

DYNAMIC FUZZY-PROBABILISTIC MODEL USING A CHANCE-CONSTRAINED PROGRAMMING APPROACH FOR OPTIMISING THE PERFORMANCE OF WASTEWATER TREATMENT SYSTEMS

SUTRISNO*, WIDOWATI, SUNARSIH, KARTONO & TOSPORN ARREERAS

ABSTRACT

This study presents an innovative optimisation approach using a chance-constrained probabilistic-fuzzy optimization model to improve the performance of facultative ponds, which play a crucial role in wastewater treatment systems. The study focuses on optimising the treatment process based on the biological oxygen demand parameter. In this framework, several parameters, including the biological oxygen demand degradation rate, are modelled as random variables with specified probability distributions, while others, such as wastewater load, are treated as fuzzy variables with decision-maker-defined membership functions. Under these uncertainties, the objective is to maximise the treated wastewater volume while ensuring adequate safety margins for both objective and constraint functions through chance-constrained approaches. A practical case study of a wastewater treatment facility located in Bantul, Indonesia, is used to determine the optimal wastewater flow rates and processing durations. The findings show that the proposed model effectively addresses the optimization problem by producing the optimal volume of wastewater entering the inlet per day and the detention time, offering a valuable decision-support tool for similar wastewater treatment settings.

Keywords: facultative pond; probabilistic-fuzzy optimization; sewage treatment facility; uncertain programming

1. Introduction

Proper stabilisation of wastewater before disposal is essential for maintaining environmental and water sustainability. Given the limited number of treatment plants, optimising their operational efficiency is crucial for increasing treatment capacity. Various indicators, such as biological oxygen demand (BOD), are employed to evaluate water quality after treatment (Ye *et al.* 2024). Domestic wastewater, which originates from households, hotels and general industrial activities, constitutes the most common category of wastewater (Onu *et al.* 2023). Facultative ponds are widely used in wastewater treatment facilities due to their effectiveness in reducing pollutant concentrations until they reach standard levels, as indicated by BOD values. In addition, mathematical optimisation techniques are applied to improve the efficiency and processing capacity of facultative ponds in wastewater treatment systems. More broadly, advanced analytical and computational methods have also been developed for solving nonlinear and fractional integro-differential problems, demonstrating the growing role of mathematical techniques in modelling complex dynamic systems (Eshkuvatov *et al.* 2025; Alhawarat *et al.* 2021).

A range of models has been introduced for wastewater treatment management, each designed according to the specific challenges faced by decision-makers (Zhang *et al.* 2021; Mahajan *et al.* 2021; Zhang & Li 2021). A straightforward model was introduced by Huang *et al.* (2017) to manage pollutant concentration using a quantitative prototype. Other simplified models include a linear model with deterministic parameters (Sunarsih *et al.* 2018), a model focusing on adsorbents for wastewater treatment analysis (Gopakumar *et al.* 2018), a quantitative approach for sewage treatment (Kogo *et al.* 2017), and a model for estimating wastewater treatment plant construction costs (Olukanni & Ducoste 2011). Beyond wastewater treatment optimisation, additional models have been developed for sewage management (Recio-Garrido *et al.* 2018),

energy analysis (Borzooei *et al.* 2019), effluent and sludge analysis (Benito *et al.* 2020; Khatri *et al.* 2023), microplastic removal (Zhang *et al.* 2020), and (bio)energy generation from wastewater (Shahid *et al.* 2019; Zhang & Liu 2020; Tobin *et al.* 2020; Tawfik *et al.* 2022; Koffi & Okabe 2022; Caligan *et al.* 2022). To the best of the authors' knowledge, no model has been formulated under a chance-constrained approach that permits probabilistic constraints to be applied to both random and fuzzy parameters. Only a preliminary study by the authors Armanda *et al.* (2025) is available in the literature. Some other studies have proposed similar models; however, only one type of uncertainty, either probabilistic or fuzzy, was considered (Kartono *et al.* 2024a,b). Nevertheless, the preliminary study dealt with a static case, in which the problem was solved for only one observation period, and the model could not be used for multiple optimisation time periods. Thus, this study addresses this research gap.

In response to this need, a new approach is developed to improve the performance of facultative ponds in wastewater treatment by accounting for uncertainties, including variable inlet pollutant concentrations. The proposed framework allows decision-makers to set probabilistic constraints that control the likelihood of exceeding uncertain limits. These uncertainties are represented as fuzzy parameters specified by membership functions elicited from decision-maker expertise. The model is applied to BOD analysis using the facultative pond system in Yogyakarta Province, Indonesia, to obtain optimal operational decisions.

2. Proposed Framework

2.1. Model Setting and Underlying Assumptions

The proposed model examines BOD removal in facultative ponds, with the wastewater treatment plant in Yogyakarta Province used as the case study. The plant manages domestic wastewater generated from industries, households, hospitality sectors and offices through several treatment stages, as illustrated in Figure 1. The descriptions of the symbols used in the model are provided in Table 1.

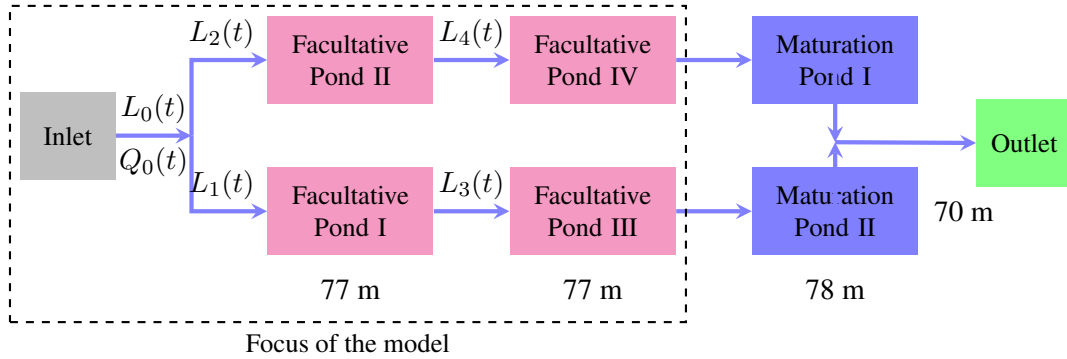


Figure 1: Layout illustration of the Sewon wastewater treatment facility located in Yogyakarta Province. Wastewater enters through the inlet, is processed in facultative ponds and maturation ponds, and is subsequently discharged into the river through the outlet

Figure 1 shows that the facility consists of four facultative ponds and two maturation ponds. Wastewater enters the treatment plant through the inlet, where it undergoes preliminary filtration to remove coarse suspended solids and debris. Each day, the wastewater volume, measured in m^3 , and the waste load, measured as the mass of pollutant in kg , are recorded. Approximately half of the filtered wastewater enters facultative pond I, while the other half enters facultative pond II. Based on probabilistic proportions, part of the wastewater load is subsequently transferred to facultative ponds III and IV, as described in the mathematical models in (5) and (6).

After a specified detention time, the treated wastewater enters the maturation ponds and is finally discharged into the river through the outlet.

Since the main pollutant degradation process occurs in the facultative ponds, this study focuses on this component of the treatment system. The primary objective is to optimise the volume of wastewater treated across all facultative ponds while satisfying the required BOD standards under various uncertain parameters. The specific assumptions and conditions considered in this study are as follows:

- Observations and optimisation were conducted over a one-day period.
- BOD was used as the primary parameter for assessing wastewater quality.
- The facility exclusively processed domestic wastewater originating from Yogyakarta Province.
- The inlet waste load, measured daily, was characterised by fuzzy uncertainty. Its membership function was determined by the decision-maker using observational and historical information.
- The BOD degradation rate was represented as a fuzzy uncertain parameter, with its membership function derived from secondary data and direct observations. BOD levels were analysed from wastewater samples obtained at designated grid points within the facultative ponds.
- An index value was defined in the mathematical model to regulate BOD degradation.
- Discrete membership functions were adopted to represent all fuzzy parameters.

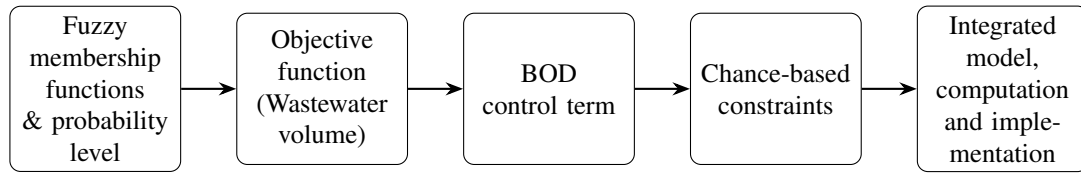


Figure 2: Research procedure adopted in this study

Table 1: Mathematical symbols

Symbol	Unit	Description	Category
$Q_0(t)$	$[m^3]$	Inflow volume of wastewater at the inlet on day t	Decision variable
$Q_i^e(t)$	$[m^3]$	Inflow volume of wastewater at facultative pond i on day t	Decision variable
$d(t)$	[day]	Wastewater processing duration on day t	Decision variable
$L_0(t)$	$[kg]$	Incoming waste load at the treatment plant inlet on day t	Fuzzy parameter
$k(t)$	[%]	BOD degradation rate per day of detention time on day t	Probabilistic parameter
$L_i(t)$	$[kg]$	Incoming waste load at the inlet of pond i on day t	Intermediate parameter
$L_i^e(t)$	$[kg]$	Amount of waste load processed in pond i on day t	Intermediate parameter
$E_i^r(t)$	[%]	Target or reference point for the BOD degradation efficiency index on day t	Deterministic parameter
BM	$[mg/L]$	Wastewater BOD concentration quality standard	Deterministic parameter
$C_i(t)$	$[mg/L]$	BOD concentration in pond i on day t	Auxiliary parameter
$E_i(t)$	[%]	BOD removal efficiency index in pond i on day t	Auxiliary parameter
p_1	[%]	Proportion of waste load entering pond III from pond I	Probabilistic parameter
p_2	[%]	Proportion of waste load entering pond IV from pond II	Probabilistic parameter
$\alpha_i, \beta, \gamma_i$	[%]	Probability of satisfying the chance-based constraints	Deterministic parameter

The methodological framework adopted in this study is outlined as follows. A summary of the research procedure is presented in Figure 2.

- Step 1.* The membership functions for the fuzzy parameters are defined by the decision-maker, who also specifies the probability of satisfying the lower-bound chance-based constraints in the constraint functions.
- Step 2.* The wastewater inflow rate is formulated as the objective function.
- Step 3.* A quadratic deviation between the actual BOD level and a reference value is used to construct the control term. The reference value is selected based on the decision-maker's knowledge and judgement of facultative pond operations.
- Step 4.* The constraint functions are established and incorporated into the model by considering the design of the wastewater treatment facility and its essential operating conditions.
- Step 5.* For computational implementation, the model is implemented in LINGO 21.0 and solved using its native solver. Chance-based programming is then applied to determine the optimal decision (LINDO Systems Inc. 2017). The computed solution is subsequently implemented in the facility.

2.2. Model

The notations used to construct the programming model are listed in Table 1. Two objective functions are considered, namely maximising the wastewater inflow and minimising the quadratic deviation of the BOD level from the target value established by the facility planner. Combining these objectives leads to the following optimisation problem, which is described component by component thereafter:

$$\min Z = \mathbb{E} \left[\sum_{t=1}^T \left(-Q_0(t) + \sum_{i=1}^4 [E_i(t) - E_i^r(t)]^2 \right) \right]. \quad (1)$$

Subject to, for all $t = 1, 2, \dots, T$,

$$L_i^e(t) = \frac{Q_i^e(t) \cdot C_i(t)}{1000}, \quad \forall i \in \{1, 2, 3, 4\}; \quad (2)$$

$$C_i(t)E_i(t) \leq BM, \quad \forall i \in \{1, 2, 3, 4\}; \quad (3)$$

$$Cr\{L_1(t) + L_2(t) = L_0(t)\} \geq \beta; \quad (4)$$

$$Cr\{L_3^e(t) = (1 - p_1)L_1(t)\} \geq \gamma_1; \quad (5)$$

$$Cr\{L_4^e(t) = (1 - p_2)L_2(t)\} \geq \gamma_2; \quad (6)$$

$$Q_1^e(t) + Q_2^e(t) = Q_0(t), \quad Q_3^e(t) = 0.5Q_1^e(t), \quad Q_4^e(t) = 0.5Q_2^e(t); \quad (7)$$

$$Cr \left\{ E_i(t) = \frac{d(t) \cdot k(t)}{d(t) \cdot k(t) + 1} \right\} \geq \alpha_i, \quad \forall i \in \{1, 2, 3, 4\}. \quad (8)$$

The explanation of the model is elaborated as follows. First, the notation $\mathbb{E}[\cdot]$ represents the expectation of the argument. Note that the term in the objective (1) is multiplied by negative one because it was originally intended to be maximised. After that, the equality and inequality constraints (2) to (8) for this model are respectively explained as follows:

- Equality (2) is used to quantify the waste load based on the inflow rate and the organic matter content.
- Inequality (3) ensures that the BOD level is less than the specified upper bound, namely the quality standard.
- Constraint (4) is formulated to represent that the total waste load processed in ponds I and II is approximately equal to the pre-treatment load at the inlet.
- The equality (5) approximates the waste load transferred from pond I to pond III, while (6) approximates the transfer from pond II to pond IV.
- The equalities in (7) describe that the total inflow to ponds I and II balances the inlet inflow, whereas the inflows to ponds III and IV are each half of those to ponds I and II, respectively.
- The chance-based inequality (8) represents the required BOD degradation efficiency index.

Merging the objective with the constraints produces a chance-based optimisation problem. The chance-based programming technique is used to identify the optimal decision, and the closedness and boundedness of the constraints ensure that an optimal solution exists whenever the feasible region is non-empty.

This optimisation was then solved using the chance programming method, see e.g. Liu (2015) for the technical details. First, note that from the chance programming theory, since all uncertain parameters $L_0(t)$, $k(t)$, p_1 and p_2 appear only in the constraints and not in the objective function, the chance-based functions are required solely for the constraint functions. Furthermore, in treating the fuzzy parameters, the calculation of weight values is implemented first. Notably, the membership functions are discrete. Discrete membership functions were employed due to their simplicity and suitability for practical applications, where uncertainty is often described by a limited number of observed or estimated values. This facilitates efficient computation and straightforward interpretation. Furthermore, this approach allows the model to capture imprecision effectively without introducing unnecessary analytical complexity, as would be the case with continuous membership functions such as triangular or trapezoidal forms.

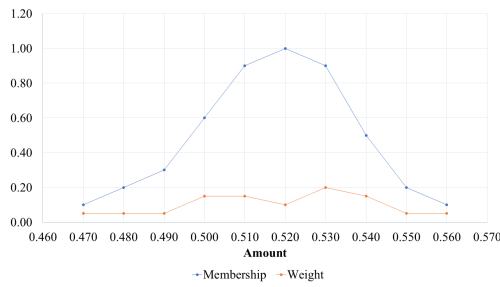
Subsequently, the obtained deterministic equivalent is solved using deterministic optimisation algorithms. In this study, the algorithm of the gradient-based method, embedded in LINGO 21.0, was employed. Finally, the optimal solution obtained from the computation is implemented.

3. Results and Discussion

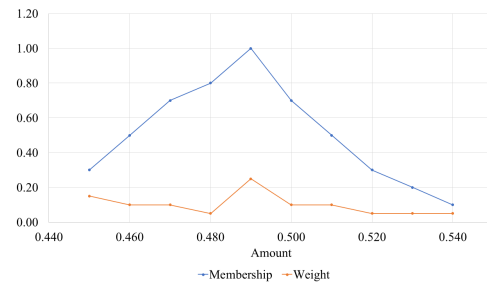
This study is based on the layout of the wastewater treatment site located in Bantul, Indonesia, where the treatment flow is presented in Figure 1. The parameters and findings of the chance-constrained probabilistic-fuzzy programming model are provided in the subsections that follow.

3.1. Parameter Setting

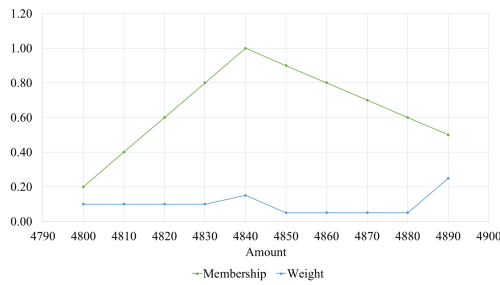
The fuzzy membership functions were constructed by randomly sampling values near the mean of the observational data reported in (Sunarsih *et al.* 2018). The corresponding membership function values and their weights are depicted in Figures 3a, 3b and 3c. In accordance with the local government regulations of Yogyakarta Province, the upper bound for the BOD level in treated wastewater is 50 mg/L . In particular, the BOD degradation rate k was assumed to follow a normal distribution with mean 1.1 and variance 0.02, c.f. Sunarsih *et al.* (2018), with the probability density function illustrated in Figure 3d. In addition, the decision-maker set a target efficiency index of $E_i^r = 0.5$ for each pond. All computations were carried out in LINGO 21.0 using the generalised reduced gradient algorithm; see LINDO Systems Inc. (2017) and Liu (2015) for technical details.



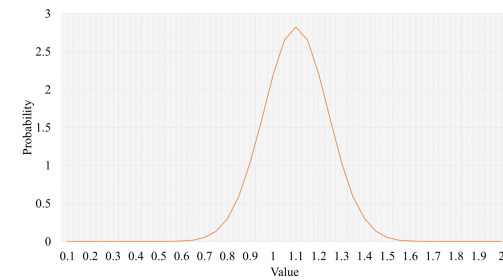
(a) Membership function for waste load percentage for ponds I and II



(b) Membership function for waste load percentage for ponds III and IV



(c) Membership function for wastewater volume at the inlet



(d) Probability density function for the BOD degradation rate

Figure 3: Graphical representation of the membership functions for the fuzzy parameters and the probability density function for the probabilistic parameter. The membership functions follow the discrete-time form, while the probability density function follows a normal distribution

3.2. Discussion

Figure 4 illustrates the optimal decisions obtained from the optimisation process. The inflow rate at the inlet was determined to be $26,950.00 \text{ m}^3$ per day. The distribution for each pond is shown in Figure 4a. Based on these calculations, the expected BOD concentration after treatment is around 50 mg/L . Compared to its target, the error was 5%. The total BOD degraded is shown in Figure 4d.

As shown in Figure 4c, detention time is not strictly required to last a full day. However, choosing a full-day detention period is expected to reduce the BOD concentration below 50 mg/L . The efficiency index varies across the four ponds due to fluctuations in parameters, with an average value of 32.04%. This suggests that the performance of the facultative ponds can

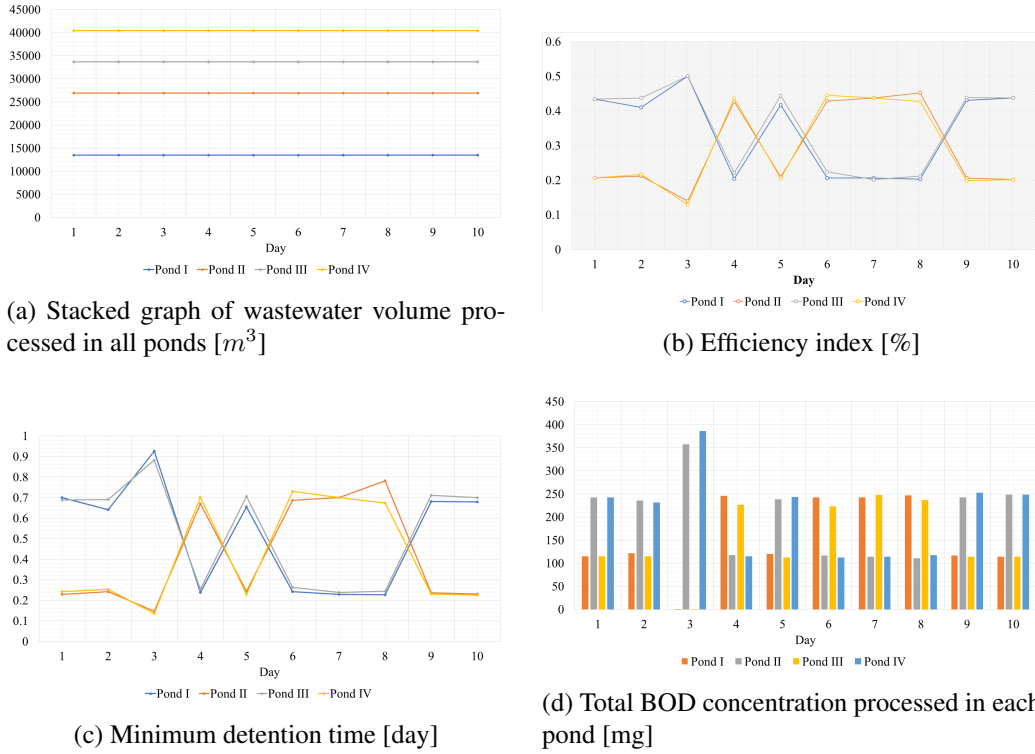


Figure 4: The optimal decisions regarding the management of the Sewon wastewater treatment facility. These decisions suggest the optimal wastewater volume to be processed, its detention time, the corresponding BOD degradation efficiency index, and the processed BOD concentration

be improved, for example, by removing accumulated sludge. It can also be seen that Pond II exhibits the highest efficiency index and must be maintained, whereas Pond I has the lowest value and improvement is needed, for example, by adding more aerators.

The obtained results are consistent with previous studies on facultative pond systems, particularly in terms of effluent BOD levels and treatment behaviour, see e.g. (Sunarsih *et al.* 2018; Ali & Üçüncü 2023a; Pichardo-Zarate & Román-Martínez 2025). However, compared to deterministic approaches, the proposed model provides additional flexibility in handling uncertainty through probabilistic-fuzzy parameters. Differences in efficiency values may be attributed to varying assumptions regarding detention time, environmental conditions, and system parameters. Furthermore, the operational insights obtained, such as the need for aeration and sludge removal, are in line with established recommendations in the literature, see e.g. (Sunarsih *et al.* 2023; Ali & Üçüncü 2023b; Armanda *et al.* 2025; Wang & Wang 2025), reinforcing the practical relevance of the proposed approach.

Table 2: Percentage change of decision variables relative to the medium uncertainty scenario

Scenario (ϵ)	Q_0 (%)	BOD (%)	Efficiency (%)
Low ($\epsilon = 0.5$)	0.00	-0.03	-8.881784e-12
Slightly Low ($\epsilon = 0.75$)	0.00	-0.02	0.00
Medium ($\epsilon = 1$)	0.00	0.00	0.00
Slightly High ($\epsilon = 1.25$)	0.00	0.03	0.00
High ($\epsilon = 1.5$)	0.00	0.06	2.353673e-12

Table 2 illustrates the impact of uncertainty levels on key decision variables and system per-

formance. The level of uncertainty is controlled by a scaling parameter ϵ , which adjusts the spread of the distributions while preserving their mean values. Lower values of ϵ represent reduced variability, whereas higher values correspond to increased uncertainty. The results indicate that the optimal inflow rate remains relatively stable across scenarios, while the effluent BOD shows a slight increase under higher uncertainty. The efficiency index exhibits very small variation. The percentage deviation from the medium uncertainty scenario remains within a small range, indicating that the model decisions are relatively insensitive to changes in uncertainty levels.

The findings yield several practical insights for decision-makers in managing facultative ponds, namely:

- i) The decision-maker can assign different confidence levels to each constraint in the mathematical model, enabling adjustments that reflect experience and professional judgement.
- ii) Some parameter values remain unknown during computation, implying that decisions are made under uncertainty. Consequently, actual outcomes may deviate from those predicted by the model, and depending on the probability levels considered, results may be either better or worse than expected.
- iii) The optimisation procedure may be performed iteratively with varying parameter values, such as alternative membership functions and probability density functions, until sufficient confidence in the decision is achieved. Nevertheless, computational time should be considered, particularly when time constraints limit the opportunity for extensive analysis.

4. Conclusion

A novel multi-period chance-constrained probabilistic-fuzzy programming approach is presented in this study to improve the efficiency of facultative wastewater stabilisation ponds. Based on the case study results, the findings show that the model effectively enhances system performance, achieving an optimal inlet inflow rate of 26,950.00 m^3 per day.

Future work should explore more sophisticated modelling frameworks to capture complex treatment processes, including pollutant degradation in maturation ponds. Furthermore, investigating sludge characteristics could provide deeper insights into their effects on pond efficiency.

Acknowledgement

This work was supported by the Faculty of Science and Mathematics, Universitas Diponegoro, under the Riset Utama 2024 Research Grant Contract No. 25.II.E/UN7.F8/PP/II/2024.

References

- Alhawarat A., Alolaiyan H., Masmali I.A., Salleh Z. & Ismail S. 2021. A descent four-term of liu and storey conjugate gradient method for large scale unconstrained optimization problems. *European Journal of Pure and Applied Mathematics* **14**(4): 1429–1456.
- Ali H.Q. & Üçüncü O. 2023a. Mathematical modeling and optimization of wastewater stabilization ponds using nonlinear programming. *Mehran University Research Journal of Engineering and Technology* **42**(1).
- Ali H.Q. & Üçüncü O. 2023b. Modeling and optimizing wastewater stabilization ponds for domestic wastewater treatment. *Civil Engineering Journal* **9**(11): 2834–2846.
- Armanda A.M.D., Sutrisno S., Sunarsih S., Widowati W. & Kartono K. 2025. Probabilistic-fuzzy programming model with chance-constrained to optimize wastewater treatment plants: A case study with the bantul wastewater treatment plant layout. *E3S Web of Conferences*, p. 03036. EDP Sciences.
- Benito M., Menacho C., Chueca P., Ormad M.P. & Goñi P. 2020. Seeking the reuse of effluents and sludge from conventional wastewater treatment plants: Analysis of the presence of intestinal protozoa and nematode eggs. *Journal of Environmental Management* **261**: 110268.

- Borzooei S., Campo G., Cerutti A., Meucci L., Panepinto D., Ravina M., Riggio V., Ruffino B., Scibilia G. & Zanetti M. 2019. Optimization of the wastewater treatment plant: From energy saving to environmental impact mitigation. *Science of The Total Environment* **691**: 1182–1189.
- Caligan C.J.A., Garcia M.M.S., Mitra J.L. & San Juan J.L.G. 2022. Multi-objective optimization for a wastewater treatment plant and sludge-to-energy network. *Journal of Cleaner Production* **368**: 133047.
- Eshkuvatov Z., Laadjal Z. & Ismail S. 2025. Solving class of mixed nonlinear multi-term fractional volterra–fredholm integro-differential equations by new development of HAM. *Mathematical Modelling and Analysis* **30**(1): 52–73.
- Gopakumar A., Narayan R., Nagath S.A., Nishanthan P., Mohammed R.S. & Chandran S.S. 2018. Waste water treatment using economically viable natural adsorbent materials. *Materials Today: Proceedings* **5**(9): 17699–17703.
- Huang B.S., hong Hong C., huan Du H., Qiu J., Liang X., Tan C. & Liu D. 2017. Quantitative study of degradation coefficient of pollutant against the flow velocity. *Journal of Hydrodynamics, Ser. B* **29**(1): 118–123.
- Kartono, Sutrisno, Sunarsih & Widowati 2024a. Chance-constrained fuzzy optimization model for enhancing facultative ponds: A case study at bantul wastewater treatment plant. *AIP Conference Proceedings*, p. 020009. AIP Publishing LLC.
- Kartono K., Sutrisno S., Sunarsih S., Widowati W., Arreeras T. & Syukur M. 2024b. Dynamic-chance-constrained-based fuzzy programming approach for optimizing wastewater facultative ponds for multi-period case. *WSEAS Trans. Syst.* **23**: 24–30.
- Khatri N., Vyas A.K., Abdul-Qawy A.S.H. & Rene E.R. 2023. Artificial neural network based models for predicting the effluent quality of a combined upflow anaerobic sludge blanket and facultative pond: Performance evaluation and comparison of different algorithms. *Environmental Research* **217**: 114843.
- Koffi N.J. & Okabe S. 2022. High electrical energy harvesting performance of an integrated microbial fuel cell and low voltage booster-rectifier system treating domestic wastewater. *Bioresource Technology* **359**: 127455.
- Kogo B.K., Biamah E.K. & Langat P.K. 2017. Optimized design of a hybrid biological sewage treatment system for domestic wastewater supply. *Journal of Geoscience and Environment Protection* **5**(5): 14–29.
- LINDO Systems Inc. 2017. *LINGO: The Modeling Language and Optimizer*. LINDO Systems Inc., Chicago, Illinois.
- Liu B. 2015. *Uncertainty Theory*. Berlin, Heidelberg: Springer.
- Mahajan S., Gupta S.K., Ahmad I. & Al-Homidan S. 2021. Using concave optimization methods for inexact quadratic programming problems with an application to waste management. *Journal of Inequalities and Applications* **2021**(60).
- Olukanni D.O. & Ducoste J.J. 2011. Optimization of waste stabilization pond design for developing nations using computational fluid dynamics. *Ecological Engineering* **37**(11): 1878–1888.
- Onu M.A., Ayeleru O.O., Oboirien B. & Olubambi P.A. 2023. Challenges of wastewater generation and management in sub-saharan africa: A review. *Environmental Challenges* **11**: 100686.
- Pichardo-Zarate L. & Román-Martínez A. 2025. Deterministic vs. stochastic optimization in municipal wastewater treatment plants: A comparative analysis. *SSRN Electronic Journal*.
- Recio-Garrido D., Kleiner Y., Colombo A. & Tartakovsky B. 2018. Dynamic model of a municipal wastewater stabilization pond in the arctic. *Water Research* **144**: 444–453.
- Shahid A., Ishfaq M., Ahmad M.S., Malik S., Farooq M., Hui Z., Batawi A.H., Shafi M.E., Aloqbi A.A., Gull M. & Mahmood M.A. 2019. Bioenergy potential of the residual microalgal biomass produced in city wastewater assessed through pyrolysis, kinetics and thermodynamics study to design algal biorefinery. *Bioresource Technology* **289**: 121701.
- Sunarsih, Sasongko D.P., Khabibah S. & Sutrisno 2023. Integrated fuzzy and probabilistic programming for optimizing wastewater pollutant degradation processes: A static case. *11TH INTERNATIONAL SEMINAR ON NEW PARADIGM AND INNOVATION ON NATURAL SCIENCES AND ITS APPLICATION (ISNPINSA): Developing Science and Technology Toward the New Normal Era*, p. 020020. AIP Publishing LLC.
- Sunarsih, Widowati, Kartono & Sutrisno 2018. Mathematical analysis for the optimization of wastewater treatment systems in facultative pond indicator organic matter. *E3S Web Conf.* **31**: 05008.
- Tawfik A., Niaz H., Qadeer K., Qyyum M.A., Liu J.J. & Lee M. 2022. Valorization of algal cells for biomass and bioenergy production from wastewater: Sustainable strategies, challenges, and techno-economic limitations. *Renewable and Sustainable Energy Reviews* **157**: 112024.
- Tobin T., Gustafson R., Bura R. & Gough H.L. 2020. Integration of wastewater treatment into process design of

- lignocellulosic biorefineries for improved economic viability. *Biotechnology for Biofuels* **13**: 24.
- Wang M.H.S. & Wang L.K. 2025. Waste stabilization pond fundamentals and innovations. *Industrial Water and Waste Treatment*, pp. 231–272. CRC Press.
- Ye G., Wan J., Deng Z., Wang Y., Zhu B., Yan Z. & Ji S. 2024. Machine learning-based prediction of biological oxygen demand and unit electricity consumption in different-scale wastewater treatment plants. *Journal of Environmental Chemical Engineering* **12**(2): 111849.
- Zhang Q. & Li Z. 2021. Data-driven interval credibility constrained quadratic programming model for water quality management under uncertainty. *Journal of Environmental Management* **293**: 112791.
- Zhang Q., Li Z. & Huang W. 2021. Simulation-based interval chance-constrained quadratic programming model for water quality management: A case study of the central grand river in ontario, canada. *Environmental Research* **192**: 110206.
- Zhang Q. & Liu L. 2020. A microbial fuel cell system with manganese dioxide/titanium dioxide/graphitic carbon nitride coated granular activated carbon cathode successfully treated organic acids industrial wastewater with residual nitric acid. *Bioresource Technology* **304**: 122992.
- Zhang X., Chen J. & Li J. 2020. The removal of microplastics in the wastewater treatment process and their potential impact on anaerobic digestion due to pollutants association. *Chemosphere* **251**: 126360.

Department of Mathematics

Diponegoro University

Jalan Prof. Jacub Rais

Semarang 50275, INDONESIA

E-mail: s.sutrisno@live.undip.ac.id, widowati@lecturer.undip.ac.id,
sunarsih@lecturer.undip.ac.id, kartono@lecturer.undip.ac.id*

School of Management

Mae Fah Luang University

Chiang Rai, 57100

THAILAND

E-mail: tosporn.arr@mfu.ac.th

Received: 30 July 2025

Accepted: 29 April 2026

*Corresponding author