

A PROBABILISTIC AGE-AND-GENDER SPECIFIC MALAYSIAN POPULATION PROJECTION: INTEGRATING MORTALITY, FERTILITY AND NET MIGRATION

*(Unjuran Populasi Malaysia Mengikut Umur dan Jantina Secara Kebarangkalian:
Pengintegrasian Kematian, Kelahiran dan Migrasi Bersih)*

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ABSTRACT

Population forecasting has become increasingly crucial for national planning, particularly in addressing demographic shifts including rising life expectancy, declining fertility rates and changing in migration flows. This study integrates three population components, including mortality, fertility and net-migration rates, into the probabilistic cohort component model in order to generate age-and-gender specific population forecasts. This study extends the existing projection methods by incorporating the Generalised Log Gamma model for fertility and the Heligman-Pollard model for mortality into the Lee-Carter model, along with the residual method for migration. Malaysian historical data, including populations, fertility rates and mortality rates by age group and gender spanning over 44 years from 1980 to 2023, were collected from the Department of Statistics Malaysia (DOSM), whereas net migration data were estimated from the demographic balance equation. These data were fitted into the proposed probabilistic cohort component method. Subsequently, the population projections were performed over the ten-year horizon. The results revealed that the Generalised Log-Gamma and the Heligman-Pollard models provide the best fit for Malaysia's fertility and mortality data, respectively, as indicated by a low sum of squared errors. In terms of forecast accuracy, the GLG-Lee-Carter model produced highly accurate fertility projections, while the HP-Lee-Carter model performed best for male mortality based on mean absolute percentage errors. The projected population pyramids for Malaysia indicate that substantial demographic shifts resulted from a growing proportion of older adults and a shrinking young population. These trends suggest critical implications for national policy, particularly concerning labour market supply and the sustainability of social protection and retirement systems.

Keywords: Malaysia population forecast; probabilistic cohort model; Lee-Carter; mortality model; fertility model; net migration

ABSTRAK

Unjuran populasi telah menjadi semakin penting untuk perancangan negara, terutamanya dalam menangani corak migrasi yang kompleks, di samping jangka hayat yang semakin meningkat dan kadar kesuburan yang semakin menurun. Kajian ini mengintegrasikan tiga komponen populasi utama iaitu kadar kematian, kesuburan dan migrasi bersih ke dalam model komponen kohort secara kebarangkalian bagi menghasilkan unjuran populasi mengikut umur dan jantina yang lebih kukuh. Kajian ini memperluas kaedah unjuran sedia ada dengan menggabungkan model Generalised Log-Gamma untuk kesuburan dan model Heligman-Pollard untuk kematian ke dalam model Lee-Carter, bersama kaedah residual bagi migrasi. Data sejarah Malaysia termasuk populasi, kadar kesuburan dan kadar kematian mengikut kumpulan umur dan jantina bagi tempoh 44 tahun dari 1980 hingga 2023 telah dikumpulkan daripada Jabatan Perangkaan Malaysia, manakala data migrasi bersih dianggarkan melalui persamaan imbalan demografi. Data-data ini dimuatkan ke dalam kaedah komponen kohort kebarangkalian yang dicadangkan,

seterusnya unjuran populasi dijalankan bagi jangka masa 10 tahun. Hasil kajian menunjukkan bahawa model Generalised Log-Gamma dan model Heligman-Pollard memberikan padanan terbaik bagi data kesuburan dan kematian Malaysia, masing-masing, seperti yang ditunjukkan oleh jumlah ralat kuasa dua yang rendah. Dari segi ketepatan unjuran, model GLG–Lee-Carter menghasilkan unjuran kesuburan yang sangat tepat, manakala model HP–Lee-Carter menunjukkan prestasi terbaik bagi kematian lelaki. Piramid populasi yang dijangka bagi Malaysia menunjukkan perubahan demografi yang ketara, hasil daripada peningkatan peratusan penduduk warga tua dan pengurangan populasi muda. Tren ini membawa implikasi kritikal terhadap dasar negara, khususnya berkaitan bekalan tenaga kerja dan kelestarian sistem perlindungan sosial serta persaraan.

Kata kunci: unjuran populasi Malaysia; model kohort kebarangkalian; Lee-Carter; model kematian; model kesuburan; migrasi bersih

1. Introduction

Age-and-gender-specific population forecasts are essential for government and policymakers in designing effective policies related to socioeconomics and social benefits, particularly in formulating the nation's development objectives. These projections are also used by insurance companies for strategic product development, decision-making, and targeted market analysis.

The most widely used method for population projections is the cohort-component model, a deterministic model that stratifies the population by age and gender. This model operates through a demographic accounting equation, whereby a population at a future time is equivalent to the current population plus births and net migration (immigrants minus emigrants) and subtracting deaths (Ševčíková *et al.* 2016). This deterministic cohort-component method has been employed in a few national official statistics department. However, this model has some limitations. One key disadvantage of deterministic cohort-component method is its inability to quantify uncertainty in future demographic trends, resulting in single-point estimates without confidence intervals. This limitation becomes more pronounced in long-term projections, where key demographic rates such as fertility, mortality, and migration are often random with some uncertainties in future values. In addressing the uncertainty of forecasts, deterministic methods often rely on expert judgement to predefine high, medium and low scenarios. Nonetheless, these ranges provide no clear justification, resulting in potential bias and under-predicted forecast values in certain low-mortality countries (Keilman 2019; Lee & Tuljapurkar 1994).

Acknowledging the limitations of the deterministic population projection method, there is a growing transition of population forecasting techniques from deterministic to stochastic. Stochastic approaches incorporate statistical models to measure the uncertainty of forecasts. These models treat fertility, mortality, and migration as stochastic processes with associated probability distributions, enabling the prediction of intervals and more robust forecasting. The probabilistic cohort component models have been used in several countries, including the United States (Lee & Tuljapurkar 1994), Norway (Keilman 2019), Australia (Hyndman & Booth 2008), the United Kingdom (Wright 2010), and South Korea (Kim & Kim 2020). These methods allow for more flexible assumptions regarding future changes in vital rates. For example, Hyndman and Booth (2008) proposed a probabilistic cohort component model that incorporates smoothed mortality rates, fertility rates and net migrants to forecast the Australian population. Moreover, Yu *et al.* (2023) demonstrated that probabilistic cohort component models produce narrower prediction intervals and more accurate subnational projections compared to deterministic models.

Despite experiencing significant demographic transitions, Malaysia's population projection remains deterministic. The official statistics primarily rely on the traditional cohort component method, which lacks integration with advanced modelling techniques capable of accounting for long-term uncertainty (Department of Statistics Malaysia 2024a). This creates a critical gap in evidence-based planning, particularly for sectors such as healthcare and labour market policy. To address this limitation, this study proposes a probabilistic cohort component population projection framework for Malaysia. This study adopts the model structure proposed by Hyndman and Booth (2008), with key modifications in the modelling of each demographic component. While Hyndman employed a functional data approach to generate smoothed single age rates for fertility, mortality, and migration, this study instead follows the Kim and Kim (2020) work that used component-specific statistical distributions, namely the Generalised Log-Gamma for fertility and the Heligman-Pollard model for mortality. The selected model is useful to better capture the unique characteristics of each demographic component. In contrast to Kim and Kim (2020), who focused solely on estimating the single-year population, this study incorporates the Lee-Carter model to forecasts both fertility and mortality rates over time. These forecast values are integrated into the cohort component projection model to generate multi-year, age-structured population forecasts.

Therefore, the aims of this research include as follows: (1) to model age- and gender-specific fertility and mortality rates using selected mathematical models including the Generalised Log Normal and the Heligman-Pollard to generate single age data from the five-year age group data, (2) to estimate the age- and gender-specific net migrations using the growth balance residual approach, (3) to perform ten-year forecasts for each component fertility and mortality using the Lee-Carter model and time series for net migrations, and (4) to incorporate all forecasted components into the cohort component framework subsequently construct Malaysia's projected population pyramids. The findings provide valuable insights into Malaysia's future demographic rates and offer refined population projection methods for data-driven national policies.

2. The Three Components of the Malaysian Population

Future population forecasts are influenced by the three population components: mortality, fertility and migration. Thus, understanding these three components is essential for making informed decisions in fields such as healthcare, social security and economic development. The following sections outline each population component's historical trends in Malaysia and discuss the corresponding modelling techniques. These components are then integrated into a probabilistic cohort-component framework for age and gender-specific population projection.

2.1. Mortality

Mortality is one of the crucial determinants for population dynamics, affecting the size and age structure of a population. Over the past decades, Malaysia has experienced a steady decline in mortality rates, which has contributed to significant improvements in life expectancy. According to the Department of Statistics Malaysia (2024b), the life expectancy at birth for the Malaysian population increased from 71.2 years in 1991 to 75.2 years in 2024, with female life expectancy consistently higher than that of males.

Declining mortality, coupled with a continued reduction in fertility, has accelerated Malaysia's demographic transition towards an ageing society. Malaysia will be classified as an ageing population when 15% of the nation's total population consists of those aged 60 years and above, which is projected to be realised by 2029 (Malaysia Population Research Hub n.d.).

As of 2024, statistics revealed that the proportion of older persons aged 60 in Malaysia reached 11.6% of the population, approaching closer to this ageing benchmark. In comparison to high-income countries, such as Japan, Germany, and Italy, which have already entered the super-aged societies, where more than 20% of the population is aged 65 and older (Population Reference Bureau 2020). Although Malaysia is still in the early stages of ageing, its rapid pace of demographic changes, combined with moderate economic readiness, highlights the urgency for proactive policy responses on social protection systems, healthcare, and elderly home care.

To analyse the age-specific mortality rates, the Heligman-Pollard (HP) model is one of the most widely used parametric approaches. It is particularly useful for transforming age-grouped mortality data into single-age estimates. The model comprises an eight-parameter model that decomposes mortality into three key phases: early childhood, accidental hump, and ageing-related mortality. It has been successfully applied in countries such as South Korea (Kim & Kim 2020), Malaysia (Ibrahim 2008) and Indonesia (Sari *et al.* 2025). A study from Edrus *et al.* (2021) compared two mortality rate smoothing approaches, including the parametric Heligman-Pollard model and the non-parametric P-spline method using Malaysian data. While the P-spline approach demonstrated a better fit with lower fitting errors, the Heligman-Pollard model was observed to more effectively capture the underlying mortality patterns of the population. Furthermore, the spline-based approach poses the risk of overfitting, especially when the smoothing parameters are not optimally selected. This could reduce predictive accuracy when applied to future or out-of-sample data.

To account for time-varying uncertainty in mortality forecasting, this study employs the Lee-Carter model, a widely adopted stochastic approach that could capture uncertainties and temporal changes. This model incorporates two factors, including an age-specific parameter and a time-varying mortality index. There were few modifications have been proposed in the literature in which some were proven to be more accurate than that of the original version such as functional data model (Hymdman & Ullah 2007), Poisson log bilinear model (Brouhns *et al.* 2002), and cohort adjustment (Cairns *et al.* 2006). Still, the original Lee-Carter model remains favoured due to its simplicity, interpretability, and proven robustness across diverse populations (Lee & Tuljapurkar 1994; Booth *et al.* 2006).

2.2. Fertility

Fertility also plays a vital role in shaping population growth and age structure. In Malaysia, the Total Fertility Rates (TFRs) have demonstrated a significant decline from 4.9 children per woman in 1970 to 1.6 children in 2022, falling below the replacement level of 2.1 (United Nations 2023). The decline in fertility is associated with advancements in healthcare, higher education among women, and access to birth control (Zulqarnain & Yusuf 2022). Moreover, the Age-specific Fertility Rates (ASFRs) of the Malaysian population exhibit a shift in the peak of the unimodal fertility curve toward older maternal ages, indicating a trend toward delayed childbearing and the postponement of first births (Shair *et al.* 2024a). These changes are expected to lead to a decline in the working-age population, thus putting pressure on the nation's economic support systems. Hence, to overcome this issue, the Malaysian government has implemented several initiatives to encourage childbearing among Malaysian women. This includes providing one-off cash assistance to women from low-income groups who give birth, aiming to reduce the financial burden of having children while encouraging a more balanced demographic structure (Bernama 2022). In addition, the Malaysian government is currently investing in reproductive health services, such as in-vitro fertilisation and has allocated funds for the construction of the National Subfertility Centre to address the decline in the country's total fertility rate (CodeBlue 2023).

Modelling ASFRs requires robust statistical approaches due to their typical unimodal and right-skewed distributions. ASFR represents the annual number of births among every 1,000 women within specific age groups, particularly those aged 15 to 49 years. Classical parametric models include the Hadwiger model and its extensions, such as the Gamma, Beta, Coal Trussel, Brass, and Gompertz models (Shang 2012; Gaire *et al.* 2022). In addition, a more flexible skew-normal distribution (Mazzuco & Scarpa 2015) and skew-logistic distribution (Shair *et al.* 2024b) have been proven to better reflect the fertility data with bimodal and unimodal ethnic specific fertility patterns. Kim and Kim (2020) employed the Generalised Log Gamma (GLG) model to model South Korea's fertility data, as the generalised property makes the model flexible enough to be applicable to diverse countries. These mathematical models work to transform five-year age group data into single-year age data for one particular year.

Consistent with its age-specific mortality forecasting, this study employs the Lee-Carter model to forecast Malaysian age-specific fertility rates. Lee (1993) employed the Lee-Carter model to forecast age-specific fertility rates. Hyndman and Ullah (2007) successfully demonstrated the model's extension for both mortality and fertility data projections. Shair *et al.* (2024b) applied both the original Lee-Carter model and its functional data variant to Malaysian fertility. Although the functional data model performed slightly better in capturing certain ethnic-specific trends, the research proved that the Lee-Carter model still provides a solid basis for modelling age-specific fertility in Malaysia.

2.3. Net migrations

Net international migration (the difference between immigration and emigration) is the third component in population projections. There were approximately three million migrants living in the country as of July 2023, representing 8.9% of the population (International Organization for Migration 2023). These migrants are predominantly from Indonesia, Bangladesh, Myanmar, Nepal, and other Asian nations. As Malaysia transitions toward an ageing society, the proportion of the elderly population is increasing, while the working-age population is gradually declining. This demographic shift, combined with rising educational levels, has reduced domestic interest in low-skilled jobs (World Bank 2023). Consequently, migrants have become essential, especially in agriculture, manufacturing, and construction. Migrant workers are often hired at younger ages, making them critical to offsetting the shrinking labour force and supporting economic productivity. However, Malaysia's Twelfth Plan outlines the aim to reduce the dependency on low-skilled foreign labour by introducing a multi-tiered levy system. This policy seeks to encourage industrial transformation toward automation and high-skilled employment. Following these changes, net international migration declined from 10.4% in 2020 to 7.4% in 2022 (Department of Statistics Malaysia 2023).

Modelling net migration remains complex due to its sensitivity to changes in national policies. Although annual data on net migration are available, detailed data disaggregation by age and gender on a yearly basis remains scarce. Several methods exist to estimate net migration. However, the residual method is particularly valuable due to its applicability across various levels of disaggregation (United Nations 2023). This method uses the demographic accounting equation such that population at time $t + n$ is equal to population at time t , plus births, minus deaths, and adjusted for net migrations (Raymer *et al.* 2023). If population data, birth rates, and death rates are available at two points in time (census years), the net migration can be calculated as the residual component, balancing the population equation (United Nations 2023).

3. Data and Methodology

3.1. Data

This study comprises three forms of demographic data. First, Malaysia's age-specific fertility rates are classified into five-year age groups: 15-19, 20-24, 25-29, 30-34, 35-39, 40-44, and 45-49. Second, Malaysia mortality data generated from abridged life tables for age groups 0, 1-4, 5-9, 10-14,.....65-69, 70-74, and 75+, covering both genders. Both the first and second were collected from the Department of Statistics Malaysia (DOSM). Third, Malaysian population data segregated by five-year age groups for both genders was collected from the United Nations. These datasets were set to be consistent, covering a 44-year period, from 1980 to 2023. The in-sample fitting period for both fertility and mortality models employs datasets from 1980 to 2013, whereas the out-sample forecast or evaluation period is from 2014 to 2023. The future forecasts of the age-specific population will be performed for ten years from 2024 to 2033.

3.2. The cohort component method

Following Hyndman and Booth (2008), this study adopts the cohort-component method for projecting age- and gender-specific population structure by considering the separate impacts of three main components: fertility, mortality, and migration. The method was developed based on the demographic balance equation, in which the population at time t equals the population at time $t - 1$, plus those who came through birth or immigration, minus those who departed due to death or emigration. The mathematical equations for both males' and females' populations are described below:

Female populations:

$$\begin{aligned} {}^fP_{t+1}(x+1) &= {}^fP_t(x) - {}^fD_t(x, x+1) + {}^fG_t(x, x+1), \\ {}^fP_{t+1}(0) &= {}^fB_t - {}^fD_t(B, 0). \end{aligned} \quad (1)$$

Male populations:

$$\begin{aligned} {}^mP_{t+1}(x+1) &= {}^mP_t(x) - {}^mD_t(x, x+1) + {}^mG_t(x, x+1), \\ {}^mP_{t+1}(0) &= {}^mB_t - {}^mD_t(B, 0). \end{aligned} \quad (2)$$

where

- ${}^fP_{t+1}(x+1)$ = number of female populations aged $x+1$ in calendar year $t+1$.
- ${}^mP_{t+1}(x+1)$ = number of male populations aged $x+1$ in calendar year $t+1$.
- ${}^fP_t(x)$ = number of female populations aged x in calendar year t .
- ${}^mP_t(x)$ = number of male populations aged x in calendar year t .
- ${}^fD_t(x, x+1)$ = deaths of female age x in year t , occurred during a year.
- ${}^mD_t(x, x+1)$ = deaths of male age x in year t , occurred during a year.
- ${}^fG_t(x, x+1)$ = net migration of female age x in year t , occurred during a year.
- ${}^mG_t(x, x+1)$ = net migration of male age x in year t , occurred during a year.
- ${}^fP_{t+1}(0)$ = number of female newborn in calendar year $t+1$.
- ${}^mP_{t+1}(0)$ = number of male newborn in calendar year $t+1$.
- fB_t =number of female newborn in calendar year t .
- mB_t =number of male newborn in calendar year t .
- ${}^fD_t(B, 0)$ = female infant deaths in calendar year t

- ${}^mD_t(B, 0)$ = male infant deaths in calendar year t

3.2.1. Fertility component

The segregation between male and female newborns is based on Malaysia's male-to-female sex ratio as follows:

$${}^fB_t = B_t / (1 + \rho) \text{ and } {}^mB_t = B_t \left(\frac{\rho}{1 + \rho} \right), \quad (3)$$

where

- fB_t = number of female newborn in calendar year t .
- mB_t = number of male newborn in calendar year t .
- $\rho = 1.05$, representing 105 male newborns for every 100 female newborns.
- B_t = total number of births in calendar year t obtained from equation below.

$$B_t = \sum_{x=15}^{49} {}^fP_t(x)(f_t(x)). \quad (4)$$

where

- ${}^fP_t(x)$ = number of female populations aged x in calendar year t .
- $f_t(x)$ = age-specific fertility rates of female between reproductive age 15 to 49 years old

Following Kim and Kim (2020), a generalised log gamma distribution (GLG) model is employed to estimate a single age-specific fertility rate (ASFRs) for each year. The GLG model is constituted by four parameters and is expressed as follows:

$$\gamma(x) = \frac{C|\lambda|}{b\Gamma\left(\frac{1}{\lambda^2}\right)} \left(\frac{1}{\lambda^2}\right)^{\lambda^{-2}} \exp\left[\frac{1}{\lambda}\left(\frac{x-u}{b}\right) - \frac{1}{\lambda^2} \exp\left(\lambda\left(\frac{x-u}{b}\right)\right)\right], \quad (5)$$

where

- $\gamma(x)$ = smoothed single-aged fertility rates for aged x .
- C = level of giving birth ($0 \leq C \leq 1$).
- u = average age of women when giving birth ($15 \leq u \leq 49$).
- b = standard deviation of the average age of women when giving birth ($b > 0$).
- λ = constant value indicating birth order ($-\infty \leq \lambda \leq \infty, \lambda \neq 0$).

The estimated single age-specific fertility rates for each year will be then fitted into the Lee-Carter model, a two-factor model, with age and time components, in order to obtain its future values. The equation of the Lee-Carter model is described as follows:

$$\ln \gamma_t(x) = a(x) + b(x)k(t) + \epsilon(x, t), \quad (6)$$

where

- $\gamma_t(x)$ = smoothed single-aged fertility rates for aged x in year t .
- $a(x)$ = average of $\log \gamma_t(x)$ for aged x over t years.

- $k(t)$ = the time component of fertility over the fitting period of t years.
- $b(x)$ = the age component, captures the age-specific x sensitivity to changes in $k(t)$.
- $\epsilon(x, t)$ = model's residual errors

Both $b(x)$ and $k(t)$ were estimated from the singular value decomposition (SVD) of the age x year matrix of $[\ln \gamma_t(x) - a(x)]$, with the constraint of $\sum_t k_t = 0$ and $\sum_x b_x = 1$. Once these parameters are estimated, the $k(t)$ values are forecasted using the time series forecasting techniques, in order to obtain the forecasted fertility rates. The forecasted fertility rates are incorporated into Equation (6) to obtain the projected total number of births during the forecast period.

3.2.2. Mortality component

The second component of population projection is mortality. The notations used for mortality modelling described as below:

Female Deaths:

$$\begin{aligned} {}^fD_t(x, x+1) &= {}^fP_t(x)({}^fq_t(x)), \\ {}^fD_t(B, 0) &= {}^fB_t({}^fq_t(0)). \end{aligned} \quad (7)$$

Male Deaths:

$$\begin{aligned} {}^mD_t(x, x+1) &= {}^mP_t(x)({}^mq_t(x)), \\ {}^mD_t(B, 0) &= {}^mB_t({}^mq_t(0)). \end{aligned} \quad (8)$$

where

- ${}^fD_t(x, x+1)$ = deaths of female age x in year t , occurred during a year.
- ${}^mD_t(x, x+1)$ = deaths of male age x in year t , occurred during a year.
- ${}^fP_t(x)$ = number of female populations aged x in calendar year t .
- ${}^mP_t(x)$ = number of female populations aged x in calendar year t .
- ${}^fq_t(x)$ = age-specific of deaths for female aged x in calendar year t .
- ${}^mq_t(x)$ = age-specific probability of deaths for male aged x in calendar year t .
- mB_t =number of male newborn in calendar year t .
- ${}^fq_t(0)$ = probability of death female newborn in calendar year t .
- ${}^mq_t(0)$ = probability of death male newborn in calendar year t .

Similar to fertility rates, this study converts the age group mortality data into single age rates. The common extrapolation method is the Heligman-Pollard (HP) model adopted. The model effectively smoothed age-gender-specific mortality data and captured trends across life stages. It has been used in nations such as South Korea to oversee ageing populations and anticipate future death patterns (Kim & Kim 2020). Thus, this study employs the HP model to estimate mortality trends in Malaysia for both males and females by smoothing historical data. The Heligman-Pollard eight-parameter model is described in the equation below:

$$\frac{q(x)}{(1-q(x))} = A^{(x+B)^C} + D \exp\left(-E \left(\ln\left(\frac{x}{F}\right)\right)^2\right) + GH^x, \quad (9)$$

where

- $q(x)$ = probability of death for age x in a year.
- A, B, C, D, E, F, G , and H = the model's positive parameters to reflect three age diversifications of mortality patterns. The first term represents the early childhood mortality, the second term represents the mid-life mortality or occasionally captures the accident bump trend, whereas the third term is the adult age mortality, usually more than 40 years and represents the Gompertz exponential function (Ibrahim 2008).

The estimated single age probability of death for males and females for each year are then fitted into the Lee-Carter model separately to obtain the forecasted values as below:

$$\ln {}^m q_t(x) = \mu(x) + \beta(x)\vartheta(t) + \varepsilon(x, t), \quad (10)$$

$$\ln {}^f q_t(x) = \alpha(x) + \theta(x)\gamma(t) + \varphi(x, t). \quad (11)$$

where

- ${}^m q_t(x)$ = age-specific probability of deaths for male aged x in calendar year t
- ${}^f q_t(x)$ = age-specific probability of deaths for female aged x in calendar year t
- $\mu(x)$ and $\alpha(x)$ = average log probability of death of males and females over time
- $\vartheta(t)$ and $\gamma(t)$ = the time component representing the overall trends of mortality over the fitting period for male and female populations respectively.
- $\beta(x)$ and $\theta(x)$ = the age components of the male and female populations.
- $\varepsilon(x, t)$ and $\varphi(x, t)$ = Randomness or residual error factors $\varepsilon(x, t)$.

Predicting mortality rates with the Lee-Carter model is reduced to forecasting the mortality index $\vartheta(t)$ and $\gamma(t)$ using time series approaches. Similar to fertility data, the ARIMA function in R programming was used to determine an ARIMA model that provides a better fit to Malaysia's male and female mortality data.

3.2.3. Net-migration component

Due to a Malaysian migration annual data segregation by age and gender is unavailable, both the observed and projected migration data by age and gender were estimated using the demographic growth balance equation, also known as the residual model (Raymer *et al.* 2023). This approach corresponds directly with the cohort approach of population dynamics. The formulation for age-specific net migration for both females and males is described below:

$${}^f G_t(x, x + 1) = {}^f P_{t+1}(x + 1) - {}^f P_t(x) + {}^f D_t(x, x + 1), \quad (12)$$

$$(x, x + 1) = {}^m P_{t+1}(x + 1) - {}^m P_t(x) + {}^m D_t(x, x + 1), \quad (13)$$

where

- ${}^f G_t(x, x + 1)$ = number of net migration of female age x in year t , during a year.
- ${}^m G_t(x, x + 1)$ = number of net migration of male age x in year t , during a year.
- ${}^f P_{t+1}(x + 1)$ = number of female populations aged $x + 1$ in calendar year $t + 1$.
- ${}^m P_{t+1}(x + 1)$ = number of male populations aged $x + 1$ in calendar year $t + 1$.
- ${}^f P_t(x)$ = number of female populations aged x in calendar year t .

- ${}^mP_t(x)$ = number of male populations aged x in calendar year t .
- ${}^fD_t(x, x + 1)$ = deaths of female age x in year t , occurred during a year.
- ${}^mD_t(x, x + 1)$ = deaths of male age x in year t , occurred during a year.

Although each demographic component is modelled separately as to best capture its unique dynamics, the outputs are subsequently combined within the Cohort Component Method (CCM). Specifically, the forecasted age-specific fertility rates, mortality rates, and net migration values are input into the CCM framework to project population changes year by year.

4. Results and Analysis

The results of the analysis and projections for each population component are presented in the following sections. The forecasted values of fertility, mortality, and net migration are then integrated into the cohort-component model to generate comprehensive age- and sex-specific population projections.

4.1. Age- and gender-specific Malaysia fertility forecasts

The Malaysia age-group fertility rates were modelled using the GLG model described in Equation (4). This flexible parametric model is capable of capturing the asymmetric and heteroskedastic nature of fertility distributions across age groups (Kim & Kim 2020). Its application is particularly valuable for smoothing grouped fertility data and extrapolating to single-year age estimates, which are necessary inputs for detailed population projections.

The GLG model comprises four parameters: C , λ , u , and b . Table 1 reports the estimated parameter values for each year from 1980 to 2023. The parameter C denotes the overall fertility level per 100 women, thus bounded between 0 and 1 per woman. The shape parameter λ displays the skewness of the distribution, which captures asymmetry in the fertility distribution. The parameter u corresponds to the mean age of fertility, which has demonstrated an increasing trend from 15.293 in 1980 to 20.243 in 2023. Meanwhile, b is the standard deviation, indicating the dispersion of fertility around the mean age. Both the location parameter u and the scale parameter b exhibit temporal fluctuations, indicating dynamic shifts of the age structure of fertility among Malaysian women. Using these fitted parameters, the smoothed ASFR values by single-year ages were produced, thereby reducing irregularities and improving estimation reliability. This smoothing process enhanced interpretability and enabled accurate interpolation of fertility rates from conventional five-year age groups to finer age intervals.

The evaluation fitting model involved minimising the sum of squared errors (SSEs) between observed and fitted values, where a lower SSE signifies a better fit. The fitted values were then evaluated against the observed data to assess model performance. The SSE percentages were between 0.91% and 4.49% for data between 1980 and 2023, indicating a strong model fit over these years, as the fitted ASFR appears to closely match the observed data. See Figure 1 for the smoothed single age-specific fertility rates over the observed period. Overall, the application of the GLG model demonstrated robust capability in capturing Malaysia's fertility dynamics, yielding a refined single-age fertility estimate suitable for further demographic projections. This approach aligns with recent advancements in demographic modelling that prioritise flexibility and accuracy techniques that could minimise bias and uncertainty in forecast values (Agostinelli *et al.* 2016).

Table 1: Estimated Parameters of Generalised Log Gamma model from the Malaysian age group of fertility rates, age 15- 45, from the year 1980 to 2023

Year	C	λ	u	b	Year	C	λ	u	b
1980	92.577	- 0.423	15.293	2.666	2002	79.682	- 0.435	18.230	2.540
1981	62.818	- 0.407	14.812	2.581	2003	74.421	- 0.434	18.338	2.507
1982	92.459	- 0.423	15.348	2.692	2004	81.141	- 0.439	18.634	2.514
1983	87.840	- 0.422	15.477	2.667	2005	97.244	- 0.449	18.992	2.561
1984	55.709	- 0.404	15.148	2.535	2006	64.568	- 0.431	18.618	2.447
1985	78.403	- 0.416	15.735	2.604	2007	52.059	- 0.423	18.595	2.379
1986	65.997	- 0.411	15.504	2.603	2008	94.400	- 0.450	19.184	2.567
1987	89.622	- 0.425	16.111	2.678	2009	90.508	- 0.448	19.150	2.576
1988	81.982	- 0.421	16.276	2.602	2010	70.366	- 0.438	19.039	2.505
1989	67.996	- 0.417	15.740	2.677	2011	81.350	- 0.444	19.335	2.538
1990	81.982	- 0.423	16.150	2.663	2012	78.006	- 0.442	19.376	2.515
1991	69.430	- 0.417	16.013	2.599	2013	59.907	- 0.434	19.176	2.455
1992	80.441	- 0.422	16.174	2.639	2014	81.538	- 0.447	19.564	2.550
1993	97.469	- 0.431	16.572	2.645	2015	82.758	- 0.449	19.730	2.565
1994	56.660	- 0.410	16.022	2.517	2016	92.340	- 0.457	19.871	2.638
1995	80.264	- 0.425	16.610	2.574	2017	97.560	- 0.461	19.962	2.682
1996	79.825	- 0.425	16.620	2.579	2018	77.610	- 0.450	19.788	2.628
1997	60.798	- 0.416	16.571	2.477	2019	80.915	- 0.454	19.866	2.649
1998	96.256	- 0.437	17.099	2.646	2020	55.748	- 0.437	19.495	2.554
1999	86.852	- 0.434	17.563	2.585	2021	92.966	- 0.464	20.249	2.576
2000	74.922	- 0.427	17.826	2.502	2022	60.797	- 0.444	19.699	2.421
2001	92.653	- 0.440	18.050	2.619	2023	83.919	- 0.458	20.243	2.504

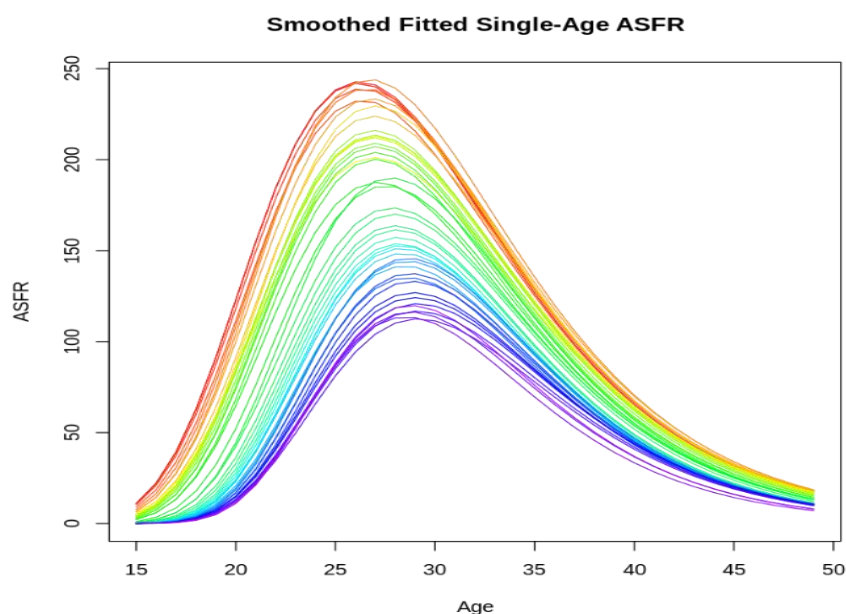


Figure 1: Malaysia Smoothed Single-Age Fertility Rates from 15 to 49 years old using GLG distribution function from 1980 to 2023

The smoothed, single-year Age-Specific Fertility Rates (ASFR) for Malaysia from 1980 to 2023, as displayed in Figure 1, exhibit a consistent unimodal bell-shaped distribution, with the highest value occurring between ages 25 and 35 years and declining at both ends, at ages 15 and 49 years old. Over the observed period, there is a marked decline in fertility levels, with a noticeable downward shift from the 1980 curve (red line) to that of 2023 (purple line), reflecting an overall fertility reduction. These trends also reveal a subtle rightward shift in peak fertility, indicative of delayed childbearing, possibly attributable to increasing educational attainment among Malaysian women and greater access to reproductive health services. This finding is consistent with sub-Saharan Africa, where Bongaarts (2020) highlighted that enhanced educational attainment and family planning initiatives contributed significantly to shifts in fertility preferences.

These smoothed ASFRs were subsequently incorporated into the Lee-Carter model to generate fertility forecasts. Figure 2 presents the estimated parameters $a(x)$, $b(x)$ and $k(t)$. The first graph on the left illustrates $a(x)$, the average log-fertility rates across age groups over time. The second panel, displaying $b(x)$, the age-specific sensitivity parameter that reflects how responsive each age group's fertility is to temporal changes. A concave pattern emerges, indicating that fertility at younger ages is more sensitive to temporal changes, while fertility at older ages exhibits comparatively less volatility. Lastly, the temporal component $k(t)$, known as the fertility index, captures the general decline in fertility levels over the projection horizon, indicating the trend of decreasing fertility across the observed period from 1980 to 2023.

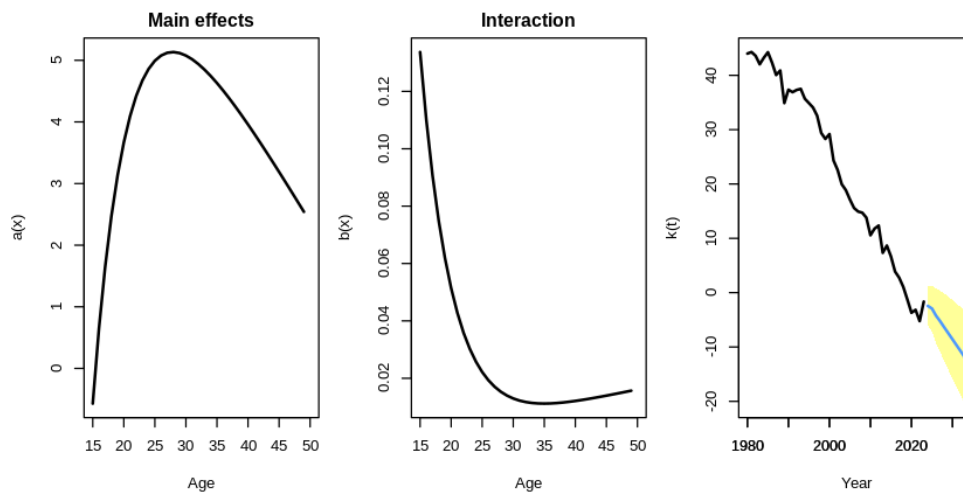


Figure 2: Estimated Parameters of the Lee-Carter Model $a(x)$, $b(x)$, $k(t)$ for Malaysia's Age-Specific Fertility Rates from 1980 to 2023.

The fertility index values are then forecasted ten years ahead using a time series approach, represented within the confidence intervals shaded in yellow. The model selection is based on the Arima function, which identifies the optimal model specification based on minimum forecast errors. The Lee-Carter model performed best in modelling the observed fertility rates over the 44-year horizon with a high coefficient determination and an R-squared value of 99.5%, capturing most of the variation in the fertility data. The out-of-sample forecast accuracy of the GLG-Lee-Carter model was evaluated by comparing predicted values for the period 2014 to 2023 with observed fertility data over the same time frame, using the Mean Absolute Percentage Error (MAPE). The GLG-Lee-Carter exhibited MAPE values ranging from 7.26%

to 14.70%, with an average error of 10.44% across the ten-year evaluation period. These results indicate an accurate forecasting performance, though some variability in accuracy may reflect the influence of unexpected demographic or socioeconomic shifts.

The forecasted future value of ASFRs from 2024 to 2033 is illustrated in Figure 3. Each colored curve represents a specific forecast year, ranging from red (2024) to purple (2033). The projections exhibit a persistent decline in fertility levels over time while maintaining the characteristic unimodal distribution. Fertility rates consistently peak between ages 25 and 30 and decline at the lower (15-19) and upper (40-49) reproductive age intervals. The consistent downward shift in the ASFR curves across the ten-year forecast horizon reflects Malaysia's ongoing trend of below-replacement fertility. The continued decline in fertility is largely driven by socioeconomic pressures, particularly rising living costs, delayed marriage, and increasing female labour-force participation (Zulqarnain & Yusuf 2022).

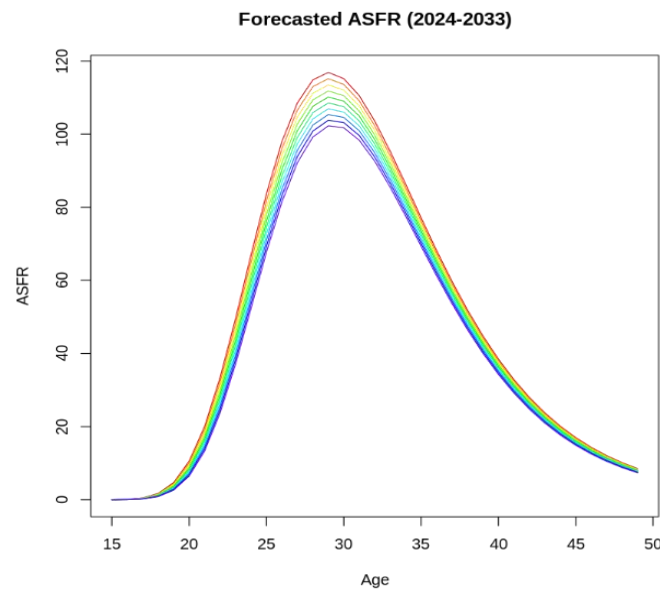


Figure 3: 10-year Malaysia Forecasted Single-Age Fertility Rates (15-49) from 2024 to 2033 using the Lee-Carter Model

4.2. Age- and gender-specific Malaysia mortality forecasts

This study adopts the Heligman-Pollard eight-parameter model to estimate the single age mortality rates given the age group rates. The parameters of the Heligman-Pollard (HP) model offer a thorough estimation of mortality at all phases of life. While parameter *A* represents early childhood mortality or the likelihood of dying between years, parameter *B* represents the difference in mortality between ages 0 and 1. Parameter *C* represents the rate of death decline during childhood. The larger the value, the faster the rate of mortality reduction. The height of the mortality hump, which is usually observed in adolescence or early adulthood, is represented by parameter *D*. Parameter *E* regulates the breadth of the mortality hump. The bigger the values, the smaller the age range for mortality increase. Parameter *F* controls the mortality hump's position on the age axis. The parameters *G* and *H*, indicate the pace at which death rises with age, which are used to calculate older age mortality. These eight parameters were estimated through the fitting of Malaysia age group mortality data from 1980 to 2023 using Equation (9).

Table 2: The average values of the Heligman-Pollard parameters A, B, C, D, E, F, G , and H for male and female Malaysian mortality data. The average is obtained over a 44-year period from 1980 to 2023

Gender	A	B	C	D	E	F	G	H
Male	0.149	4.476	0.652	0.298	34.447	95.114	0.001	1.068
Female	0.003	0.729	0.383	0.338	21.067	94.635	0.001	1.074

Table 2 reports the average values of the eight parameters from the Heligman-Pollard mortality model that were estimated separately for Malaysian males and females over a 44-year period (1980-2023). In total, the parameter estimation comprises 704 parameters ($44 \text{ years} \times 8 \text{ parameters} \times 2 \text{ genders}$), from which the average values were computed to provide a summary of mortality patterns. From Table 2, differences can be observed between male and female parameters. For example, the higher values of A, B , and C among males suggest greater early-age mortality risk. At the same time, the accident-related parameters D and E also differ, potentially reflecting gender differences in risk exposure during young adulthood. Old-age mortality parameters F, G , and H are relatively similar across genders, indicating convergence in elderly mortality risk. The smoothed single age mortality rates using the fitted Heligman-Pollard were presented in Figure 4.

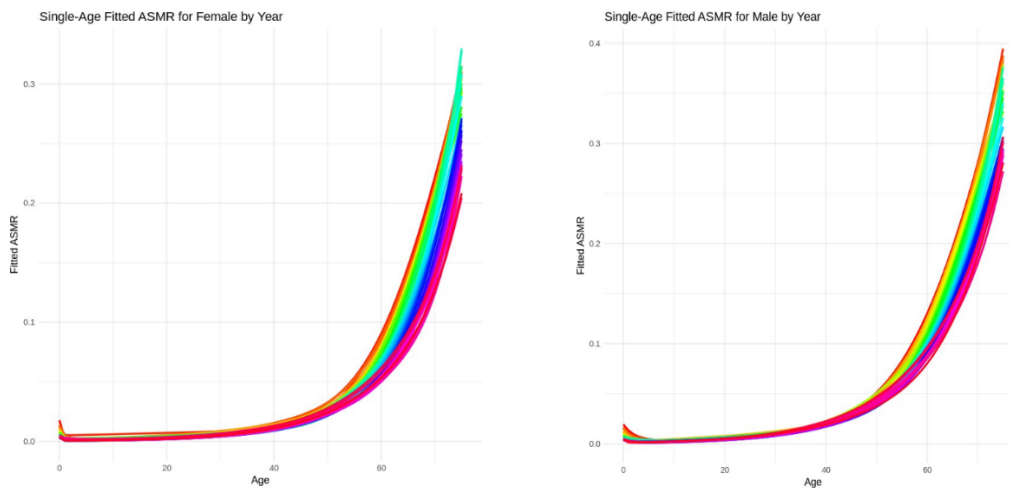


Figure 4: Malaysia Smoothed Single-Age Mortality Rates (0-75), using the Heligman-Pollard Eight-Parameter Model from 1980 to 2023

Figure 4 illustrates the smoothed single-age mortality rates (ASMRs) for Malaysian males and females aged 0 to 75 for the observation period from 1980 to 2023. The left and right panels present fitted ASMR curves for females and males, respectively. Each curve represents a specific year over the same observation period. The gap between the earliest years (red curves) and the latest years (maroon curves), which is substantial among older age groups, indicates significant mortality improvement by year. The curves exhibit the classic J-shaped pattern, reflecting common patterns of human mortality, in which high rates in infancy are followed by a decline through childhood and early adulthood, and then a progressive increase in older ages. Additionally, male mortality consistently exceeds female mortality, particularly at older ages.

Figure 5 presents the fitted Lee-Carter mortality model parameters, which were estimated from 1980 to 2023, together with forecasted values of the mortality index from 2024 to 2033. The top-left plots in both panels denote the fitted $\mu(x)$ and $\alpha(x)$, representing the average log

mortality rate by age over time for males and females. Both curves represent higher mortality at infancy and an exponential increase at older ages. The basis function $\beta(x)$ and $\theta(x)$ captures how sensitive each age group's mortality is to changes in the mortality index. Higher values of $\beta(x)$ and $\theta(x)$ at older ages suggests that senescent mortality is more responsive to temporal mortality shifts. The lower plots indicate the temporal trend of $\lambda(t)$ and $\gamma(t)$, which reflects the overall level of mortality over time. Both male and female indices demonstrate a general downward trend, indicating significant mortality improvements from 1980 to 2023. The shaded yellow areas (2024-2033) represent forecast values of the mortality index, demonstrating a continuous decreasing pattern of mortality. The declining $\lambda(t)$ and $\gamma(t)$ values suggest a continued mortality decline in Malaysia. The forecasted $\lambda(t)$ and $\gamma(t)$ values were incorporated into the Lee-Carter model, along with the other estimated parameters, to generate forecast values of age-specific mortality rates. These rates will then be multiplied by the corresponding population size to estimate the expected number of deaths.

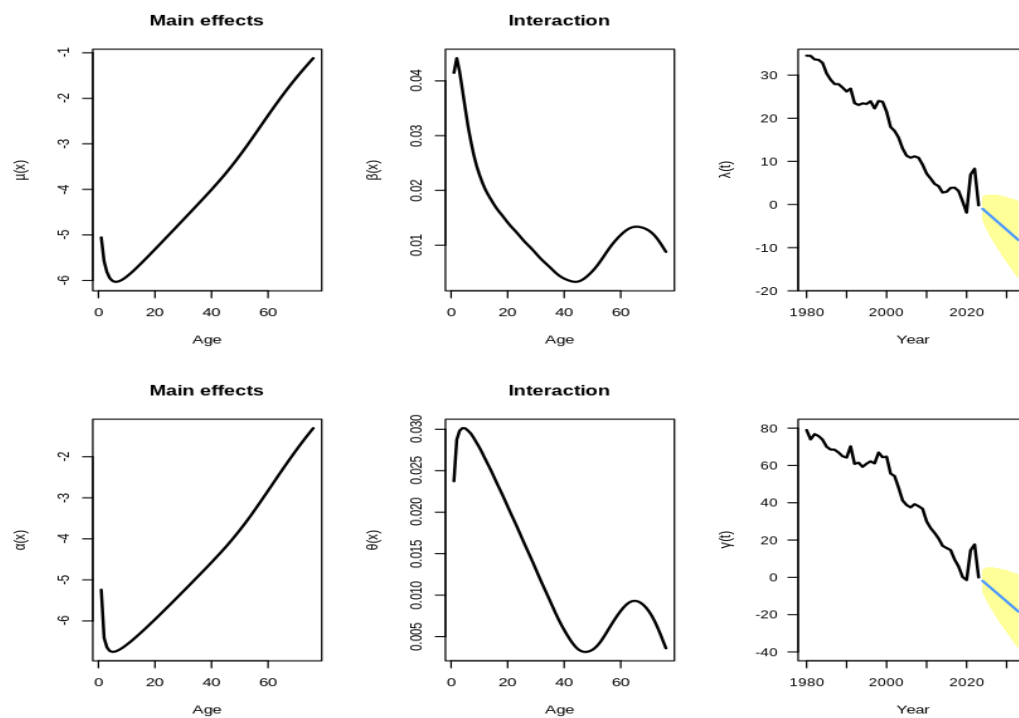


Figure 5: The fitted values of the Lee-Carter parameters from 1980 to 2023, and forecasted values of the mortality index from 2024 to 2033 for males (left) and females (right)

The out-sample forecast accuracies of the HP-Lee-Carter model were evaluated over the evaluation period from 2014 to 2024 using the Mean Absolute Percentage Error (MAPE). The error values are recorded separately for males and females, indicating the predictive accuracy of the model for each gender across the years. The age-specific mortality forecast values obtained using Equations (10) and (11) using the estimated parameters $\mu(x)$, $\alpha(x)$, $\beta(x)$ and $\theta(x)$ and forecasted values of $\lambda(t)$ and $\gamma(t)$. Table 3 shows the forecast values of mortality rates by age and gender and reports the rate of improvement over the forecast period. The mortality rates for both males and females show consistent declines between 2024 and 2033 across all age groups, reflecting continued improvements in Malaysia population health. Female mortality improvements are generally higher than those of males, particularly at younger ages

where reductions reach over 30 percent. The rate of improvement slows at middle ages but increases again at older ages, indicating improvement in chronic disease management and elderly care.

Table 3: The forecasted mortality rates by age and gender in 2024 and 2033 with percentage of mortality improvement over the forecast years.

Age	Mortality Rates in 2024		Mortality Rates in 2033		Percentage of Improvement (%)	
	Male	Female	Male	Female	Male	Female
0	0.00305	0.00173	0.00226	0.00117	25.90%	32.37%
10	0.00194	0.00041	0.00166	0.00026	14.43%	36.59%
20	0.00412	0.00107	0.00373	0.00077	9.47%	28.04%
30	0.00871	0.00305	0.00823	0.00249	5.51%	18.36%
40	0.01829	0.00868	0.01781	0.00797	2.62%	8.18%
50	0.03779	0.02081	0.03622	0.01958	4.15%	5.91%
60	0.08145	0.04382	0.07447	0.03807	8.57%	13.12%
70	0.18419	0.12284	0.16850	0.10851	8.52%	11.67%

4.3. Age- and gender-specific Malaysia net-migration estimates

Taking into consideration single-age birth and death estimates over the observed period, the adjusted net migrations were estimated using Equation (12) for females and Equation (13) for males. No smoothing technique was applied to the net migration data. Figure 6 presents the estimated age-specific net migration trends for Malaysian males (left) and females (right).

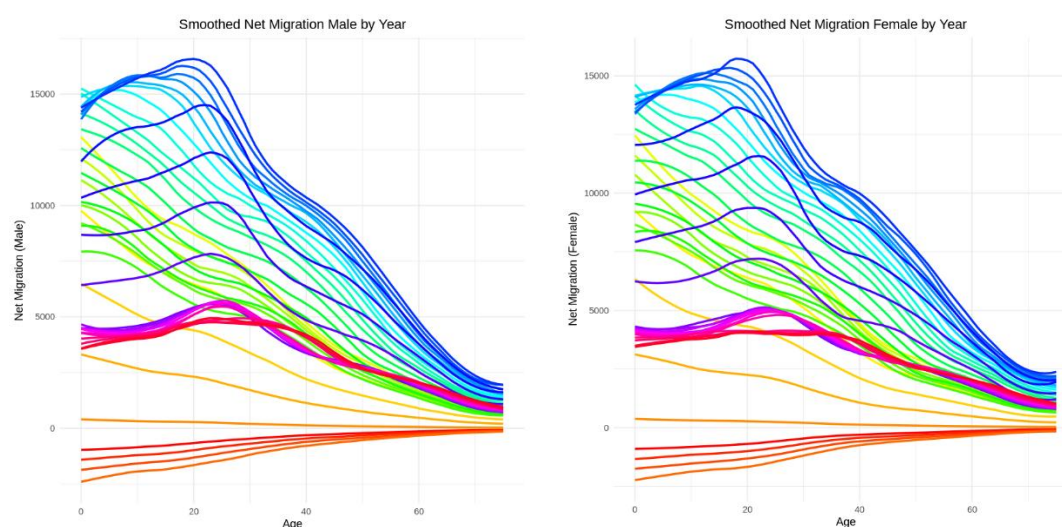


Figure 6: The Smoothed Estimated Values of Net Migration for males (left) and females (right) using the demographic growth balance model from 1980 to 2023

In both panels, the highest Malaysian net migration occurs between ages 15 and 35, which is typical of labour-driven migration and this trend is widely documented across countries globally (UN DESA Population Division 2024). Earlier years (red, orange, and yellow lines) present lower levels of net migration across all age groups. From the mid-1990s to the 2010s (green to blue lines), there is a noticeable increase in net migration, especially among young adults. The maroon line (2023) suggested a modest decline from previous peaks, potentially

reflecting policy changes, economic shifts, or post-pandemic adjustments. While net migration remains concentrated among youth, older age groups (40+) have demonstrated slight increases in more recent years. The persistent peak in youth migration highlights the continued reliance on labour migration and underscores the importance of migration policy tailored to this demographic.

Forecasting net migration by age is challenging due to data limitations and high volatility in migration flows. In this study, a time-series forecasting approach was used to estimate future net migration by age, employing a separate time-series model for each age to capture unique migration patterns over time. Once the models are fitted, they are used to forecast future values for each respective age group. These individual forecasts can then be aggregated to obtain the overall projected net migration across all ages.

4.4. Age- and gender-specific Malaysia population projection

Estimated births, deaths, net migration, and the present population are all considered when projecting age-specific population using the Cohort Component Model (CCM). In these sections, the three components of the population are considered to estimate the age population structure during the observation and forecast years. The results are explained below. Beyond being a useful visual aid, population pyramids are crucial instruments for understanding the underlying patterns of demographic changes.

The population pyramids in Figure 7 illustrate the demographic structure of Malaysia by age and gender for two years, including an observed year (1980) and a forecasted year (2033). Malaysia has undergone substantial changes in its population structure, driven by shifts in fertility and mortality patterns over the decades. From 1980 to 2033, a clear demographic shift can be observed from a young population structure to an ageing population society, a demographic transition previously experienced by many developed nations.

In 1980, Malaysia's population pyramid displayed an expansive pyramid with a classic triangle look characterised by a broad base and a narrow apex. This structure reflects a high birth rate in 1980, resulting in a high proportion of young age groups. The birth rates, however, have been decreasing over the years. In addition, the size of the elderly group was low due to relatively low life expectancy at birth in 1980, which was 68.24 years. These factors led to Malaysia being considered a young population, typical for developing countries during that period.

In contrast, the forecasted Malaysian population pyramid in 2033 exhibits a markedly different shape toward a more stationary or even constrictive form. The base of the pyramid has narrowed, indicating a projected decline in birth rates, while the upper and middle sections have widened significantly. This shift reflects both a reduction in fertility and an increase in life expectancy, contributing to population ageing. Similar demographic transitions have been observed in countries such as South Korea, where population ageing is driven by sustained low fertility rates and rising longevity (Kim & Chung 2024). Notably, the population share of individuals aged 60 and above was projected to increase substantially, highlighting the ageing transition in Malaysia's demographic profile. This transformation suggests an increase in old-age dependency ratios, with a shrinking proportion of working-age individuals relative to the elderly. Furthermore, the aging index, defined as the ratio of individuals aged 60 and above to those under the age of 15, was projected to increase significantly by 2033. The projected population structure for 2033 already provides clear evidence that population aging is a significant and rapidly emerging reality for Malaysia.

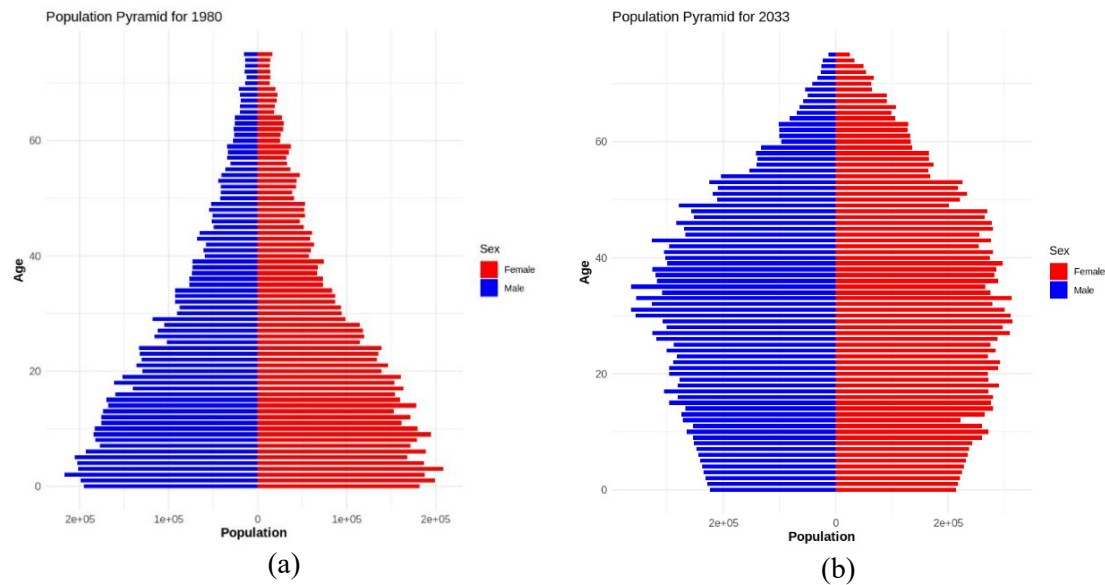


Figure 7: Malaysia Population Pyramids during an Observed Year (1980) and a Forecast Year (2033) with age and gender distribution

As Malaysia aspires to reach developed nation status, the continued decline in fertility and an increase in the number of older persons result in demographic challenges. These trends pose risks to economic growth due to potential labour shortages and reduced productivity, and at the same time increase the financial burden to support healthcare and social benefits to the elderly. To address this issue, the government and all sectors must take proactive steps. Key policy recommendations include expanding affordable and accessible childcare services while promoting flexible work arrangements to support a better work-life balance, especially for working women. Simultaneously, the government could enhance national productivity by extending the economic participation of older adults. This includes providing targeted incentives to encourage older individuals to remain in or re-enter the workforce. It is also recommended to develop a National Active Ageing Framework that promotes lifelong learning, preventive healthcare, and age-friendly workplace practices by benchmarking frameworks from advanced ageing nations. These integrated strategies are essential to maintaining economic resilience and ensuring economic sustainability in Malaysia.

5. Conclusions

This study presents an integrated demographic approach for population forecasting, combining fertility, mortality, and migration components within a probabilistic Cohort Component Model framework. This study enhances existing research by employing model that better reflect the behaviour of each demographic component which are more suitable for capturing the unique age patterns in Malaysia's demographic data. In addition, this study incorporates the Lee-Carter model to produce long-term forecasts for fertility and mortality. These enhancements result in a more flexible and realistic forecasting framework compared to previous approaches

Results reveal a significant demographic transition in Malaysia towards an ageing population by 2030, as evidenced by a shrinking number of young age groups and an expansion of older age groups, following patterns previously observed in more developed economies. This research provides notable findings for Malaysia's future, particularly in areas such as workforce

availability, pension sustainability, healthcare demand, and social support systems. Future research could extend this work by incorporating socioeconomic scenarios, policy shock simulations, or health-adjusted life expectancy measures to provide more comprehensive insights. Moreover, extending comparative forecasting approaches for each population component such as exponential smoothing and Bayesian time-series models would provide a more comprehensive performance evaluation.

Acknowledgements

The authors would like to express their sincere gratitude to the Ministry of Higher Education (MOHE), Malaysia, for the financial support provided under the Fundamental Research Grant Scheme (FRGS) (Grant No: FRGS/1/2023/STG06/UITM/02/6).

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Received: 25 June 2025

Accepted: 14 January 2026

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