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MODELLING THE DEPENDENCE STRUCTURE BETWEEN BITCOIN AND MAJOR CURRENCIES USING COPULA DYNAMIC DURING EXTREME PERIOD

(Permodelan Struktur Pergantungan antara Bitcoin dan Mata Wang Utama Dunia Menggunakan Copula Dinamik Semasa Tempoh Ekstrem)

GOH PEI SHAN, R. NUR-FIRYAL* & SAIFUL IZZUAN HUSSAIN

ABSTRACT

This article studies the bivariate dependence structure for six pairs of daily returns Bitcoin with Euro (EURO), Pound sterling (GBP), Japan yen (JPY), Canadian dollar (CAD), Australian dollar (AUD) and Chinese renminbi (CNY) from 1 January 2017 until 31 December 2019. A time-varying copula approach is employed to explore the dependence between Bitcoin and major currencies during extreme period. The Autoregressive-Generalized Autoregressive Conditional Heteroscedastic-t (AR-GARCH-t) model is applied to estimate the marginal distributions whereas Gaussian and Symmetric Joe-Clayton (SJC) copula models are used to analyse the joint distributions. The results showed there is no tail dependence for all pairs. Overall, Bitcoin has a strong dependence with GBP and lowest dependence with CNY within the pairs using time-varying Gaussian. However, the overall time-varying dependency are very low, which indicates that Bitcoin can acts as a hedge asset against the risk of currency market. Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) have been employed in selection of the best model and conclude that time-varying Gaussian copula is the most appropriate method in measuring the dependence between the currencies.

Keywords: Bitcoin; foreign exchange market; Gaussian copula; Symmetric Joe-Clayton copula

ABSTRAK

Artikel ini mengkaji struktur pergantungan bivariat untuk enam pasangan pulangan harian Bitcoin dengan Euro (EURO), Pound Sterling (GBP), yen Jepun (JPY), dolar Kanada (CAD), dolar Australia (AUD) dan renminbi China (CNY) daripada 1 Januari 2017 hingga 12 Disember 2019. Pendekatan kopula yang berbeza masa digunakan untuk memahami kebergantungan antara Bitcoin dan mata wang utama dunia semasa tempoh ekstrem. Model *Autoregressive-Generalized Autoregressive Conditional Heeroscekastic-t* (AR-GARCH-t) digunakan untuk menganggar taburan marginal manakala model kopula Gaussian dan Simetri Joe-Clayton (SJC) digunakan untuk menganalisis taburan bersama. Keputusan menunjukkan tiada kebergantungan ekor untuk semua pasangan. Secara keseluruhan, Bitcoin mempunyai kebergantungan yang kuat dengan GBP dan kebergantungan paling rendah dengan CNY dalam pasangan menggunakan Gaussian yang berbeza masa. Walau bagaimanapun, kebergantungan keseluruhan masa yang berbeza adalah sangan rendah, menunjukkan bahawa Bitcoin boleh bertindak sebagai asset lindung nilai terhadap risiko pasaran mata wang semasa tempoh ekstrim. Kriteria Maklumat Akaike (AIC) dan Kriteria Maklumat Bayesian (BIC) telah digunakan dalam pemilihan model terbaik dan membuat kesimpulan bahawa kopula Gaussian yang berubah masa adalah kaedah yang paling sesuai dalam mengukur pergantungan antara mata wang.

Kata kunci: Bitcoin; pertukaran mata wang asing; kopula Gaussian; kopula Simetri Joe-Clayton

1. Introduction

Cryptocurrency market received significant attention and has grown rapidly to become a new type of investment assets. One of the features that makes cryptocurrency different that other markets is it decentralized nature of blockchain makes cryptocurrency is a self-governing system (Ismail *et al.* 2020). A cryptocurrency acts as a medium of exchange because it is used to protect the transactions when dealing with the other unit of currencies. Bitcoin is the most well-known and most valuable cryptocurrency in the world since it is the first decentralized cryptocurrency created in 2009. Other cryptocurrencies like Litecoin, Ethereum, Tether, Tron and so on have been created after Bitcoin.

Eventhough Bitcoin is traded in US dollar, it executted as an alternative payment system which is not controlled by the authorities. Bitcoin is the first decentralized currency system that works worldwide and depends on digital signatures, cryptographic proofs of work, and peer-to-peer networking to contribute a distributed ledger involving transactions known as blockchain. Bhatia *et al.* (2020) has explained the process and function of blockchain where blockchain can record transactions and provide independent verification using digital recording. Decentralized blockchains ensure high uptime, immutability for future auditing, and transaction speed variability due to varying velocities (Hughes *et al.* 2019). The percentage of total market capitalization of the dominance cryptocurrencies from 1st January 2017 until 31st December 2019 shown in Figure 1 prove that Bitcoin is the most valuable cryptocurrency. It is inevitable to say that the value of Bitcoin is very valuable because rising out of expectation at the end of year 2020 until 2021 and achieved US $28,2994.01 on 1st January 2021 (CoinMarketCap).

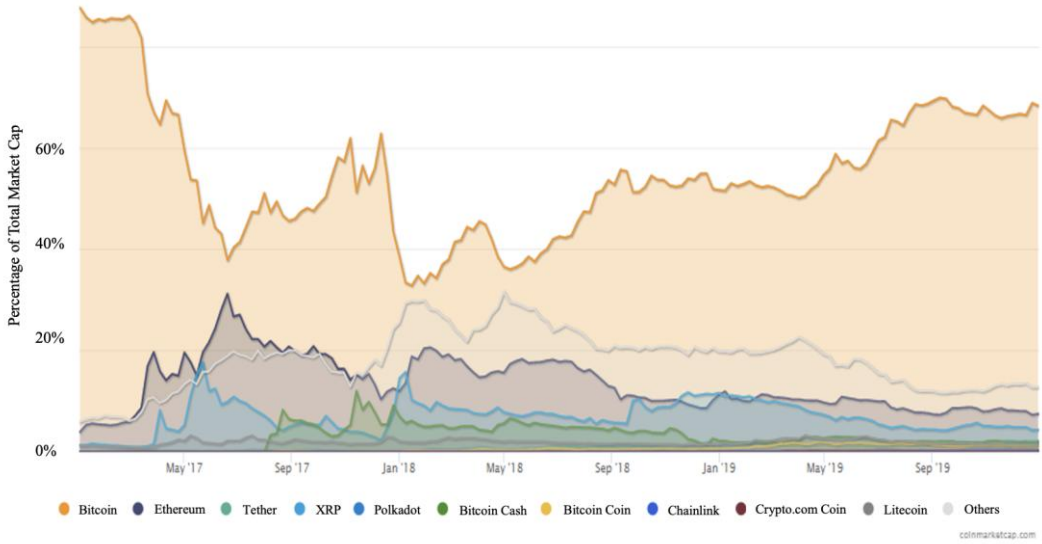


Figure 1: Percentage of total market capitalization of cryptocurrencies

The biggest currency market in this world is foreign exchange market that include several major currencies 6 of these are chosen in this research, which are Euro (EURO), Pound sterling (GBP), Canadian dollar (CAD), Japanese yen (JPY), Australian dollar (AUD) and Chinese renminbi (CNY). All these currencies have their specialised characteristics and are able to affect the value and price of other currencies. Bitcoin are traded in US dollars (USD) which is the top traded currency globally issued by the United States are the main reason we did not include US dollars as one of the currency. The Special Drawing Rights (SDR) created by International

Monetary Fund (IMF) includes USD, EURO, GBP, JPY and CNY (Trumen 2022). All these include CAD and AUD are the most traded currency in the world.

Many research studies the dependence or relationship between Bitcoin and assets categories like bonds, stocks and currencies (Bouri *et al.* 2018; Hussain & Li 2018a, Pho *et al.* 2021; Tiwari *et al.* 2019). The copula-GARCH approach has been employed to test the dependence structure because of its flexibility which simplify the process. One of the advantages is the transformation of random variable would not cause changes on copula model and it allows the multivariate distribution function to be decomposed into marginal distribution function and dependence function (Papaefthymiou *et al.* 2008).

Osterrieder *et al.* (2016) studied tail dependence between cryptocurrencies such as Bitcoin, MaidSafeCoin, LiteCoin, Dash, Monero, and Ripple using Gaussian copula and found strong non-normal characteristics with Bitcoin and Litecoin being highly correlated, while Ripple had the lowest correlation with other cryptocurrencies. Boako *et al*. (2019) studied vine copula-based dependence on Ethereum and Bitcoin, finding strong dependence and recommending Ethereum as the optimal choice for portfolio investors. Bouri *et al.* (2018) analyze the daily dependence of global financial stress on Bitcoin returns and found low global financial stress index (GFSI). In order to assess the risk related to extreme events, it is essential to measure tail dependence which is significant to analyze potential asymmetric model (Shamiri *et al.* 2011)

Tiwari *et al.* (2019) applied the copula-ADCC-EGARCH model on S&P 500 index and six cryptocurrencies (Ripple, Stellar, Ethereum, Litecoin, Dash and Bitcoin) and found that the overall time-varying dependencies are very low and the most effective hedge asset against risk of the S&P 500 index is Litecoin. Guesmi *et al.* (2019) found that ARMA(1,1)–DCC–GJR–GARCH model best describes Bitcoin's joint dynamics with other financial assets and short position of Bitcoin combine with gold, oil and equities reduced the portfolio's risk. Bouri *et al.* (2018) studied the effects of gold prices and aggregate commodity index on Bitcoin's using advanced autoregressive distributed lag (ARDL) models and found that gold prices and aggregate commodity index will affect Bitcoin price movements. Xu *et al.* (2019) found that the overall correlation generally fluctuare when large amount of liquidity begins to flow into commodity market and often associated with geopolitical and economic events that affects the structure between oil and exchange rate.

Kjærland *et al.* (2018) studied the relationship between technological factor Hashrate and Bitcoin price using ARDL and GARCH approaches and show that S&P 500 and Google searches' returns influence Bitcoin price prove that Hashrate acts as a crucial independent variable. Al-Yahyaee *et al.* (2018) compared Bitcoin with stock, currency, and gold using multifractal detrended fluctuation analysis (MF-DFA) and found that Bitcoin is the most inefficient market among these markets.

Szetela *et al.* (2016) employed ARMA and GARCH models to model the conditional mean and variance shows that there is no relations between Bitcoin and certain global currencies. Hussain and Li (2018b) found low dependence between the Chinese stock market and developed markets. The strongest dependence was found between China and Australia, while the weakest was between Chinese and US stock markets based on static and time-varying copula. Hu (2010) applied a time-varying conditional copula approach to study the dependence structures of Chinese and US stock markets on other financial markets and found lower tail dependence is weaker in some periods and tail dependence is higher in US compared to China. Mensah and Premaratne (2014) examined trends and tail dependence in Asian banking sector indices from 2000 to 2012 found high volatility persistence in all twelve markets with time-varying copulas are the most suitable method for modeling the dependence structure.

Qiao *et al.* (2011) used a flexible nonlinear multivariate Markov-switching-VAR model (MS-VAR) to study US, New Zealand, and Australian stock market relationships and found higher correlations in bear regimes compared to bull regimes, highlighting the importance of

correlation in measuring stock dependence. Cajueiro and Tabak (2008) studied long-range dependence in equity returns and volatility of 41 indices using R/S, V/S, and Wald statistic and found no significant long-range dependence, with strong volatility memory and weak mean returns, and higher volatility dependence in developed economies.

Bitcoin prices influence macroeconomics factor such as money supply, consumer price index (CPI) and economic policy uncertainty (EPU) in both the short and long run, subsequently impact the foreign exchange rate (Wang *et al.* 2023). Zhou (2021) found that Bitcoin functions as a hedge asset during low uncertaininty and as a safe haven during high economic uncertainty but it is ineffective as either hedge or safe haven asset during stock market crashes. However, studies also shows different result based on different countries such as safe haven for Venezuela, diversifier for Japan and China and weak hedge for Sweden and Estoni which report that Bitcoin might work differently for different countries (Kliber *et al.* 2019).

This research examines the dependency of Bitcoin and other major currencies using the copula approach. Cryptocurrencies especially Bitcoin become more valuable, while the other major currencies play an important role in developing the global economies. Dependence is crucial in economics and mathematics as it affects portfolio diversification. In order to understand the relationship, copula approach are becoming more attractive for investors to understand the dependence structure of the market. Most research focuses on tail dependence on financial asset returns, using data such as exchange rates, market indexes, cryptocurrencies, bond yields, and commodities market. This research measures the dependence between Bitcoin and cryptocurrencies using the copula approach, focusing on their financial aspects. Bitcoin's growing popularity and global economy role make it a fascinating topic to explore.

Numerous measures and models investigate stock market dependence, but many have limitations due to distribution characteristics and non-linear data. This research aims to model the dependence between Bitcoin and six major currencies using dynamic copula. While research on Bitcoin and stock market dependence is extensive, the sharp increase in Bitcoin market values in November 2019 suggests it may have an impact on the economy. The copula approach models currency dependence using marginal distributions allows more accurate bivariate information.

The period chosen for this study, from January 2017 to December 2019, is particularly significant as it covers a period of extreme volatility and rapid development in the cryptocurrency market. Bitcoin, the best-known cryptocurrency, experienced several critical fluctuations during this period, driven by both market speculation and increasing adoption. In particular, the value of Bitcoin rose to an all-time high in December 2017, followed by a significant correction throughout 2018, before stabilising in 2019. This period is representative of the increased instability in the cryptocurrency market, making it an ideal time window to examine the dynamic relationship between Bitcoin and major global currencies. Bitcoin's extreme behaviour during this period, characterised by its price volatility, allows for a robust analysis of the potential dependency between Bitcoin and traditional currencies. This analysis is crucial for investors and economists who want to understand how Bitcoin, a decentralised and highly speculative asset, interacts with established financial systems under conditions of market stress (Bouri *et al.* 2018). By focusing on this specific time period, the study is able to gain important insights into whether Bitcoin can act as a hedge against currency market risks or whether it is influenced by similar market forces.

In summary, this study aims to model the dynamic dependence between Bitcoin and six major currencies (EUR, GBP, JPY, CAD, AUD and CNY) during a period of extreme market conditions, using the time-varying copula approach. The novelty of this study lies in the use of both the Gaussian copula and the Symmetrised Joe-Clayton (SJC) copula to capture not only the general dependence but also the tail dependencies that are often overlooked in traditional

financial analyses. By applying this advanced statistical method, the study offers new insights into how Bitcoin interacts with established currencies, especially in times of market volatility. The remainder of this paper is organised as follows: Section 2 discusses the methodology used, including model specifications and estimation techniques. Section 3 presents the empirical results, including the descriptive statistics and the copula-based dependence results. Finally, Section 4 contains the conclusions, summarising the main findings and discussing the implications for portfolio diversification and financial risk management.

2. Methodology

This section will discuss the measure employed to model the dynamic dependence between Bitcoin and other major currencies using Gaussian Copula and Symmetric-Clayton Copula (SJC). Gaussian Copula will be able to explore absolute dependence and SJC is use to model both lower and upper tail dependencies. Both of the methods will be use to outline the dependence structures for financial returns.

The returns are calculated using geometric series since the daily returns are continuous. The formula below is used:

$$r_{i,t} = ln(\frac{P_{i,t}}{P_{i,t-1}}) \tag{1}$$

where $P_{i,t}$ denotes the currency prices, i denotes the currency, and t denotes the days.

2.1. *Marginal distribution estimation*

Before computing a bivariate copula model, a model for each of the marginal distributions for every currency must be specified following an AR(p)–GARCH(1,1)–skew t process. The model is proposed by Bollerslev (1987) is used.

$$x_{i,t} = \alpha_i + \sum_{j=1}^{p} \beta_j \, x_{i,t-j} + \varepsilon_{i,t} \quad \text{for} \quad i = 1,2 \tag{2}$$

$$\sqrt{\frac{v}{\sigma_{i,t}(v-2)}} . \varepsilon_{i,t} | I_{t-1} \sim t(v) \tag{3}$$

$$\sigma_{i,t}^2 = a_i + b_i + \sigma_{i,t-1} + c_i \varepsilon_{i,t-1}^2 \tag{4}$$

where $x_{i,t}$ indicates univariate currencies' return series, $\varepsilon_{i,t}$ represents error term in conditional mean equation, v represents the degree of freedom of Student's t distribution, $\sigma_{i,t}^2$ is the variance, I_{t-1} is the information set at time *t-1*.The standardized residuals follow Student's t-distribution.

2.2. *Copula functions*

A copula function is a multivariate cumulative distribution function in which the marginal distributions is a uniform distribution with interval [0,1]. Sklar's theorem contributes to the link between a joint distribution and the corresponding copula (Vogiatzoglou 2016) where the joint

distribution is separated into its univariate marginal distributions and a copula function. Therefore, dependence structure of copula between two variables, variables X_1 and X_2 is defined as:

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)) \tag{5}$$

The copula defined in (2) is unique if all marginal distributions are continuous. From (2) one can achieve the copula according to the following formula:

$$C(u_1, u_2) = C(F_1^{-1}(u_1), F_2^{-1}(u_2)) \tag{6}$$

where $u_i = F_i(x_i)$, *i=1,2.*

The general copulas usually used in finance and economics are Gaussian copula, Clayton copula, Symmetrized Joe-Clayton (SJC) copula and student-T copula.

2.3. *Gaussian copula*

Since the cumulative density functions of the standardized residuals are from the marginal models, the below copula function defines the dependence structure between the margins (Mensah & Premaratne 2014)

$$C(u, v) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} exp\{-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)} dxdy\} \tag{7}$$

$$= \Phi_p(\Phi^{-1}(u), \Phi^{-1}(v)) \ , -1 \leq \rho \leq 1 \tag{8}$$

where Φ is standard normal for the univariate cumulative distribution; Φ^{-1} represents the inverse of the standard normal; $\Phi_p$is standard normal for the bivariate cumulative distribution; ρ is the correlation coefficient between two variables.

The time-varying Gaussian copula parameters is modelled as follow (Patton Andrew 2006):

$$\rho_t = \lambda(w + \beta\rho_{t-1} + \alpha\frac{1}{10}\sum_{j=1}^{10}[\Phi^{-1}(u_{t-j}), \Phi^{-1}(v_{t-j})] \tag{9}$$

where $\lambda = \frac{1-e^{-x}}{1+e^{-x}}$ is a modified logistic transformation to ensure ρ_t is within (-1,1). From Eq. (9), copula dependence parameters assume to follow ARMA (1,10) process and $\beta\rho_{t-1}$ corresponds to the persistence effect while the last term $\alpha\frac{1}{10}\sum_{j=1}^{10}[\Phi^{-1}(u_{t-j}), \Phi^{-1}(v_{t-j})]$ describe the variation effect of dependence.

2.4. *Symmetrised Joe-Clayton copula (SJC copula)*

The symmetrized Joe-Clayton (SJC) copula is modified from the original Joe-Clayton (JC) copula (Hu 2010). The JC copula is given by

$$C_{JC}(u,v|\tau^u,\tau^v) = 1-\left\{1-\left([1-(1-u)^k]^{-\gamma}+[1-v^k]^{-\gamma}-1^{-\frac{1}{\gamma}}\right)\right\}^{1/k} \quad (10)$$

where $k=\frac{1}{log_2(2-\tau^u)}, \gamma=-\frac{1}{log_2(\tau^L)}$ and $\tau^u\epsilon(0,1), \tau^L\epsilon(0,1)$.

The JC copula has certain attributes, which captures upper tail (τ^u) and lower tail (τ^L) dependence. The upper tail dependence could be written as below:

$$\tau^U = \lim_{\varepsilon\to 1} Pr[U>\varepsilon|V>\varepsilon] = \lim_{\varepsilon\to 1} Pr[U>\varepsilon|V>\varepsilon] = \lim_{\varepsilon\to 1}\frac{(1-2\varepsilon+C(\varepsilon,\varepsilon))}{1-\epsilon} \quad (11)$$

The copula C will show upper tail dependence as long as the above limit exists and $\tau^U\epsilon(0,1)$ and $\tau^u=0$ is used if no upper tail dependence. At the same time, the lower tail dependence is exhibited as

$$\tau^L = \lim_{\varepsilon\to 0} Pr[U\le\varepsilon|V\le\varepsilon] = \lim_{\varepsilon\to 0} Pr[U\le\varepsilon|V\le\varepsilon] = \lim_{\varepsilon\to 0}\frac{C(\varepsilon,\varepsilon)}{\epsilon} \quad (12)$$

In case of the above limit exists, then the copula C will show lower tail dependence and $\tau^L\epsilon(0,1)$ and $\tau^L=0$ is used if no lower tail dependence.

However, when both of the tail dependence is the same, $\tau^U=\tau^L$, the functional form still emphasizes degree of asymmetry. Patton Andrew (2006) introduced the Symmetrized Joe Clayton (SJC), so that the tail dependence is employed to determine the existence of asymmetry. According to him, the SJC is expressed as following

$$C_{SJC}(u,v|\tau^U,\tau^L) = 0.5\left[C_{JC}(u,v|\tau^U,\tau^L)+C_{JC}(1-u,1-v|\tau^U,\tau^L)+u+v-1\right] \quad (13)$$

The SJC becomes symmetric only when $\tau^U=\tau^L$ but the symmetric dependence on the variables are not forced by its specification. The parameters of upper and lower tail dependence for SJC are described as follow:

$$\tau_t^U = \delta(\omega U+\beta_u\tau_{t-1}^U+\alpha_U\frac{1}{10}\sum_{j=1}^{10}|u_{t-j}-v_{t-j}|) \quad (14)$$

$$\tau_t^L = \delta(\omega U+\beta_L\tau_{t-1}^L+\alpha_L\frac{1}{10}\sum_{j=1}^{10}|u_{t-j}-v_{t-j}|) \quad (15)$$

where $\delta=\frac{1}{1+e^{-x}}$ is the logistic transformation which retains the parameters (τ^U,τ^L) within (0,1).

The time-varying parameter has two type of parameters which are upper and lower tail dependence. These dynamics of τ_t^U and τ_t^L followed an ARMA(1,10) process while the autoregressive terms, $\beta_u\tau_{t-1}^U$ and $\beta_L\tau_{t-1}^L$ captures persistence whereas a forcing variables, $\alpha_U\frac{1}{10}\sum_{j=1}^{10}|u_{t-j}-v_{t-j}|$, captures variation in the dependence.

2.5. *Parameter estimation and testing*

Financial returns are usually non-Gaussian distributed and the serial correlation and heteroscedasticity has been modelled in each marginal distribution using AR(p)–GARCH(1,1)–t model. The maximum likelihood estimation (MLE) is adopted to estimate copula dependence parameters after estimating the marginal distributions. Since there are many parameters to estimate and the distribution can be very complex, Inference for the Margins (IFM) method has been chosen as estimation method. According to Heinen and Valdesogo (2012), IFM method is employed to estimate parameters of copula because it is a general method that had been applied in other copula research although it will cause loss of efficiency. IFM is similar to MLE of the dependence parameter given the estimated marginal distributions proposed by Joe and Xu (1996) and Shih and Louis (1995). The two steps in IFM are as follows:

Step 1: The parameters of the marginal distribution is estimated by using MLE:

$$\theta_{m,i} = argmax \sum_{i=1}^{T} logf_i(x_{i,t}|x_i^{t-1}; \theta_{m,i}) \tag{16}$$

where parameters from the first step are collected in a vector $\theta_m = (\theta_{m,i}, \ldots\ldots, \theta_{m,n})$.

Step 2: The estimation of copula parameters is shown as below:

$$\theta_c = argmax_{\theta_c} L_c(X; \theta_m, \theta_c) \tag{17}$$

2.6. *Selecting the best model*

After estimating parameters using two different copula functions, the most adequate model for the research would be selected using Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The best model will be chosen based on the lowest value which represents less information lost. The AIC formula is:

$$AIC = 2k - 2log(L) \tag{18}$$

where k is the number of parameters estimated by the model and L is maximum likelihood estimate of the model. The BIC formula is:

$$BIC = klog(n) - 2log(L) \tag{19}$$

where n is the total number of daily returns while k is the number of parameters estimated by the model.

3. Results

3.1. *Descriptive statistics*

The daily closing prices of currencies from 1st January 2017 to 31st December 2019 are collected. The relative daily price is described as the price return by using the starting price for each currency. Because the daily relative price of Bitcoin and major currencies have a huge

different, the graphs are separated into two figures (Figure 2 and Figure 3) to observe it more precisely. The vertical dotted lines are drawn to observe the overall trend from year to year.

In order to compare the performance of all the currencies, relative price have been defined as a standard index that will be used for every financial market with the true index. Figure 2 shows the relative price of Bitcoin gradually increased until it reached the peak of 18.5755 on 12 December 2017 throughout the three years. It fluctuated in 2018 and undergoes recovery at the middle year of 2019. Obviously, the values move drastically throughout these three years.

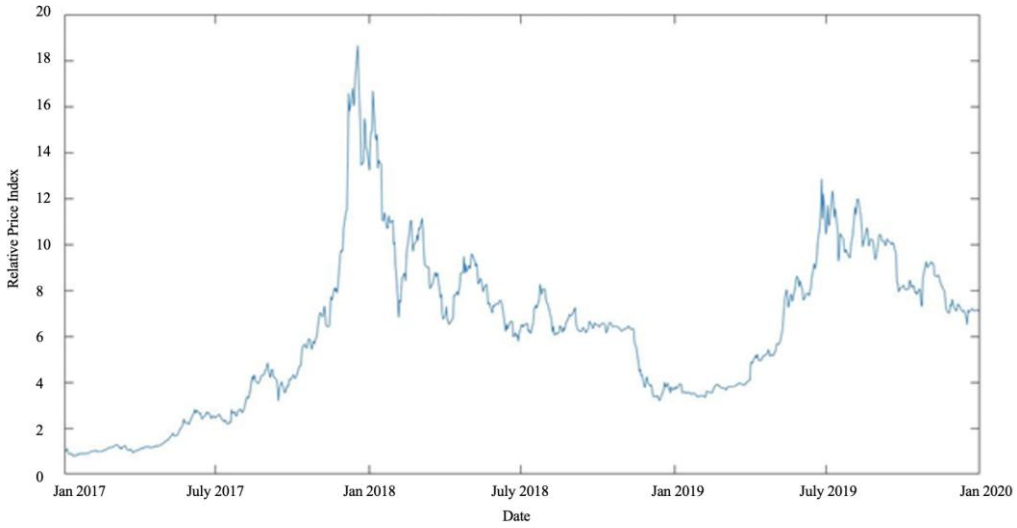


Figure 2: Daily relative price of bitcoin

Figure 3 shows that almost all the currencies have similar trend during the three years. They reached the peak during January 2018 and July 2018 with EURO has the highest peak followed by GBP, AUD, JPY, CNY and CAD. However, in 2019, JPY outperformed the other currencies while AUD has the lowest daily relative price in 2019 although it had the highest daily relative price at the beginning of 2017. Overall, the relative price of major currencies is higher on 31 December 2019 compared to 1 January 2017 except for CNY and AUD.

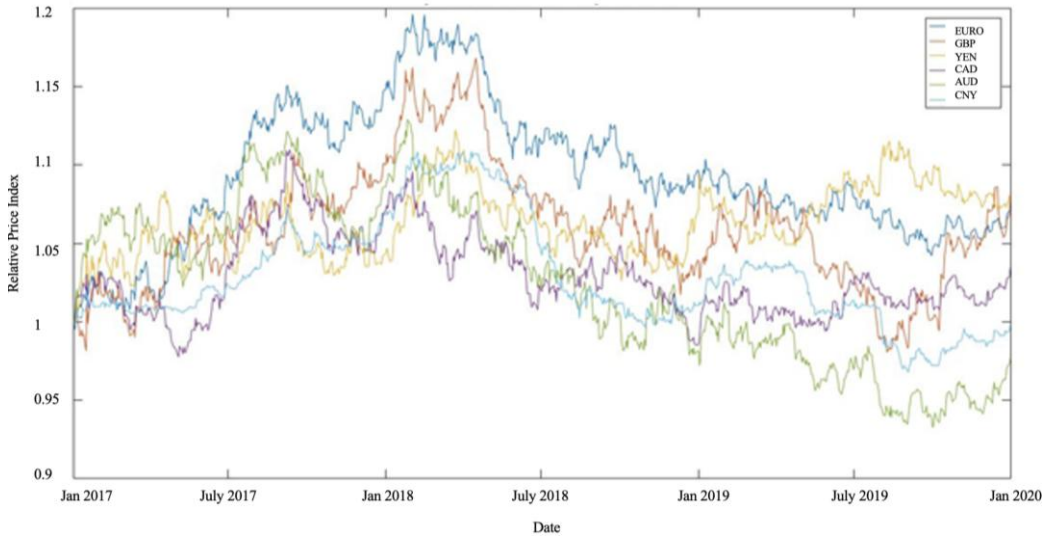


Figure 3: Daily relative price of major currencies

The difference in daily relative price between Bitcoin and major currencies is extremely large. The range of daily relative price of major currencies is around the range of 1.2 but for Bitcoin's range goes to maximum of 18.5755. This may because of price of Bitcoin is rising at a high rate compared to the major currencies. However, the trend of relative price for both Bitcoin and major currencies are almost the same, that is, the price increased in 2017, followed by decreased in 2018 with a slight different trend in 2019. The relative price trends of major currencies are uneven where JPY, EURO, CAD have similar trend to Bitcoin but others currencies such as GBP, CNY, AUD are inversed to Bitcoin.

Table 1 summarize the return series for each currency and reveal that all the daily average returns of the currencies are positive over the sample period except for AUD and CNY. All the currencies daily average are lower than Bitcoin. However, Bitcoin is more volatile with standard deviation of 5.0322% while all the currencies standard deviation are less than 0.5251%. GBP is the most volatile major currency with standard deviation of 0.5251% while CNY is the less volatile major currency with standard deviation of 0.2576%. CNY has the highest value of kurtosis meaning that it has the highest frequencies of extreme returns. Bitcoin, AUD and CNY have the negative skewness which means that the long tail is shifted to the left and EURO, GBP, JPY and CAD have the positive skewness which means that the long tail of the returns is shifted to the right. Based on the Jarque-Bera test (JB Stat) in Table 1, all the market returns are not normal. Theoretically, the higher the standard deviation, the higher the mean returns. So, Bitcoin has the highest mean returns and standard deviation while CNY is vice-versa because both of the mean returns and standard deviations are the lowest.

Table 1: Descriptive stastistics

	BTC	EURO	GBP	JPY	CAD	AUD	CNY
Mean	0.2514	0.0089	0.0099	0.0090	0.0044	-0.0030	-0.0003
Median	0.3811	0.0092	0.0000	0.0000	-0.0133	0.0000	0.0000
Standard Deviation	5.0322	0.4100	0.5251	0.4518	0.4059	0.4857	0.2576
Kurtosis	3.0605	0.7866	2.2728	1.1474	0.9920	0.7915	3.6386
Skewness	-0.0550	0.0676	0.5147	0.1476	0.1980	-0.0082	-0.2276
Range	48.4072	3.2950	4.6921	3.7019	3.3860	3.7922	2.7163
Minimum	-24.6852	-1.9007	-1.7076	-1.6503	-1.7234	-1.8275	-1.6090
Maximum	23.7220	1.3943	2.9846	2.0516	1.6627	1.9647	1.1073
JB Stat	298.6344	20.0010	198.4079	44.3774	36.0689	19.6658	428.6037
Count	778	778	778	778	778	778	778

Table 2 displays the correlation coefficients between the currencies' return series. Bitcoin has the highest correlation with GBP and lowest correlation with CNY which are 0.0575 and -0.0565 respectively. Hence, Bitcoin has positive relationship with GBP and negative relationship with CNY. The highest correlation between two major currencies belongs to CAD and AUD the lowest correlation between two major currencies are AUD and JPY. Although the existence of weakest correlation is beneficial in the diversification portfolio, the dependence between Bitcoin and major currencies should be analysed using copula model since upper and lower tail information employed in copula can provide more detail information.

Table 2: Linear correlation table

	BTC	EURO	GBP	JPY	CAD	AUD	CNY
BTC	1						
EURO	0.0315	1					
GBP	0.0575	0.5620	1				
JPY	0.0200	-0.0293	0.0181	1			
CAD	0.0095	0.4258	0.3262	-0.0239	1		
AUD	0.0029	0.5491	0.3746	-0.0277	0.6073	1	
CNY	-0.0565	0.2848	0.2285	0.0392	0.2513	0.3356	1

3.2. *Empirical analysis and results*

3.2.1. *Marginal models*

The estimated parameter and t-values to model the conditional mean using ARMA(1,0) model, while the parameter estimate for the conditional variance using GARCH(1,1) model along with the parameter estimated for the standardized distribution from skewed-t model. Overall, there are 7 parameters of marginal models obtained for each currency consists of (μ, ϕ) for the conditional mean, (ω, α, β) for conditional variance and (ν, ξ) for distributional parameters.

Table 3 represents the estimated parameters for ARMA(1,0)–GARCH(1,1) model fitted to all the return series of Bitcoin and all currencies. Based on Bollerslev (1986), in order to assure that the GARCH(1,1) model is accurately specified, there are a few conditions about the inequality constraints that must be fulfilled, $\alpha, \beta, \omega \geq 0$, and the sum of the ARCH and GARCH coefficients $\alpha + \beta \leq 1$. All the parameters in the table shown satisfies these conditions. The volatility updating parameter α ranges between 0.0110 to 0.0981 while the autoregressive variance parameter β ranges between 0.7696 and 0.9862. Based on the high β values, the GARCH model captures high volatility persistence in all the currencies. The parameters, ν, for the innovations is significant for all currencies.

Table 3: Estimation parameter marginal model

	BTC	EURO	GBP	JPY	CAD	AUD	CNY
				Conditional Mean			
μ	0.2193	0.0054	0.0034	0.0050	0.0085	-0.0019	0.0068
	(0.1590)	(0.0130)	(0.0180)	(0.0150)	(0.0140)	(0.0170)	(0.0080)
φ	-0.0045	-0.0390	-0.0433	-0.0941	0.0014	-0.0179	-0.0868
	(0.0340)	(0.0380)	(0.0380)	(0.0340)	(0.0360)	(0.0350)	(0.0330)
				Condition Variance			
ω	0.3963	0.0000	0.0494	0.0015	0.0000	0.0015	0.0011
	(0.2680)	(0.0090)	(0.0450)	(0.0020)	(0.0000)	(0.0020)	(0.0010)
α	0.0981	0.0130	0.0482	0.0454	0.0130	0.0110	0.0871
	(0.0000)	(0.0670)	(0.0320)	(0.0180)	(0.0040)	(0.0050)	(0.0240)
β	0.9019	0.9858	0.7696	0.9470	0.9862	0.9821	0.9129
	(0.0000)	(0.1210)	(0.1840)	(0.0240)	(0.0040)	(0.0050)	(0.0270)
$\alpha + \beta$	1.0000	0.9988	0.8178	0.9924	0.9992	0.9931	1.0000
				Distributional Parameters			
ν	3.2395	14.4293	6.4774	8.0720	8.3547	12.7568	3.2420
	(0.3120)	(16.2110)	(1.3110)	(2.0700)	(3.2280)	(4.4070)	(0.3230)
ξ	-0.0361	0.0500	0.0823	0.1024	0.1092	-0.0334	0.0626
	(0.0420)	(0.0530)	(0.0550)	(0.0480)	(0.0430)	(0.0570)	(0.0390)
AIC	4535.2043	788.4964	1161.4934	906.3117	764.6863	1080.4692	-70.9779
BIC	4567.8014	821.0935	1194.0905	938.9088	797.2834	1113.0663	-38.3808
LL	-2260.6020	-387.2480	-573.7470	-446.1560	-375.3430	-533.2350	42.4890

3.2.2. *Copula models*

Both Gaussian and SJC copula have been employed in order to get the overview of dependency. Table 4 shows the parameter estimated and asympototic variance in parentheses for the copula approaches. α and β are the coefficients of the time-varying Gaussian copula. The parameter ω^U and ω^Lcorrespond to upper and lower dependence tails of the SJC copula with parameters α^U and β^U apply to the coefficient for the time-varying SJC copula at upper dependence tail and parameters α^L and β^L correspond to the lower dependence tail. The values of the optimum copula log-likelihood (*LL*), Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) are also displayed in the table.

Figure 4 represents the pairwise time-varying copula dependence of Bitcoin and major currencies from 1 January 2017 to 31 December 2019. The top table show results for the Gaussian copula which is general dependence while the middle and below table are the lower and upper tail dependence respectively, estimated from the time-varying of SJC copula. The evolution of the copula dependence for every pair are defined by the parameters α and β, α is smaller than β for all the pairs, meaning that persistent effect is higher than variation coefficients.

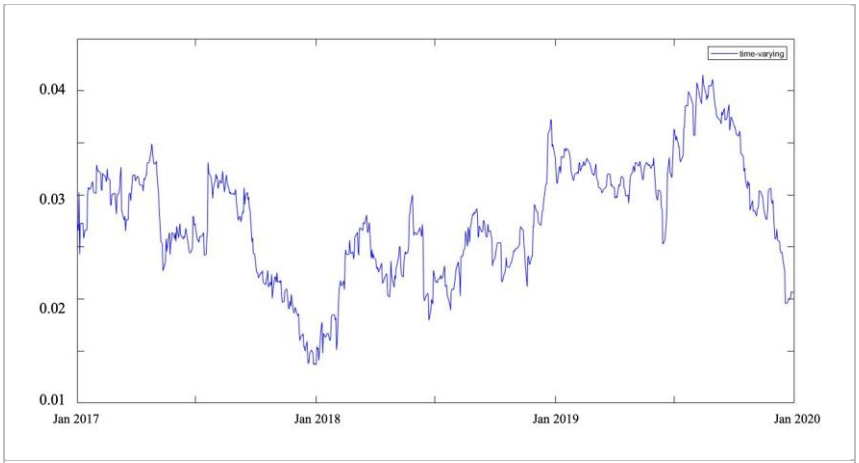
Table 4: Parameters time-varying for Gaussian and SJC copula

	BTC-EURO	BTC-GBP	BTC-JPY	BTC-CAD	BTC-AUD	BTC-CNY
			Time-varying Gaussian Copula			
α	0.0011	0.0000	0.0436	0.0010	0.0000	0.0000
	(0.0070)	(0.0000)	(0.0270)	(0.0070)	(0.0000)	(0.0000)
β	0.9820	0.8684	0.7183	0.9805	0.5581	0.8419
	(0.0140)	(0.5620)	(0.1940)	(0.0220)	(0.4730)	(0.4080)
AIC	3.3991	2.2932	0.1390	3.9525	3.9953	2.8049
BIC	12.7125	11.6066	9.4525	13.2659	13.3088	12.1183
LL	0.3000	0.8530	1.9300	0.0240	0.0020	0.5980
			Time-varying SJC			
ω^U	-9.7907	-9.6639	-9.4514	-9.3599	-9.5808	-9.7430
	(7.2990)	(8.82220)	(11.7080)	(7.0780)	(6.5120)	(8.8330)
α^U	0.4493	0.0374	-7.8870	0.8961	0.5543	0.3989
	(11.4060)	(8.7330)	(15.8240)	(59.8230)	(30.2140)	(11.7010)
β^U	9.9883	9.9869	0.1260	9.8721	9.9997	9.9759
	(1.0850)	(5.7670)	(0.5560)	(2.9140)	(0.0130)	(3.1710)
ω^L	-9.9996	-9.9998	-8.8632	-10.0000	-10.0000	-9.9923
	(4.0760)	(0.1320)	(4.1680)	(0.0000)	(0.0000)	(0.2700)
α^L	-8.97890	-8.5200	-6.9859	-7.1358	-8.5553	-8.7719
	(50.9890)	(55.1900)	(18.3820)	(65.4680)	(22.0120)	(20.6260)
β^L	9.2227	9.5917	3.7465	1.6439	6.4342	3.2394
	(23.7820)	(20.4440)	(7.1120)	(88.4560)	(2.9120)	(48.6320)
AIC	10.5888	11.0484	15.9590	13.6877	15.1818	21.0134
BIC	38.5291	38.9887	43.8993	41.6280	43.1222	48.9538
LL	0.7060	0.4760	-1.9790	-0.8440	-1.5910	-4.5070

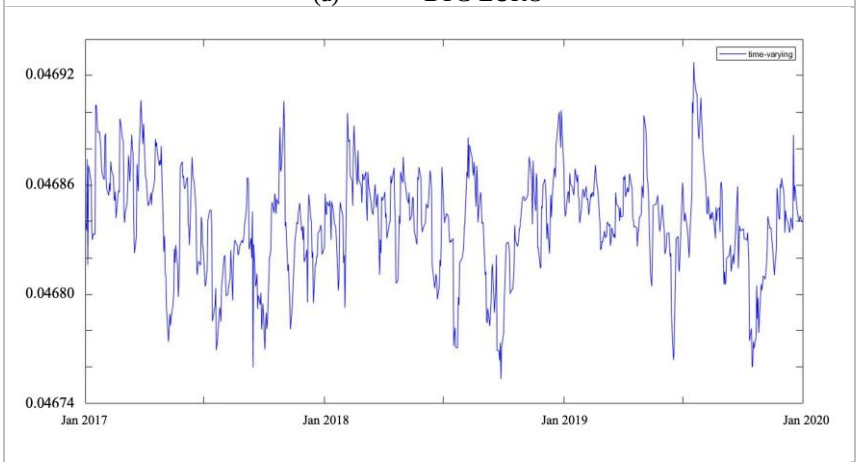
For BTC-EURO, the parameter ρ of time-varying Gaussian copula are between 0.01 and 0.04. the value of ρ decrease in 2017, start to rise in January 2018 and drop with relative price has a peak of 0.0415 around June 2019. From Figure 4(a), there is a sudden drop of ρ but rise again at the end of year 2019 but the return is lower than January 2017, which is the lowest dependence throughout the three years. For BTC-GBP, the value of dependence parameter, ρ revolve between 0.04 and 0.05 which makes the path of ρ very stable and the behaviour changes throughout the three years are the same based on Gussian copula diagram in Figure 4(b). The dependence between these market remains lower eventhough it has the strongest dependence pair compared to other pairs.

For BTC-JPY combination, the ρ are very volatile as it range between -0.24 and 0.24. Based on Figure 4(c), it peaked at 0.2330 in July 2019 and had the negative value of -0.2394 around January 2017. During the period with negative ρ, there is an opposite dependency between Bitcoin and Japanese yen and after June 2019, all the values are almost positive which means that the relative price of Bitcoin might change to dependency with Japanese yen. Figure 4(d), Gaussian copula diagram shows that for BTC-CAD, the ρ ranges between -0.01 and 0.01. in the last quarter of 2017, the value decreased and reached the lowest at -0.0099, but start the

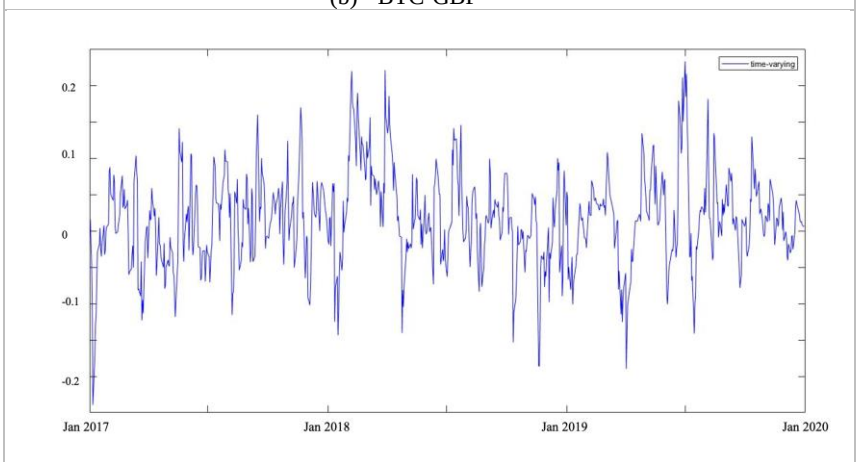
increase in 2018 when the Bitcoin relative price decrease. In 2019 all the values are positive even though the values are small compared to BTC-JPY.



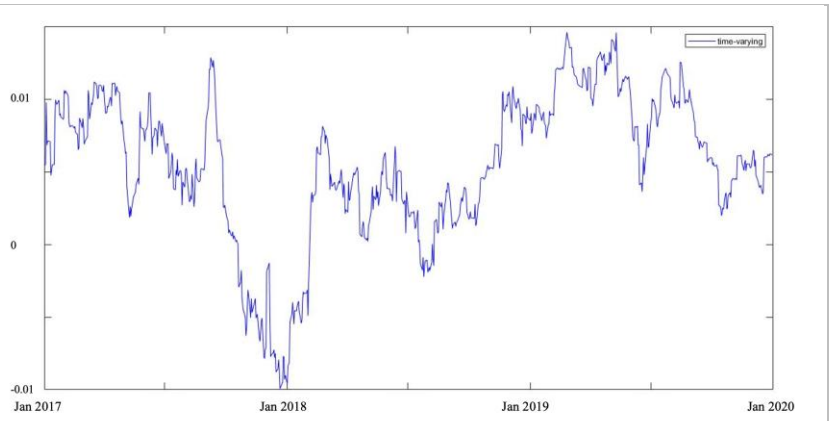
(a) BTC-EURO



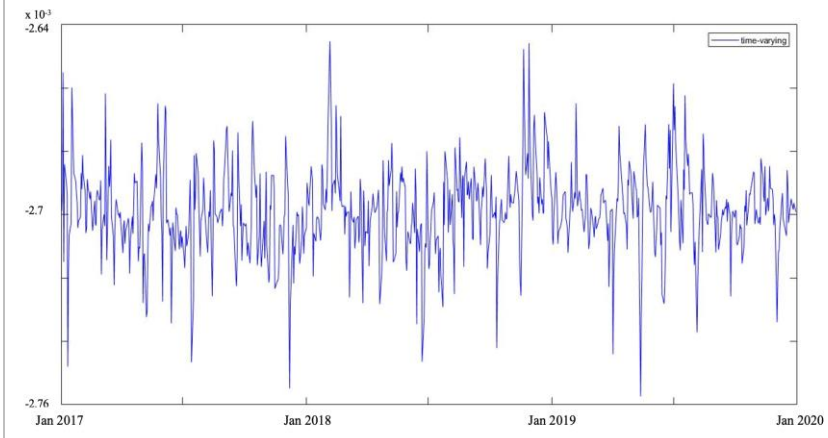
(b) BTC-GBP



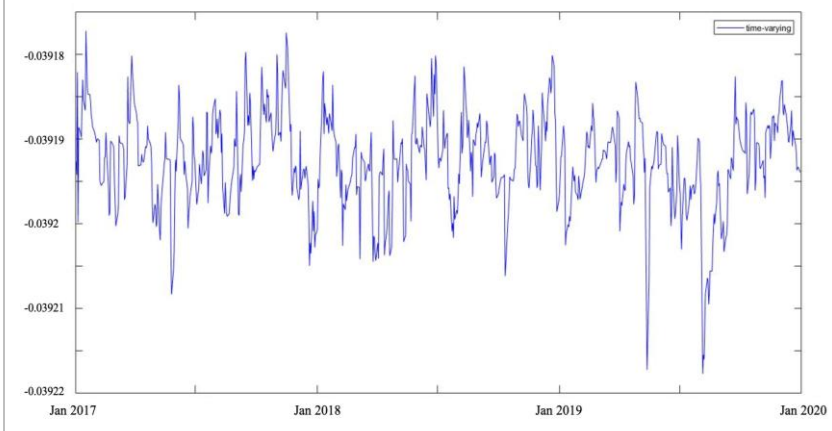
(c) BTC-JPY



(d) BTC-CAD



(e) BTC-AUD



(f) BTC-CNY

Figure 4 Dependence path for the time-varying Gaussian copula

Lastly, in Figure 4(e), for BTC-AUD, the values of ρ are negative between -0.0026 and -0.0028 similar to BTC-CNY in Figure 4(f),where the values of ρ fluctuate around -0.0392.

Hence, both AUD and CNY have negative correlation with Bitcoin. Since CNY is more negative than AUD, CNY has weaker dependence than AUD with Bitcion.

The results show GBP has the strongest dependence with Bitcoin even though the parameter values are low as it reached the highest value for dependence parameter and the most stable among all the pairs. AUD and CNY have the opposite dependence with Bitcoin since the values of ρ are negative. Based on Figure 4, the values of τ_t^U and τ_t^L for all of the pairs are very low, which shows that the dependence for both upper and lower tail dependence between both currencies is low. Overall, we can assume that there is no tail dependence between Bitcoin and the currencies. Thus, the overall dependence for six pairwise of currencies are ranked as: BTC-GBP > BTC-EURO > BTC - JPY > BTC-CAD > BTC-AUD > BTC-CNY and there is no tail dependence between Bitcoin and the currencies.

3.2.3. *Selection of the best model*

The values for the AIC and BIC are employed to select the most appropriate model. Table 4 shows the values of AIC for Gaussian copula are smaller with range around 4 compared to SJC copula with range around 10 for all pairs. Similar result from BIC where the range for Gaussian copula are around 15 and SJC copula are around 38. Hence, based on both AIC and BIC, the time-varying Gaussian copula is the best approach in modelling the dependence between the Bitcoin and major currencies.

4. Conclusion

Dependence structure is a crucial issue in finance and using only linear correlation is not an appropriate measurement because the data can be highly biased in non-Gaussian conditions and complex dynamic characteristics that exists in multivariate distributions. Copula function is one of useful and interesting tool to explore the dependence structure between the financial time series data as the distribution does not necessary assumed to be Gaussian distributed. The dependence parameter would change over time since there is still a lot of ongoing research related to time-varying conditional copula model.

This article examines the dependence structure of Bitcoin and major currencies including EURO, GBP, JPY, CAD, AUD and CNY. The marginal distribution for each series is modelled, then copula approach is use to model the dependence structure. The standard AR(1,0)–GARCH (1,1) models with a skewed-t distribution is applied to estimate the conditional marginal distributions. Two copula models, time-varying Gaussian copula and SJC copula are employed to model the overall dependence structure as well as the tail dependence and the time paths of dependence for each pairwise are shown and analysed.

In SJC copula, Bitcoin is independent from the impacts of all of the major currencies in both upper and lower tail. However, time-varying Gaussian Copula shows that Bitcoin has the strongest dependence with GBP followed by EURO, weaker dependence with JPY and CAD and totally independent with CNY and AUD. Based on the AIC and BIC values, time-varying Gaussian copula is the most appropriate method to model the dependence structure of Bitcoin and these major currencies. In conclusion, the overall time-varying dependencies are very low and Bitcoin can act as a hedge asset compared to the currency market with CNY is the most effective hedge asset against Bitcoin.

The significant benefit from this study is to shows that the asymmetry volatility between Bitcoin and currency market can provide better guidance for portfolio diversification to help portfolio investors. Second benefit from this study is Bitcoin can serve as a hedge asset against the risk of the currency market when constructing optimal portfolio. Therefore, Bitcoin might be included in portfolio diversification.

During the research period, there are many factors that might have an affect to this research such as using data from secondary data that may be not accurate with short period of data. Some solutions that we propose are data taken from more accurate platform such as Datastream and a longer data period to understand the changes of the dependence structure. Moreover, studies on other currencies especially on developing countries can help investors understand more about the dependence of Bitcoin on other countries and this might give more information related to other countries.

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Department of Mathematical Sciences
Faculty of Science and Technology
Universiti Kebangsaan Malaysia
43600 UKM Bangi
Selangor, MALAYSIA
E-mail: a163872@ukm.edu.my, nurfiryal@ukm.edu.my[*]*, sih@ukm.edu.my*

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[*]Corresponding author