime graph of G ,

$$\varepsilon(\Gamma_{copr}(H_2, G)) = 6.686.$$

Proof. By Definition 2.1, if there is a connection between vertex i and j , the element $a_{ij} = 1$. Or else, it is 0. From $\Gamma_{copr}(H_2, G)$ of Figure 1, the central vertex connected to all other vertices and no other edges between peripheral vertices. For the adjacency matrix, the characteristic polynomial of $E(\Gamma_{copr}(H_2, G))$ is as follows:

$$f(\lambda) = \lambda^2(\lambda^4 - 7\lambda^2 - 4\lambda + 4).$$

As a result, the eigenvalues of the relative co-prime graph of S_3 are $\lambda = 0$ with two repetitions and $\lambda = -2$, $\lambda = -1.343$, $\lambda = 0.529$ and $\lambda = 2.814$. Hence, by Eq. (2), $\varepsilon(\Gamma_{copr}(H_2, G)) = 2|0| + |-2| + |-1.343| + |-0.529| + |2.814| = 6.686$. $\square$

Next, the adjacency matrix of $\Gamma_{copr}(H_3, G)$ is presented below, along with the energy of the graph.

$$E(\Gamma_{copr}(H_3, S_3)) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

Proposition 3.4. Let G be the symmetric group of order six, S_3 and $\Gamma_{copr}(H_3, G)$ is the relative co-prime graph of G with respect to a subgroup $H_3 = \{(1), (123), (132)\}$. Then, the energy of the relative co-prime graph of G ,

$$\varepsilon(\Gamma_{copr}(H_3, G)) = 7.533.$$

Proof. By Definition 2.1, if there is an edge between vertex i and j , the element $a_{ij} = 1$. Otherwise, it is 0. From $\Gamma_{copr}(H_3, G)$ of Figure 1, the central vertex connected to all other vertices and no other edges between peripheral vertices. For the adjacency matrix, the characteristic polynomial of $E(\Gamma_{copr}(H_3, G))$ is as follows:

$$f(\lambda) = \lambda^3(\lambda^3 - 11\lambda - 12).$$

As a result, the eigenvalues of the relative co-prime graph of S_3 are $\lambda = 0$ with multiplicity three and $\lambda = -2.484$, $\lambda = -1.283$ and $\lambda = 2.766$. Hence, by Eq. (2), $\varepsilon(\Gamma_{copr}(H_3, G)) = 3|0| + |-2.484| + |-1.283| + |2.766| = 7.533$. $\square$

3.3. The Laplacian energy of the relative co-prime graph of S_3

In this section, starting with the subgroup H_1 , the Laplacian energies of the relative co-prime graph of S_3 are calculated.

$$L(\Gamma_{copr}(H_1, S_3)) = \begin{bmatrix} 5 & -1 & -1 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Proposition 3.5. Let G be the symmetric group of order six, S_3 and $\Gamma_{copr}(H_1, G)$ is the relative co-prime graph of G with respect to a subgroup $H_1 = \{(1)\}$. Then, the Laplacian energy of the relative co-prime graph of G ,

$$L\varepsilon(\Gamma_{copr}(H_1, G)) = \frac{26}{3}.$$

Proof. By Definition 2.5, for $i \neq j$, if there is an edge between vertex i and j , then $a_{ij} = -1$. If vertex i and vertex j are not connected by an edge, then $a_{ij} = 0$. From $\Gamma_{copr}(H_1, G)$ of Figure 1, the central vertex connected to all other vertices and no other edges between peripheral vertices. For the Laplacian matrix, the characteristic polynomial of $L(\Gamma_{copr}(H_1, G))$ is as follows:

$$f(\mu) = \mu(\mu - 6)(\mu - 1)^4.$$

As a result, the Laplacian eigenvalues of the relative co-prime graph of S_3 are $\mu = 0, \mu = 1$ with four repetitions and $\mu = 6$. Hence, by Eq. (5),

$$L\mathcal{E}(\Gamma_{copr}(H_1, G)) = \left| -\frac{10}{6} \right| + 4 \left| \frac{2}{3} \right| + \left| \frac{13}{3} \right| = \frac{26}{3}. \square$$

Next, the Laplacian energy is calculated for the relative co-prime graph of G with respect to a subgroup H_2 .

$$L(\Gamma_{copr}(H_2, S_3)) = \begin{bmatrix} 5 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & 0 & 0 & -1 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 2 & 0 \\ -1 & -1 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

Proposition 3.6. Let G be the symmetric group of order six, S_3 and $\Gamma_{copr}(H_2, G)$ is the relative co-prime graph of G with respect to a subgroup $H_2 = \{(1), (12)\}$. Then, the Laplacian energy of the relative co-prime graph of G ,

$$L\mathcal{E}(\Gamma_{copr}(H_2, G)) = \frac{32}{3}.$$

Proof. By Definition 2.5, if $i \neq j$, then $a_{ij} = -1$ if there is a connection between vertex i and vertex j . If vertex i and vertex j are not connected by an edge, then $a_{ij} = 0$. For the Laplacian matrix, the eigenvalue polynomial of $L(\Gamma_{copr}(H_2, G))$ is as follows:

$$f(\mu) = \mu(\mu - 6)(\mu - 4)(\mu - 2)(\mu - 1)^2.$$

As a result, the Laplacian eigenvalues of the relative co-prime graph of S_3 are $\mu = 0, \mu = 1$ with two repetitions, $\mu = 2, \mu = 4$ and $\mu = 6$. Hence, by Eq. (5),

$$L\mathcal{E}(\Gamma_{copr}(H_2, G)) = \left| -\frac{14}{6} \right| + \left| -\frac{8}{3} \right| + \left| -\frac{1}{3} \right| + \left| \frac{5}{3} \right| + \left| \frac{11}{3} \right| = \frac{32}{3}. \square$$

Next, the Laplacian energy of the relative co-prime graph of G , with respect to a subgroup H_3 is computed.

$$L(\Gamma_{copr}(H_3, S_3)) = \begin{bmatrix} 5 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & 0 & 0 & -1 & -1 \\ -1 & 0 & 3 & 0 & -1 & -1 \\ -1 & 0 & 0 & 3 & -1 & -1 \\ -1 & -1 & -1 & -1 & 4 & 0 \\ -1 & -1 & -1 & -1 & 0 & 4 \end{bmatrix}.$$

Proposition 3.7. Let G be the symmetric group of order six, S_3 and $\Gamma_{copr}(H_3, G)$ is the relative co-prime graph of G with respect to a subgroup $H_3 = \{(1), (123), (132)\}$. Then, the Laplacian energy of the relative co-prime graph of G ,

$$L\varepsilon(\Gamma_{copr}(H_3, G)) = 10.$$

Proof. By Definition 2.5, if $i \neq j$, then $a_{ij} = -1$ if there is a connection between vertex i and vertex j . If vertex i and vertex j are not connected by an edge, then $a_{ij} = 0$. For the Laplacian matrix, the eigenvalue polynomial of $L(\Gamma_{copr}(H_3, G))$ is as follows:

$$f(\mu) = \mu(\mu - 6)^2(\mu - 4)(\mu - 3)^2.$$

As a result, the Laplacian eigenvalues of the relative co-prime graph of S_3 are $\mu = 0, \mu = 3$ with multiplicity 2, $\mu = 4$ and $\mu = 6$ with multiplicity 2. Hence, by Eq. (5),

$$L\varepsilon(\Gamma_{copr}(H_3, G)) = \left| -\frac{22}{6} \right| + \left| -\frac{4}{3} \right| + \left| \frac{1}{3} \right| + \left| \frac{14}{3} \right| = 10. \quad \square$$

4. Conclusion

In this study, the adjacency and Laplacian matrices of the relative co-prime graph of the symmetric group S_3 are successfully constructed. Through the analysis of their characteristic polynomials, the eigenvalues of these matrices are determined. Using these eigenvalues, the corresponding graph energies and Laplacian energies are computed. The eigenvalues not only characterize the graph but also serve as a foundation for computing key metrics such as graph energy, which plays a significant role in various applications in chemistry and network theory. Future work may involve exploring similar analyses for larger symmetric groups or other types of algebraic structure to determine how the spectral properties and energies evolve.

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References

- Abd Rhani N., Mohd Ali N.M., Sarmin N.H. & Erfanian A. 2017. The relative co-prime graph of a group. *Proceedings of the 4th Biennial International Group Theory Conference (4BIGTC2017)*, pp. 155–158.
- Ahmad Fadzil A.F., Sarmin N.H. & Birkia R.M. 2018. The energy of conjugacy class graphs of some metabelian groups **1974**(1): 030014.
- Ahmad Fadzil A.F., Sarmin N.H. & Erfanian A. 2020a. Energy of Cayley graphs for alternating groups. *Southeast Asian Bulletin of Mathematics* **44**(6): 789–795.
- Ahmad Fadzil A.F., Sarmin N.H. & Erfanian A. 2020b. The energy of Cayley graphs for symmetric groups of order 24. *ASM Science Journal* **13**: 1–6.
- Akbari S. & Hosseinzadeh M.A. 2020. A short proof for graph energy is at least twice of minimum degree. *MATCH Commun. Math. Comput. Chem* **83**: 631–633.
- Alimon N.I., Sarmin N.H. & Ahmad Fadzil A.F. 2018. The energy of four graphs of some metacyclic 2-groups. *Malaysian Journal of Fundamental and Applied Sciences* **14**(1): 59–66.
- Bondy J.A. & Murty U.S.R. 2008. *Graph Theory*. New York: Springer Publishing Company, Incorporated.

- Chattopadhyay S. & Panigrahi P. 2015. On Laplacian spectrum of power graphs of finite cyclic and dihedral groups. *Linear and Multilinear Algebra* **63**(7): 1345–1355.
- Das K.C., Alazemi A. & Andelic M. 2020. On energy and Laplacian energy of chain graphs. *Discrete Applied Mathematics* **284**: 391–400.
- Dorbidi H.R. 2016. A note on the coprime graph of a group. *International Journal of Group Theory* **5**(4): 17–22.
- Estrada E. & Benzi M. 2017. What is the meaning of the graph energy after all? *Discrete Applied Mathematics* **230**: 71–77.
- Fraleigh J.B. 2003. *A First Course in Abstract Algebra*. 7th Ed. Upper Saddle River, NJ: Pearson Education.
- Friedberg S.H., Insel A.J. & Spence L.E. 2013. *Linear Algebra: Pearson New International Edition*. 4th Ed. Essex: Pearson Education Limited.
- Gutman I. 1978. Bounds for Hückel total π -electron energy. *Croatica Chemica Acta* **51**(4): 299–306.
- Gutman I. & Furtula B. 2017. Survey of graph energies. *Mathematics Interdisciplinary Research* **2**(2): 85–129.
- Gutman I. & Zhou B. 2006. Laplacian energy of a graph. *Linear Algebra and its Applications* **414**(1): 29–37.
- Ma X., Wei H. & Yang L. 2014. The coprime graph of a group. *International Journal of Group Theory* **3**(3): 13–23.
- Mahmoud R., Ahmad Fadzil A.F., Sarmin N.H. & Erfanian A. 2017a. The Laplacian energy of conjugacy class graph of some dihedral groups. *Malaysian Journal of Fundamental and Applied Sciences* **13**(2): 129–131.
- Mahmoud R., Sarmin N.H. & Erfanian A. 2017b. On the energy of non-commuting graph of dihedral groups. *AIP Conference Proceedings* **1830**(1): 070011.
- Mei Y., Guo C. & Liu M. 2021. The bounds of the energy and Laplacian energy of chain graphs. *AIMS Mathematics* **6**(5): 4847–4859.
- Mutharasu S., Mohamed Rilwan N., Angel Jebitha M.K. & Tamizh Chelvam T. 2014. On generalized coprime graphs. *Iranian Journal of Mathematical Sciences and Informatics* **9**(2): 1–6.
- Muthulakshmi R.M., Premalatha D. & Lalitha N. 2024. Independence number of order prime graph. *Indian Journal of Science and Technology* **17**(18): 1824–1827.
- Rajkumar R. & Devi P. 2015. Coprime graph of subgroups of a group. *arXiv preprint arXiv:1510.00129*.
- Sattanathan M. & Kala R. 2009. An introduction to order prime graph. *Int. J. Contemp. Math. Sciences* **4**(10): 467–474.
- Selvakumar K. & Subajini M. 2017. Classification of groups with toroidal coprime graphs. *Australasian Journal of Combinatorics* **69**(2): 174–183.
- Williams J.S. 1981. The prime graph components of finite groups. *The Santa Cruz Conference on Finite Groups (Univ California, Santa Cruz, Calif, 1979). Proceedings of Symposia in Pure Mathematics Vol. 37*, pp. 195–196.

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