

RADIATION-DRIVEN FLOW AND HEAT TRANSFER OF COUPLE STRESS HYBRID NANOFLUIDS OVER A SHRINKING RIGA PLATE

(Aliran dan Pemindahan Haba Terpacu Radiasi bagi Tekanan Bergandingan Nanobendalir Hibrid Melepasi Plat Riga Mengecut)

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ABSTRACT

This study examines the flow and heat transfer characteristics of a couple-stress hybrid nanofluid over a permeable shrinking Riga plate comprising aluminium alloys (AA7072 and AA7075) with methanol (CH_3OH) as the base fluid. The flow is influenced by the presence of an electrically actuated Riga plate, which generates a Lorentz force, promoting boundary layer stabilisation. The couple stress fluid model accounts for microstructural effects, adding complexity to the momentum equations. Similarity transformations are employed to simplify the governing partial differential equations into ordinary differential equations, which are then solved numerically using the MATLAB bvp4c solver. The effects of the considered parameters are analysed in detail. The results demonstrate that the existence of the hybrid nanoparticles and the couple stress parameter considerably influence the flow behaviour and heat transfer performance. These findings provide new insight for the control of boundary layer flow in micro- and nanoscale cooling systems.

Keywords: hybrid nanofluid; couple stress; Riga plate; suction; radiative flow

ABSTRAK

Kajian ini meneliti ciri pemindahan haba bagi aliran tekanan bergandingan nanobendalir hibrid yang terdiri daripada aloi aluminium (AA7072) dan (AA7075) dengan metanol (CH_3OH) sebagai bendalir asas ke atas plat telap Riga. Aliran dipengaruhi oleh kehadiran plat Riga yang digerakkan secara elektrik, yang menjana daya Lorentz sekali gus meningkatkan kestabilan lapisan sempadan. Model bendalir aliran tekanan bergandingan mengambil kira kesan mikrostruktur yang menambah kompleksiti kepada persamaan momentum. Transformasi penjelmaan keserupaan digunakan untuk mempermudah persamaan pembezaan separa kepada persamaan pembezaan biasa, yang kemudiannya diselesaikan secara berangka menggunakan penyelesaian bvp4c dalam MATLAB. Kesan parameter yang dikaji dianalisis secara terperinci. Keputusan menunjukkan bahawa kewujudan nanozarah hibrid dan parameter tekanan bergandingan memberikan pengaruh ketara terhadap tingkah laku aliran dan prestasi pemindahan haba. Dapatan ini menawarkan pandangan baharu untuk kawalan aliran lapisan sempadan dalam sistem penyejukan skala mikro dan nano.

Kata kunci: nanobendalir hibrid; tekanan bergandingan; plat Riga; sedutan; aliran radiasi

1. Introduction

The enhancement of fluid flow and thermal transport has long been a central focus in engineering and industrial applications, particularly in cooling technologies, biomedical systems, microfluidic devices, and energy conversion systems. As the demand for miniaturized and highly efficient systems continues to increase, conventional fluid models and coolants are often inadequate to meet modern thermal management requirements. This challenge has encouraged the development of advanced fluid models and heat transfer enhancement

techniques, including non-Newtonian fluids, nanofluids, and electromagnetic boundary layer control mechanisms (Adogbeji *et al.* 2024; Riyadi *et al.* 2024; Farooq *et al.* 2025). Among the non-Newtonian fluid models, the couple stress fluid model proposed by Stokes and Stokes (1984) and further developed by Srivastava (1985) has attracted considerable attention. This model extends the classical Navier–Stokes equations by incorporating couple stresses generated by the rotation of fluid microelements. Such a formulation is particularly suitable for representing microstructured fluids such as lubricants, polymer suspensions, and biological fluids, including synovial fluid and blood. Unlike Newtonian fluids, couple stress fluids possess the ability to resist torsional motion and additional shear effects, thereby providing a more realistic description of complex fluid behavior. Several studies have investigated couple stress fluid flows in different boundary layer configurations and reported significant influences on skin friction and flow characteristics (Gajjela & Nandkeolyar 2021; Devi *et al.* 2025).

In parallel with developments in fluid rheology, the field of heat transfer has been significantly advanced by the introduction of nanofluids, pioneered by Choi and Eastman (1995). Nanofluids are formed by dispersing nanoparticles into conventional base fluids such as water or ethylene glycol, resulting in improved thermal conductivity and enhanced heat transfer performance. Common nanoparticles used in such formulations include aluminum oxide (Al_2O_3), copper (Cu), and titanium dioxide (TiO_2) (Souza *et al.* 2022; Theegala & Rachamalla 2025). More recently, hybrid nanofluids, which consist of two or more types of nanoparticles dispersed in a base fluid, have been introduced to further enhance thermophysical properties through synergistic effects. Experimental and numerical investigations have demonstrated that hybrid nanofluids exhibit superior heat transfer performance compared with conventional nanofluids in various engineering applications (Ho *et al.* 2022; Yasmin *et al.* 2023).

Despite these advancements in fluid formulation, controlling boundary layer behavior over shrinking surfaces remains a significant challenge. Unlike stretching surfaces, shrinking configurations generate adverse pressure gradients that may lead to flow reversal and multiple solution behavior. Such instability typically requires additional control mechanisms, such as suction or electromagnetic forcing, to maintain a physically realizable boundary layer. One effective method for controlling such flows is through the use of a Riga plate, which consists of alternating electrodes and magnets embedded beneath the surface. Originally introduced by Gailitis and Lielausis (1961), the Riga plate produces a Lorentz force acting tangentially to the flow direction, enabling active boundary layer control without mechanical intervention. Previous studies have demonstrated the stabilizing capability of Riga plates; for example, Ali *et al.* (2023) reported that electromagnetic forcing suppresses flow reversal in hybrid nanofluid flows, while Mahariq *et al.* (2025) showed that electromagnetic hydrodynamic effects significantly influence the velocity and heat transfer characteristics.

In recent years, increasing research attention has been devoted to the combined effects of hybrid nanofluids and electromagnetic flow control mechanisms such as the Riga plate. These studies indicate that the Lorentz force generated by the Riga plate can effectively modify velocity profiles and thermal boundary layer structures, thereby enhancing flow control and heat transfer performance (Siddique *et al.* 2023; Khan *et al.* 2024; Bilal *et al.* 2025). Nevertheless, most of the existing studies primarily focus on Newtonian fluids, conventional nanofluids, or stretching surface configurations.

The intersection of several advanced mechanisms, namely couple stress fluids, hybrid nanofluids, shrinking surfaces, and Riga plate configurations, remains relatively unexplored. While couple stress fluids provide a realistic framework for modeling non-Newtonian behavior in microstructured fluids, their interaction with hybrid nanofluids designed to enhance thermal

performance has not been extensively investigated in boundary layer flow configurations. The complexity becomes even more pronounced when shrinking surfaces are considered, as they tend to destabilize the boundary layer, potentially leading to flow reversal and multiple solution behavior (Zainal *et al.* 2021; Waini *et al.* 2022; Sarfraz *et al.* 2023). The use of a Riga plate offers an effective electromagnetic mechanism to control such flows through the Lorentz force acting along the surface, enabling stabilization of the boundary layer without mechanical intervention (Turkyilmazoglu 2025).

Despite individual studies on couple stress fluids, hybrid nanofluids, and Riga plate configurations, the combined effects of these elements over a shrinking surface remain insufficiently explored. Motivated by this gap, the present study investigates the boundary layer flow and heat transfer characteristics of a couple stress hybrid nanofluid composed of aluminium alloys (AA7072 and AA7075) dispersed in methanol (CH₃OH) as the base fluid over a shrinking Riga plate. This combination is selected due to its favorable thermophysical properties, where aluminium alloys provide high thermal conductivity and stability while methanol possesses low viscosity and high thermal diffusivity (Mostafizur *et al.* 2022). Previous studies have also reported that such hybrid combinations can significantly enhance thermal performance, making them suitable for microfluidic cooling and advanced energy systems (Salahuddin *et al.* 2022). In the present work, the governing equations are transformed using similarity transformations and solved numerically using MATLAB's bvp4c solver to analyze the effects of relevant physical parameters on the velocity and temperature distributions, as well as the skin friction coefficient and heat transfer rate.

2. Mathematical Model

Figure 1 depicts the physical configuration of the present problem. The velocity components of the fluid in the x – and y – directions are denoted by u and v , respectively. The surface velocity is defined as $U_w(x) = ax$, the wall mass flux velocity is V_w and $T_w = T_\infty + bx^2$ is the temperature of the surface variable, given that T_∞ is the surrounding temperature. The parameters a and b represent the characteristic constants associated with the Riga plate electrodes and magnets, and ξ_∞ is referred to as the couple-stress parameter. The parameter λ denotes the stretching/shrinking parameter of the surface, where $\lambda < 0$ corresponds to a shrinking surface and $\lambda > 0$ represents a stretching surface.

In this study, the electromagnetic parameter associated with the Riga plate is taken as constant. This assumption is consistent with earlier work (Ali *et al.* 2023; Zahoor Raja *et al.* 2025) and physically corresponds to maintaining a uniform magnetization and a constant electrode current density along the Riga plate. Such a configuration is realistic in controlled laboratory or microfluidic setups where steady electromagnetic forcing is imposed.

The governing equations are derived under the boundary layer approximation, assuming constant fluid properties and incompressible flow. The similarity transformation converts the governing partial differential equations into a system of ordinary differential equations. The solution of nonlinear mathematical models often requires efficient and robust computational techniques, and iterative optimization methods such as conjugate gradient approaches have been extensively developed to improve numerical efficiency for large-scale nonlinear problems (Alhawarat *et al.* 2021). Hence, the mathematical equations that govern the proposed problem are defined as (Khan *et al.* 2013; Dhlamini *et al.* 2021):

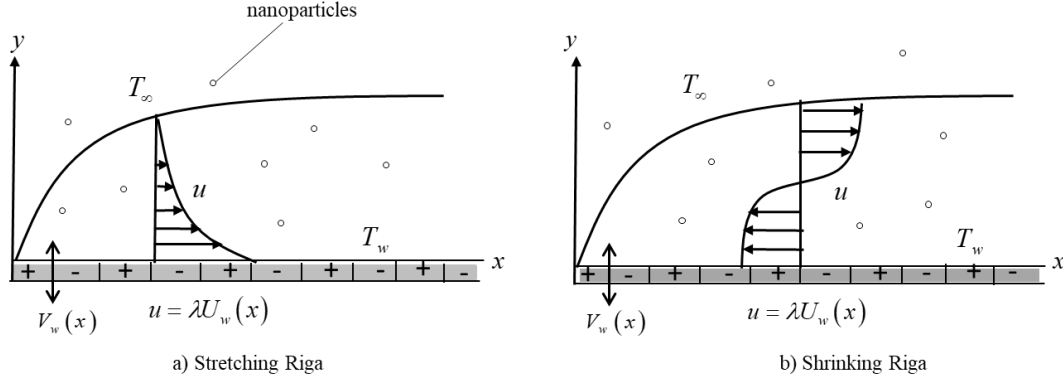


Figure 1: The schematic model

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}} \frac{\partial^2 u}{\partial y^2} - \frac{\xi_\infty}{\rho_{hnf}} \frac{\partial^4 u}{\partial y^4} + \frac{\pi j_\infty M}{8 \rho_{hnf}} \exp\left(-\frac{\pi}{a_1} y\right), \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C_p)_{hnf}} \frac{\partial q_r}{\partial y}, \quad (3)$$

$$\begin{aligned} u = \lambda U_w(x), \quad v = V_w, \quad T = T_w(x) = T_\infty + ax^2 \quad & \text{at } y = 0, \\ u \rightarrow 0, \quad u_{y,yy} \rightarrow 0, \quad T \rightarrow T_\infty \quad & \text{as } y \rightarrow \infty. \end{aligned} \quad (4)$$

The boundary conditions given in Equation (4) have clear physical interpretations. The condition $u = U_w(x)$ at the surface represents a stretching or shrinking Riga plate, where the surface velocity is externally imposed and proportional to the distance along the plate. The wall-normal velocity $v = V_w(x)$ corresponds to suction ($V_w < 0$) or injection ($V_w > 0$), which physically models mass transfer through the permeable plate. This mechanism is commonly employed to stabilize or destabilize the boundary layer. The thermal boundary condition ($T = T_w$) indicates that the plate is maintained at a constant surface temperature. Far from the surface, $u \rightarrow 0$ and $T \rightarrow T_\infty$ signify that the ambient fluid is initially at rest and remains unaffected by the surface motion and heating at sufficiently large distances. Thus, the flow configuration represents a purely surface-driven boundary layer without any imposed external free stream velocity.

Using Rosseland's estimation (Rosseland 1936), the radiative heat flux is clearly expressed as follows (Bataller 2008; Magyari & Pantokratoras 2011):

$$q_r = -\frac{4\sigma}{3k} \frac{\partial T^4}{\partial y}. \quad (5)$$

Consider that σ^* defines the Stefan-Boltzmann constant and k^* suggests the mean absorption coefficient. T^4 is extended about T_∞ to obtain $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$ by using the Taylor series and discarding higher-order terms. Then, equation (3) might be written as

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{(\rho C_p)_{hnf}} \left(k_{hnf} + \frac{16\sigma^* T_\infty^3}{3k^*} \right) \frac{\partial^2 T}{\partial y^2}. \quad (6)$$

Now, the subsequent variables of similarity were employed (Turkyilmazoglu 2014; Waini *et al.* 2023):

$$\begin{aligned} \psi &= (av_f)^{\frac{1}{2}} xf(\eta), \quad u = axf'(\eta), \quad v = -(ax)^{\frac{1}{2}} f(\eta), \\ \eta &= y(a/v_f)^{\frac{1}{2}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \end{aligned} \quad (7)$$

hence

$$V_w = -(av_f)^{\frac{1}{2}} S, \quad (8)$$

where S is the parameter of mass flux, where $S > 0$ and $S < 0$ correspond to suction and injection, respectively. Next, the fluid kinematic viscosity is ν_f , while $M = M_\circ x$ and $j_\circ$ represent the variable form of the magnet's magnetisation and the current density in the electrodes, respectively. By introducing the similarity transformations and satisfying the continuity equation (1), and utilising Eqs. (5) and (6), the governing partial differential equations (2)–(4) are reduced to the following system of ordinary differential equations.

$$-\xi f'''' + f''' + ff'' - f'^2 + Z \exp^{-\Lambda \eta} = 0, \quad (9)$$

$$\frac{1}{\text{Pr}} \left(\frac{k_{hnf}}{k_f} + \frac{4}{3} R \right) \frac{1}{(\rho C_p)_{hnf} / (\rho C_p)_f} \theta'' + f \theta' - 2 f' \theta = 0, \quad (10)$$

$$\begin{aligned} f(0) &= S, \quad f'(0) = \lambda, \quad \theta(0) = 1, \\ f'(\eta) &\rightarrow 0, \quad \theta(\eta) \rightarrow 0, \quad \text{as } \eta \rightarrow \infty, \end{aligned} \quad (11)$$

where the Prandtl number is $\text{Pr} = \nu_f / \alpha_f$ and λ is the stretching/shrinking parameter. The couple stress parameter is presented as $\xi = \xi_\circ a / \rho_{hnf} \nu_f^2$, the Riga plate (electromagnetic) parameter and the width parameter are signified as $Z = \pi j_\circ M_\circ / 8a^2 \rho_{hnf}$ and $\Lambda = \frac{\pi}{a_1} \sqrt{\frac{\nu_f}{a}}$, respectively. Finally, the thermal radiation parameter is denoted as $R = 4\sigma^* T_\infty^3 / k_f k^*$.

Table 1 illustrates the distinctive properties of a hybrid nanofluid. The effective dynamic viscosity of the hybrid nanofluid is evaluated using the Brinkman model. The density and heat capacity are determined using classical mixture formulations. The effective thermal conductivity is computed using the Maxwell model for spherical nanoparticles. These models

are widely adopted in hybrid nanofluid studies and provide reliable estimates of thermophysical properties. Meanwhile, Table 2 defines the properties of the nanoparticles and the base fluid.

The parameters ϕ_1 and ϕ_2 represent the volume fractions of the nanoparticles AA7072 and AA7075, respectively. The symbols ρ_{hnf} , μ_{hnf} , k_{hnf} , and $(\rho C_p)_{hnf}$ denote the density, dynamic viscosity, thermal conductivity, and heat capacity of the hybrid nanofluid, respectively. The corresponding properties of the base fluid are represented by ρ_f , μ_f , k_f , and $(\rho C_p)_f$.

Table 1: Characteristic properties of hybrid nanofluid (Takabi & Salehi 2014)

Characteristics	Hybrid nanofluid
Dynamic viscosity,	$\mu_{hnf} = \mu_f / (1 - \phi_{hnf})^{2.5}$
Thermal capacity,	$(\rho C_p)_{hnf} = (1 - \phi_{hnf})(\rho C_p)_f + \phi_1(\rho C_p)_{n1} + \phi_2(\rho C_p)_{n2}$
Density,	$\rho_{hnf} = (1 - \phi_{hnf})\rho_f + \phi_1\rho_{n1} + \phi_2\rho_{n2}$
Thermal conductivity, k_{hnf}	$= \left[\frac{\left(\frac{\phi_1 k_{n1} + \phi_2 k_{n2}}{\phi_{hnf}} \right) + 2k_f + 2(\phi_1 k_{n1} + \phi_2 k_{n2}) - 2\phi_{hnf} k_f}{\left(\frac{\phi_1 k_{n1} + \phi_2 k_{n2}}{\phi_{hnf}} \right) + 2k_f - (\phi_1 k_{n1} + \phi_2 k_{n2}) + \phi_{hnf} k_f} \right] \times k_f$

Table 2: The nanoparticles and base fluid properties (Tlili *et al.* 2020; Sharma *et al.* 2023)

Characteristics	C_p (J/kgK)	k (W / mK)	ρ (kg/m ³)	$\beta \times 10^{-6}$ (1/K)	Pr
AA7072	893	222	2720	23.6	
AA7075	960	173	2810	23.5	
CH ₃ OH	2545	0.2035	792	149	7.38

The physical quantities of interest are the skin friction coefficient and the local Nusselt number.

The skin friction coefficient is defined as $C_f = \frac{\tau_w}{\rho_f U_w^2}$, where the wall shear stress is given by

$\tau_w = \mu_{hnf} \left(\frac{\partial u}{\partial y} \right)_{y=0}$. The local Nusselt number is defined as $Nu_x = \frac{xq_w}{k_f (T_w - T_\infty)}$, where the wall

heat flux is $q_w = -k_{hnf} \left(\frac{\partial T}{\partial y} \right)_{y=0}$. Using the similarity transformations, these expressions reduce

to

$$\text{Re}_x^{1/2} C_f = \frac{\mu_{hnf}}{\mu_f} f''(0), \quad \text{Re}_x^{-1/2} Nu_x = - \left(\frac{k_{hnf}}{k_f} + \frac{4}{3} R \right) \theta'(0), \quad (12)$$

where the local Reynolds number is defined as $\text{Re}_x = \frac{U_w x}{\nu_f}$.

3. Results and Discussion

This section describes and analyses the numerical results concerning the flow and heat transfer characteristics of a couple stress hybrid nanofluid past a shrinking Riga plate with thermal radiation. The transformed nonlinear ordinary differential equations are solved numerically as a boundary value problem using MATLAB's bvp4c solver. The solver employs a collocation method with adaptive mesh selection to ensure numerical stability and accuracy. Suitable initial guesses are provided to facilitate convergence, and the obtained results are validated through comparison with existing literature.

In this study, the impact of the couple stress parameter (ξ), nanoparticles volume fractions (ϕ_1, ϕ_2), suction (S), and thermal radiation parameter (R), is investigated in detail. The nanoparticle volume fractions ϕ_1 and ϕ_2 correspond to aluminum alloys AA7072 and AA7075, respectively, which together constitute the hybrid nanofluid dispersed in methanol as the base fluid. The outcomes of the findings are presented in relation to the Nusselt number ($Re_x^{-1/2} Nu_x$) and skin friction coefficient ($Re_x^{1/2} C_f$), as well as velocity $f'(\eta)$ and temperature profile $\theta(\eta)$ distributions, to examine the effects of the significant governing parameters.

The ranges of the governing parameters considered in this study are selected based on physically realistic regimes reported in previous literature. The suction parameter is restricted to positive values to ensure boundary layer stability in shrinking sheet configurations. The couple stress parameter is varied within moderate values to represent realistic microstructural effects without violating boundary layer assumptions. Larger values of the couple stress parameter may produce stronger microstructural interactions and could be explored in future studies for a broader parametric investigation. Meanwhile, the electromagnetic parameter is selected within ranges commonly adopted in Riga plate studies to avoid excessive Lorentz forcing that would invalidate similarity reduction.

Commonly, shrinking surface flows are associated with dual solution behaviour due to the adverse pressure gradient and possible boundary layer separation. However, in the present study, the numerical computations produced only a unique solution within the considered parameter ranges. This behaviour may be attributed to the stabilizing influence of suction together with the electromagnetic Lorentz force generated by the Riga plate, which suppresses flow reversal and enhances boundary layer stability. Consequently, stability analysis was not applicable in the present study since no secondary solution branch was detected.

The results highlight the complex interplay between electromagnetic force (from the Riga plate), couple stress effects, and thermal phenomena (influenced by hybrid nanoparticles and radiation effects). Graphical illustrations are provided to facilitate interpretation and comparisons with existing published results are made to validate the accuracy and reliability of the present model. The dependability of the outcomes is compared to that of Waini *et al.* (2023) in Table 3. As observed in Table 3, the results from the present study exhibit excellent agreement with those reported in the literature, showing a maximum absolute discrepancy of only $\sim 10^{-8}$, which is negligible and within acceptable numerical tolerance. This consistency strongly supports the correctness and accuracy of the mathematical model, the similarity transformation, and the implementation of the numerical scheme using MATLAB's bvp4c solver. This comparative validation also reinforces the reliability of the present formulation.

Moreover, to ensure numerical accuracy and convergence, the solver was executed with a relative tolerance of 10^{-6} and an absolute tolerance of 10^{-8} . These convergence criteria were verified to be sufficiently strict by conducting sensitivity analyses and confirming that further

tightening of the tolerances produced no significant changes in the computed results. Additionally, the computational domain for the similarity variable η was extended progressively until the solutions exhibited asymptotic behavior, ensuring domain independence and numerical stability.

Table 3: Results generation of $\text{Re}_x^{1/2} C_f$ for various ϕ when $Z = 0, \xi = 0.01, \lambda = S = 1, \text{Pr} = 7.38$

ϕ_1	$\phi_2 = 0$		$\phi_2 = 0.005$	
	Waini <i>et al.</i> (2023)	Present	Waini <i>et al.</i> (2023)	Present
0	−9.392542	−9.392546	−9.513503	−9.513505
0.005	−9.510239	−9.510237	−9.634587	−9.634586

Figure 2 illustrates the variation of the reduced skin friction coefficient, $\text{Re}_x^{1/2} C_f$, with the shrinking parameter λ for different values of the couple stress parameter, ξ . It is observed that the skin friction coefficient decreases as the couple stress parameter increases from 0.05 to 0.015. Physically, a larger couple stress parameter enhances the microstructural resistance of the fluid, thereby increasing the effective resistance to the fluid motion. Consequently, the wall shear stress is reduced, leading to a lower skin friction coefficient. Moreover, for all values of ξ , the skin friction coefficient decreases as the shrinking parameter becomes less negative, indicating a gradual reduction in the wall shear generated by the shrinking surface. The clear separation between the curves demonstrates that the couple stress parameter has a significant influence on the momentum transport within the boundary layer.

Figure 3 presents the variation of the reduced local Nusselt number $\text{Re}_x^{-1/2} Nu_x$, with the shrinking parameter λ for different values of ξ . It can be seen that increasing the couple stress parameter enhances the local Nusselt number, indicating an improvement in the wall heat transfer rate. The stronger couple stress effect modifies the velocity distribution within the boundary layer, resulting in a steeper temperature gradient at the wall. Consequently, more thermal energy is transferred from the surface to the fluid, producing higher values of the reduced local Nusselt number. In addition, the local Nusselt number decreases gradually as the shrinking parameter becomes more negative, suggesting that stronger shrinking weakens the thermal transport near the surface. The distinct separation of the curves confirms that the heat transfer characteristics remain sensitive to variations in the couple stress parameter over the extended parameter range considered. The present results further demonstrate that stable numerical solutions are successfully obtained for an extended range of the couple stress parameters, thereby addressing the limitation identified in the previous study and providing a broader assessment of the influence of the couple stress parameters on the flow and heat transfer characteristics.

Figure 4 depicts the variation of the dimensionless velocity profile $f'(\eta)$, for different values of the couple stress parameter ξ . It is evident that increasing ξ from 0.05 to 0.15 suppresses the fluid velocity across the boundary layer. This behaviour is attributed to the enhanced couple stress effect, which increases the effective resistance to fluid motion and weakens the momentum transport near the shrinking surface. As a result, the momentum boundary layer decreases, allowing the velocity profile to attain the ambient condition at a shorter similarity distance. The enlarged view further confirms the gradual decrease in the velocity profile as the couple stress parameter increases.

Figure 5 illustrates the influence of the couple stress parameter ξ on the dimensionless temperature profile $\theta(\eta)$. It is observed that varying the couple stress parameters produces only a slight variation in the temperature distribution over the investigated parameter range. The temperature profiles remain very close to one another, indicating that the couple stress parameter has a relatively weak influence on the thermal field compared with its effect on the momentum field. This behaviour is clearly illustrated in the enlarged inset, where only a marginal separation between the temperature profiles can be observed. Nevertheless, the numerical results reveal a gradual increase in the local Nusselt number with increasing values of ξ , implying that the wall temperature gradient is slightly enhanced despite the relatively small changes in the overall temperature distribution.

Figure 6 illustrates the impact of ϕ_1, ϕ_2 on $\text{Re}_x^{1/2} C_f$. Three different cases are noted, which are a base fluid ($\phi_1 = \phi_2 = 0.00$), a single-component of nanofluid ($\phi_1 = 0.00, \phi_2 = 0.01$), and a hybrid nanofluid ($\phi_1 = \phi_2 = 0.01$). The presence of nanoparticles is known to increase the shear stress at the interface, which is validated by the increase in $\text{Re}_x^{1/2} C_f$ when the base fluid, single-component nanofluid, and hybrid nanofluid are compared. The base fluid represented by the solid line has the lowest value of $\text{Re}_x^{1/2} C_f$, showing minimum resistance to shear forces compared to others. The fluid's effective viscosity is increased due to the presence of nanoparticles, leading to higher momentum diffusion and thus higher values of $\text{Re}_x^{1/2} C_f$. In most cases, the hybrid nanofluid has the highest value of $\text{Re}_x^{1/2} C_f$, which suggests that, when combined, the different nanoparticles tend to increase the resistance to flow because of higher viscosity and thermal conductivity.

Figure 7 shows the relation of $\text{Re}_x^{-1/2} Nu_x$ and ϕ_1, ϕ_2 . The presence of nanoparticles greatly improves the heat transfer rate over the shrinking Riga plate as evident from the variation of the $\text{Re}_x^{-1/2} Nu_x$ with respect to the shrinking parameter, λ . The base fluid scenario, where no nanoparticles are present ($\phi_1 = \phi_2 = 0.00$), demonstrates the lowest $\text{Re}_x^{-1/2} Nu_x$ in the range of λ . When a single type of nanoparticle is added ($\phi_1 = 0.00, \phi_2 = 0.01$), a noticeable enhancement in heat transfer is observed, indicating the beneficial role of the nanoparticles in improving thermal conductivity. This suggests that the addition of the nanoparticles assists in improving thermal conductivity since heat transfer becomes noticeable. The presence of hybrid nanofluid further escalates the heat transfer when both types of nanoparticles are present in equal volume fractions, demonstrating the synergistic effect of hybrid nanoparticles as opposed to single-type nanofluids. These observations lead to the conclusion that the Nusselt number increases due to the presence of nanoparticles, which indicates that the thermal boundary layer improves with suction and electromagnetic forcing from the Riga plate.

The effect of the suction parameter S on $\text{Re}_x^{1/2} C_f$ as depicted in Figure 8, indicates that $\text{Re}_x^{1/2} C_f$ increases when suction is increased. This means that greater amounts of suction increase the frictional resistance to the flow of the couple stress hybrid nanofluid past the shrinking Riga plate, which is contrary to what is ordinarily expected, where suction is generally associated with an increase in skin friction due to the thinning of the boundary layer (Bhattacharyya 2011; Rohni *et al.* 2013). Therefore, the increase in $\text{Re}_x^{1/2} C_f$ with increasing suction indicates that the combined effect of suction and the specific behaviour of the couple stress hybrid nanofluid is that suction draws the fluid towards the plate, which augments the frictional resistance associated with the fluid motion. This observation is only valid under the set conditions developed in this problem, which showcases the particular features of couple stress hybrid nanofluids with a shrinking Riga plate.

Alternatively, Figure 9 depicts the effect of S on $\text{Re}_x^{-1/2} Nu_x$, where an increase in suction improves the heat transfer rate. This trend derives from suction removing the low-energy fluid near the surface, which decreases the thermal boundary layer and increases the temperature

gradient at the wall. This improves the heat transfer efficiency, resulting in an increase in the Nusselt number. Also, based on the results, it can be highlighted that stronger suction enhances heat transfer effectiveness, making it a beneficial mechanism for thermal control in various real-world scenarios.

The impact of S on the velocity $f'(\eta)$ and temperature profiles $\theta(\eta)$, are depicted in Figures 10 and 11, respectively. As S increases (from 2.0 to 3.5), the velocity $f'(\eta)$ is decreased, indicating a reduction in the velocity of the fluid near the surface of the plate (see Figure 10). This behavior is typical for suction, as it removes fluid from the boundary layer, thereby reducing the velocity of the fluid in that region. The steeper the negative slope of the velocity profile, the stronger the suction effect, which leads to a deceleration of the fluid near the plate. Hence, with increasing suction, the velocity decreases further, causing a thicker boundary layer with lower flow rates close to the surface. The impact of S on the temperature profile $\theta(\eta)$ is illustrated in Figure 11. It can be noted from Figure 11 that an increase in S leads to a considerable decrease in the thickness of the temperature boundary layer. The enhancement of S reduced the temperature profile, which is indicated by the decreasing curves. The temperature variations are larger closer to the surface, implying there an improvement in the heat transfer efficiency.

Figure 12 exemplifies the effects of thermal radiation (R) on $\text{Re}_x^{-1/2} Nu_x$. When R increases from 0.10 to 0.20, the heat transfer is decreased, as indicated by $\text{Re}_x^{-1/2} Nu_x$. This implies that with higher thermal radiation, heat transfer becomes less effective in the system. This also suggests that the fluid's energy balance may hinder effective heat conduction within the fluid. In many cases, thermal radiation can lead to energy being lost in the form of radiation rather than being transferred to the fluid. This, in turn, lowers the flowing medium's convective heat transfer rate in the vicinity of the plate. Thus, with the rise of R , the fluid's ability to absorb and transfer heat decreases, which is reflected in the downward trend of the heat transfer rate $\text{Re}_x^{-1/2} Nu_x$.

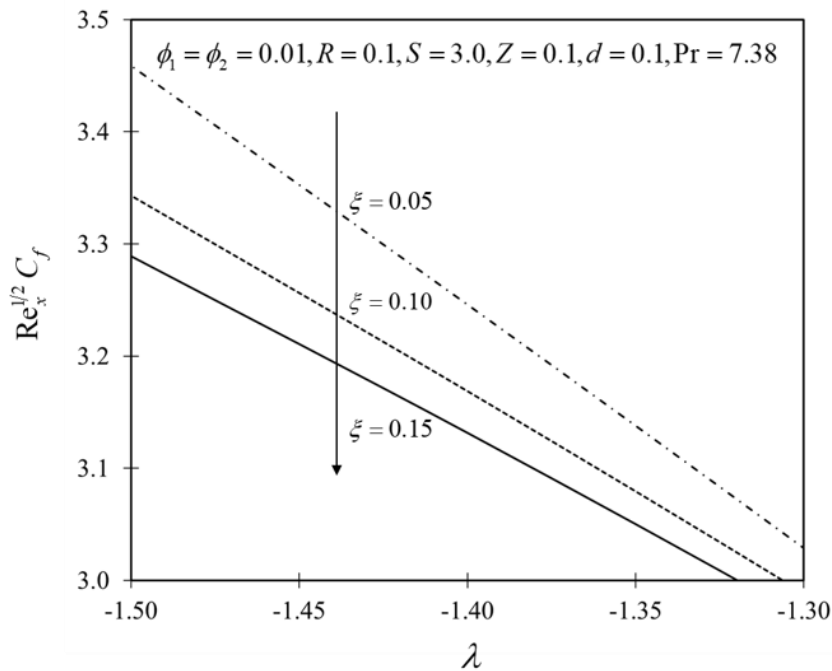


Figure 2: Trends of $\text{Re}_x^{1/2} C_f$ towards λ with different ξ

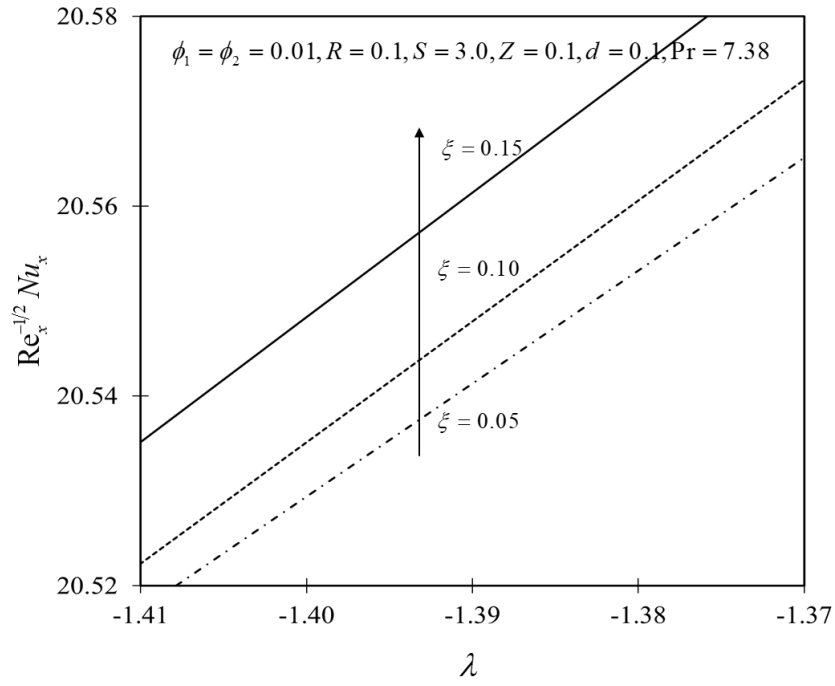


Figure 3: Trends of $Re_x^{-1/2} Nu_x$ versus λ with different ξ

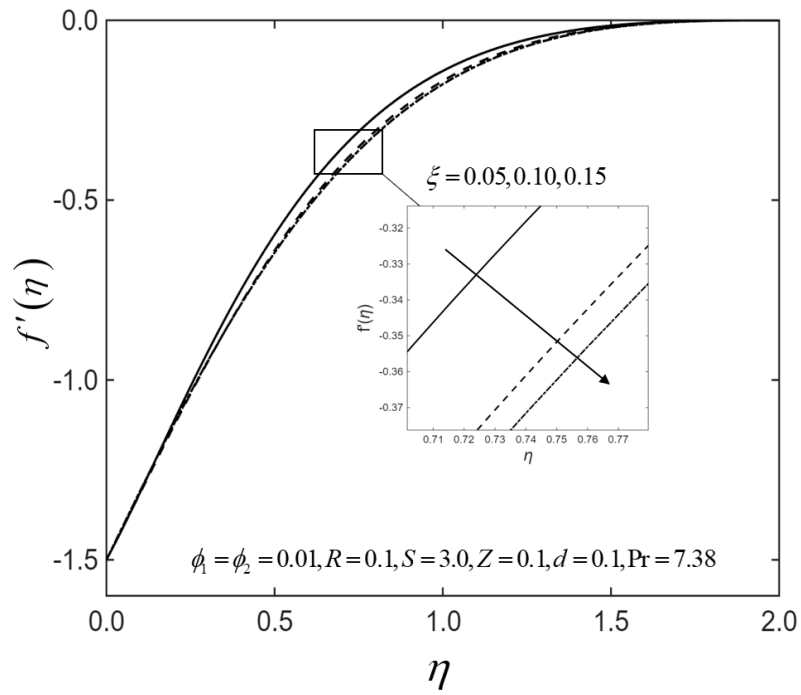


Figure 4: Trends of the velocity distribution profile $f'(\eta)$ with different ξ

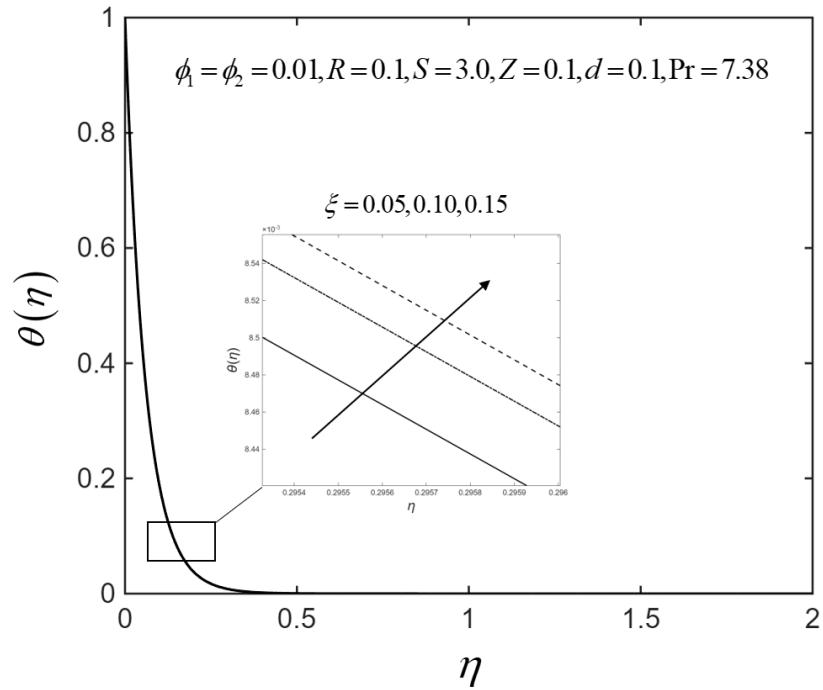


Figure 5: Trends of the temperature distribution profile $\theta(\eta)$ with different ξ

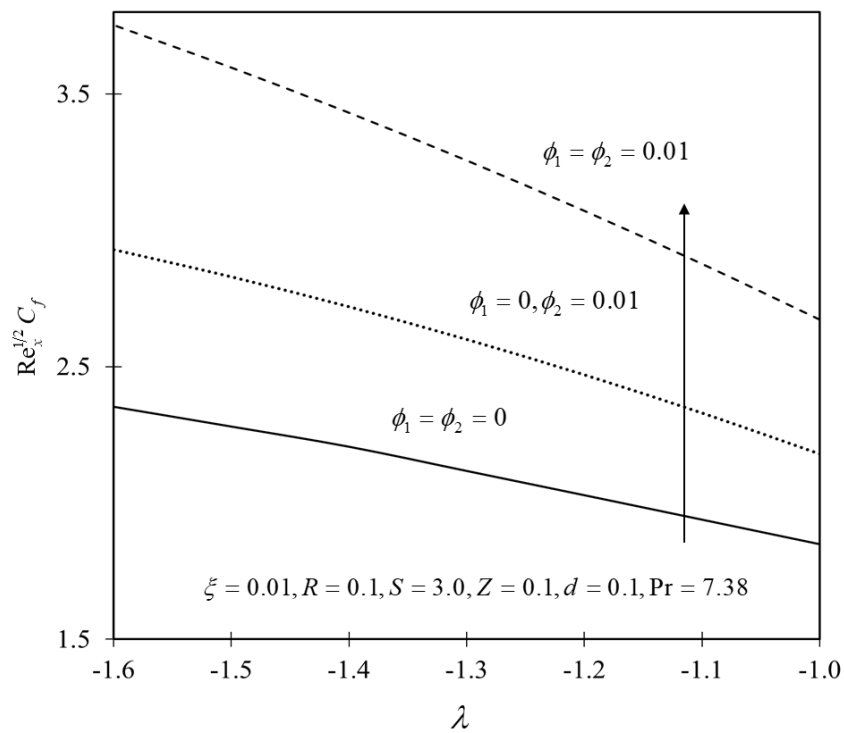


Figure 6: Trends of $\text{Re}_x^{1/2} C_f$ towards λ with different ϕ_1, ϕ_2

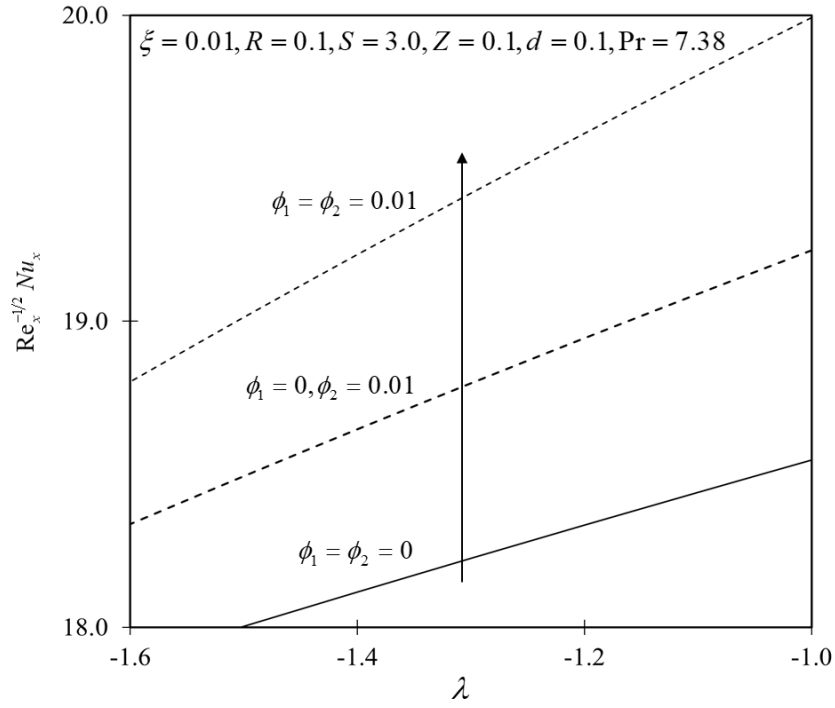


Figure 7: Trends of $\text{Re}_x^{-1/2} Nu_x$ towards λ with different ϕ_1, ϕ_2

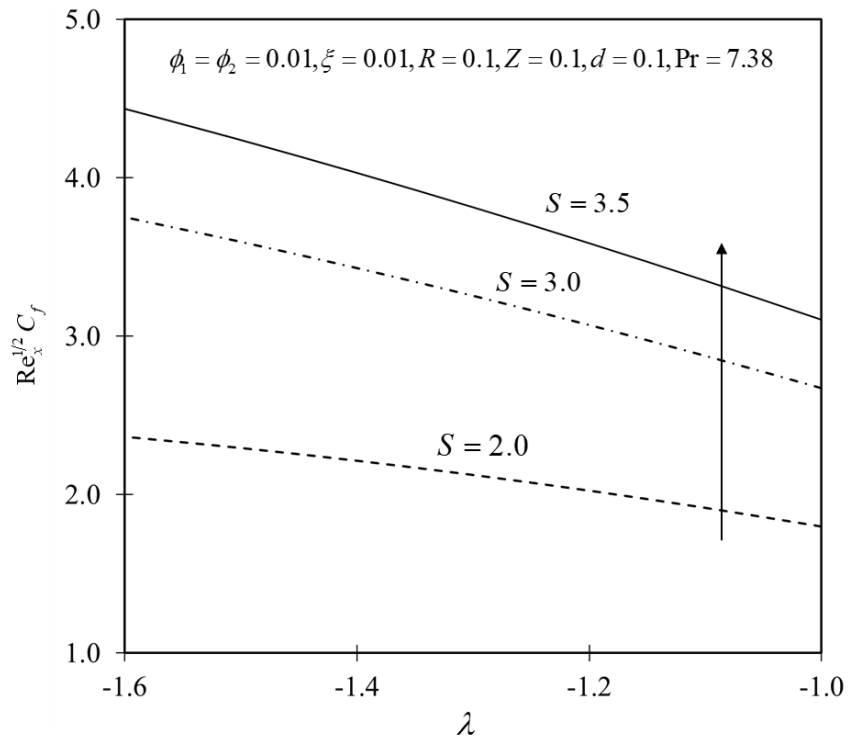


Figure 8: Trends of $\text{Re}_x^{1/2} C_f$ towards λ with different S

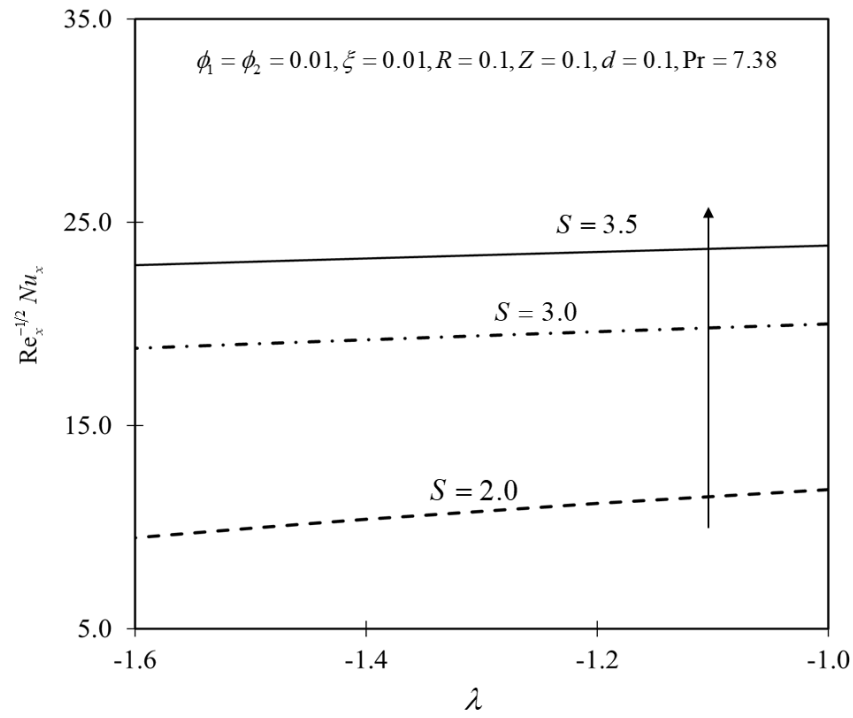


Figure 9: Trends of $\text{Re}_x^{-1/2} \text{Nu}_x$ towards λ with different S

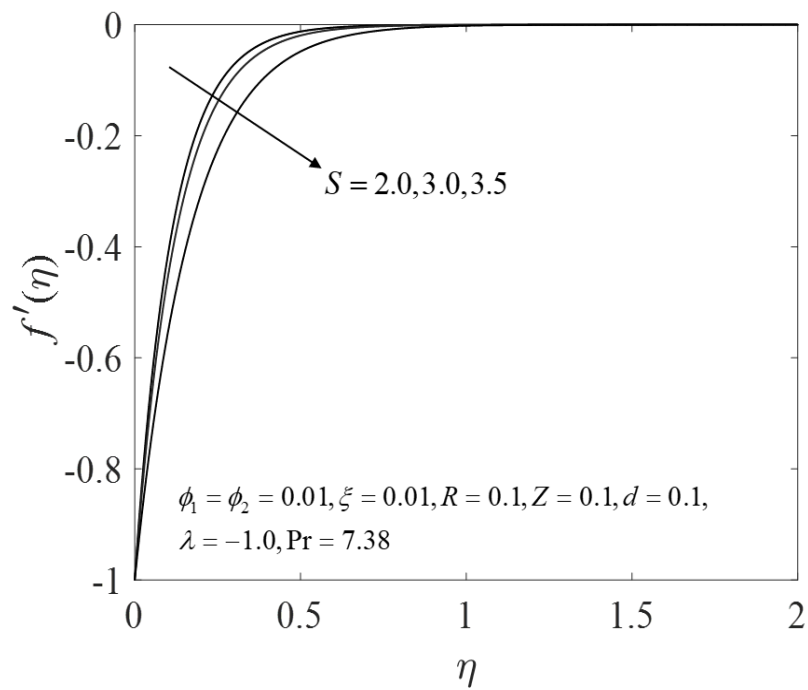


Figure 10: Trends of the velocity distribution profile $f'(\eta)$ with different S

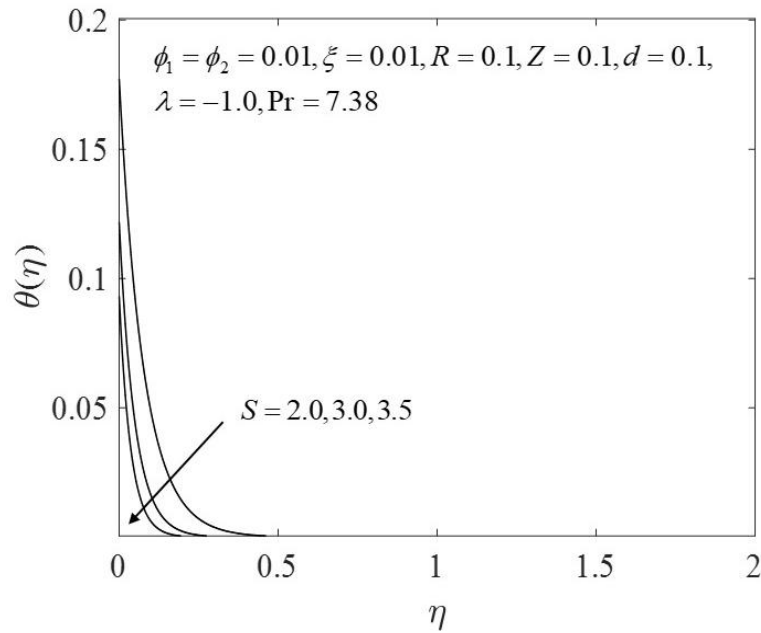


Figure 11: Trends of temperature distribution profile $\theta(\eta)$ with different S

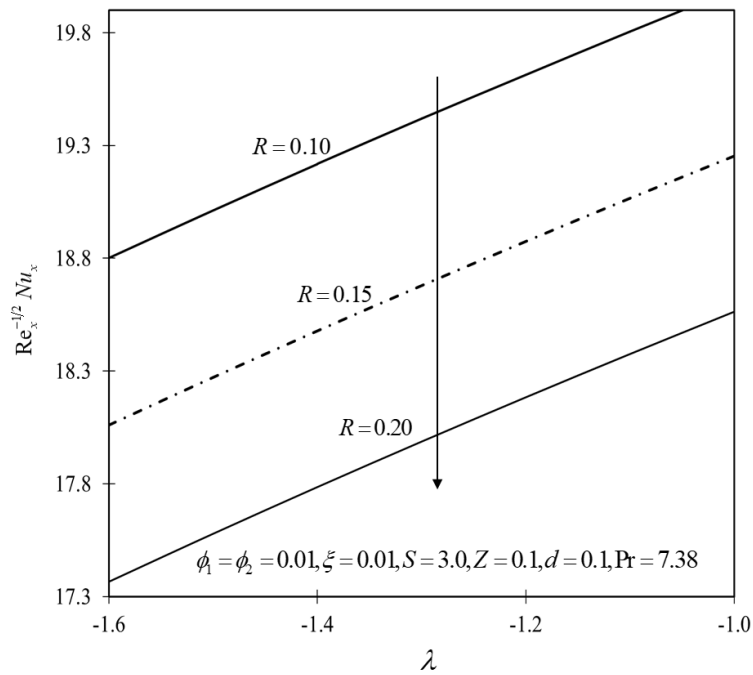


Figure 12: Trends of $Re_x^{-1/2} Nu_x$ towards λ with different R

4. Conclusion

This study investigated the radiative flow and heat transfer characteristics of a couple stress hybrid nanofluid over a permeable shrinking Riga plate using aluminum alloys (AA7072–AA7075) nanoparticles dispersed in methanol (CH_3OH) as the base fluid. The numerical results revealed that increasing the couple stress parameter decreases the skin friction coefficient while

slightly enhancing the local Nusselt number, indicating reduced wall shear accompanied by a modest improvement in the heat transfer rate. These trends demonstrate that couple stress effects exert a stronger influence on the momentum field than on the thermal field. Furthermore, increasing the suction parameter enhances both the skin friction coefficient and the local Nusselt number owing to the thinning of the momentum and thermal boundary layers, whereas increasing the thermal radiation parameter reduces the heat transfer rate. Overall, the present findings demonstrate that the combined effects of couple stress, suction, thermal radiation, and Riga plate electromagnetic forcing play important roles in controlling the flow and thermal characteristics of hybrid nanofluids over shrinking surfaces. The outcomes of this study provide useful theoretical insight for the design and optimization of advanced thermal management systems, cooling technologies, microfluidic devices, and other engineering applications involving electromagnetic flow control and hybrid nanofluids.

5. Limitations of the Study

The present analysis is subject to several modelling assumptions that may limit its applicability to real-world systems. First, the study is based on the boundary layer approximation and steady, two-dimensional laminar flow, which may not fully capture three-dimensional or turbulent effects occurring in practical applications. Second, the thermophysical properties of the hybrid nanofluid are assumed to be uniform and constant, except where specified, and nanoparticle agglomeration, sedimentation, and slip mechanisms are neglected. Third, the electromagnetic parameter associated with the Riga plate is considered constant, whereas in practical systems it may vary spatially due to non-uniform magnetization or electrode current distribution. Furthermore, the model also relies on similarity transformations, which restrict the analysis to specific flow configurations. Therefore, the results provide fundamental theoretical insight but may require experimental validation or more comprehensive numerical simulations for broader engineering implementation.

6. Future Research Directions

Future investigations may consider unsteady flow effects, three-dimensional configurations, and variable fluid properties to better represent practical engineering systems. The inclusion of additional physical mechanisms such as viscous dissipation, Joule heating, or nanoparticle agglomeration effects may further enhance model realism. Moreover, experimental validation or high-fidelity numerical simulations would be valuable to assess the applicability of the present theoretical findings in real-world thermal management and microfluidic systems.

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