

## COMPARING RCDU AND Ga STATISTICS FOR PATCH OF OUTLIERS DETECTION IN VON MISES (VM) SAMPLES

(Perbandingan Statistik RCDU dan Ga bagi Pengesanan Kelompok Nilai Pencilan dalam Sampel Von Mises (VM))

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### ABSTRACT

This paper discovers a new comparison between two outlier detection methods, the Ga statistic and the RCDU statistic, for the identification of outliers in a patch in univariate circular data. This has not been studied in the existing literature before. The RCDU statistic, being an extension from the circular median, is compared to the Ga statistic constructed using the spacing theory. We compare the detection performance of two, three, and four outliers in a patch. The proportion of detecting a correct outlier is used to measure the performance for both test statistics by Monte Carlo simulation. The result shows that the RCDU statistic slightly outperforms the Ga statistic in most cases. This finding aligns with a practical example using coastal wind direction data.

**Keywords:** RCDU statistic; Ga statistic; outliers; circular median; wind direction data

### ABSTRAK

Makalah ini memperkenalkan satu perbandingan baharu antara dua ujian ketakselarasan, iaitu statistik RCDU dan statistik Ga, bagi pengenalan nilai pencilan secara kelompok dalam data bulatan univariat. Perbandingan ini belum pernah dikaji dalam literatur sedia ada. Statistik RCDU, yang merupakan lanjutan daripada median bulatan, dibandingkan dengan statistik Ga yang berasaskan teori jarak (spacing theory). Kajian ini membandingkan prestasi pengesanan bagi dua, tiga dan empat nilai pencilan dalam satu kelompok. Peratusan pengesanan yang betul digunakan sebagai ukuran prestasi bagi kedua-dua statistik ujian melalui simulasi Monte Carlo. Hasil kajian menunjukkan bahawa statistik RCDU sedikit mengatasi statistik Ga dalam kebanyakan kes. Dapatan ini selaras dengan satu contoh aplikasi praktikal menggunakan data arah angin persisiran pantai.

**Kata kunci:** statistik RCDU; statistik Ga; pencilan; median bulatan; data arah angin

## 1. Introduction

Linear data sets are commonly seen in the real world and can manifest in diverse forms across several domains. Additionally, some variables can be measured in either degrees or radians. Data containing variables of this nature should be referred to as directional data. Directional variables include the measurements of birds' migration routes and the wind's direction. The univariate circular data is one example of directional data, which can be represented in the form of a circle. The complex arrangement of directional data, combined with a mixture of directional and linear variables, necessitates the utilization of many advanced statistical tools specifically designed for linear data.

A circular observation determines the circumference of a circle or circular object in a two-dimensional plane. The angle located between the starting point and the respective observation

point on a circle might characterize every circular observation. Circular data is mainly associated with direction and is confined to the interval  $[0, 2\pi)$  radians or  $[0, 360)$  degrees. Circular data arise across a wide range of scientific fields, including the earth sciences, meteorology, physics, biology, image analysis, psychology, and medicine, where directional or periodic measurements are central. Consequently, the analysis of circular data requires specialized statistical methods that go beyond linear statistics.

An inherent challenge in statistical analysis is the occurrence of a small number of unexpected observations, which are referred to as outliers. The issue of outliers has been a topic of interest since the earliest days of statistics. Various studies have shown that outliers, especially in modeling, forecasting, and diagnostics, greatly influence the effectiveness of general statistical methods. The existence of outliers can significantly affect parameter estimation, which may lead to inaccurate decisions (see Ramlee *et al.* (2024), Thanwiset & Srisodaphol (2023) and Alshqaq (2021). Occasionally, there are rare instances where the result is extremely disastrous and needs to be dealt with before any additional examination.

There are several methods that can be employed to identify outliers in circular data. Recently, a number of outlier detection methods have been introduced by Ramlee *et al.* (2024), Zulkefli *et al.* (2023), Zulkipli *et al.* (2022), and Satari *et al.* (2021). There are numerous methods for finding circular outliers, including the Ga statistic introduced by Mohamed *et al.* (2016) and the RCDU statistic introduced by Mahmood *et al.* (2017a), was later used by Badarisam *et al.* (2020). Among the several methods employed in analyzing circular data, both the RCDU statistic and Ga statistic are effective in identifying outliers especially in a patch. Section 3 provides additional details on the RCDU statistic and Ga statistic.

Thus, both statistics are employed in this study to detect clusters of two, three, and four outliers in univariate circular data. The RCDU statistic is proposed using a robust technique that utilizes a circular median. Therefore, its effectiveness would be significantly enhanced if the outliers are included in the circular data. This study aims to locate a cluster of unusual data points in univariate circular data that conforms to the Von Mises distribution. The identification process involves using Monte Carlo simulation experiments. The relative efficacy of RCDU statistic and Ga statistic will be comprehensively examined with their ability to accurately identify outliers, and these statistical methods will be applied to real-world datasets.

## 2. Discordancy Tests Used in The Von Mises Sample

Originally introduced by von Mises (1918), the von Mises (VM) distribution has become a fundamental model for univariate circular data. Its widespread use in circular statistics reflects its close structural relationship to the normal distribution in linear contexts, particularly in terms of symmetry and concentration about a central location. The VM distribution is defined through a mean direction,  $\mu$  and a concentration parameter,  $\kappa$ , with its probability density function expressed as

$$f(\theta; \mu, \kappa) = \frac{1}{2\pi I_0(\kappa)} e^{[\kappa \cos(\theta - \mu)]}, 0 < \theta, \mu \leq 2\pi, \kappa \geq 0, \quad (1)$$

where  $I_0(\kappa)$  is an improved Bessel function of the first kind and order zero. As the concentration parameter,  $\kappa$  approaches zero, the von Mises distribution converges to the uniform distribution on the circle. Conversely, for large values of  $\kappa$ , the distribution becomes increasingly concentrated about the mean direction and exhibits behavior analogous to that of the normal distribution.

### 2.1. RCDU statistic

Mahmood *et al.* (2017a) introduced the RCDU statistic as a procedure for identifying both single and multiple outliers in circular data. The method is motivated by two key considerations. First, outliers in circular settings do not necessarily correspond to extreme values in the linear sense. Second, it exploits the inherent symmetry of the von Mises distribution about the mean direction. Let  $\vartheta_1, \vartheta_2, \dots, \vartheta_n$  denote circular variables defined on the circumference of the unit circle. For each circular distance,  $\text{circ.dist}_{(i)}$  for  $i = 1, 2, \dots, n$ , between an observation and the circular median,  $\text{circ.med}$  is computed following the procedure described in Badarisam *et al.* (2020). The calculation are as follows:

i) For value of  $\text{circ.med}$  between  $[0, \pi]$  radians

$$\text{circ.dist}_{(i)} = \begin{cases} |\vartheta_i - \text{circ.med}| & , \quad \text{for } |\vartheta_i - \text{circ.med}| \leq \pi, \\ 2\pi - \vartheta_i + \text{circ.med} & , \quad \text{for } |\vartheta_i - \text{circ.med}| > \pi. \end{cases} \quad (2)$$

ii) For value of  $\text{circ.med}$  between  $[\pi, 2\pi]$  radians

$$\text{circ.dist}_{(i)} = \begin{cases} |\vartheta_i - \text{circ.med}| & , \quad \text{for } |\vartheta_i - \text{circ.med}| \leq \pi, \\ 2\pi - \text{circ.med} + \vartheta_i & , \quad \text{for } |\vartheta_i - \text{circ.med}| > \pi. \end{cases} \quad (3)$$

In circular data, an observation is considered a candidate outlier when its associated circular distance,  $\text{circ.dist}_{(i)}$ , attains a relatively large value. Accordingly, the RCDU statistic is formally defined as the maximum circular distance among all observations,  $\max(\text{circ.dist}_{(i)})$ . An observation is declared a statistically significant outlier if its circular distance surpasses the specified critical cut-off value.

### 2.2. Ga statistic

Mohamed *et al.* (2016) define the steps needed to obtain the Ga statistic. The procedure of outlier detection of Ga statistic highly depends on the value of 'a', the number of step spacings, which relates to the number of outliers. For example, if  $a = 1$ , then the  $G_1$  statistic can detect a single outlier in a dataset. The main concept of the  $G_a$  statistic is calculating the distance between the observation and its neighbor.

Let  $\vartheta_1, \vartheta_2, \dots, \vartheta_n$  be independent and identically distributed observations of circular from a VM distribution. Then,  $\vartheta_{(1)}, \vartheta_{(2)}, \dots, \vartheta_{(n)}$  are the sequenced circular observations. Next, based on the notation from Mohamed *et al.* (2016), a one-step ( $a = 1$ ) spacing for the  $i$ th ordered observations is given by

$$G_{1i} = \vartheta_{(i+a)} - \vartheta_{(i)}, i = 1, 2, \dots, n-1, \quad (4)$$

$$G_{1n} = 2\pi - \vartheta_{(n)} + \vartheta_{(1)}, i = n, \quad (5)$$

Let  $G_a$ , denote the gaps between observations along the circumference in ordered arrangement. This ordering remains unchanged regardless of the chosen zero (reference) direction. Consequently, for the  $i$ -th ordered observation, the  $a$ -step spacing is defined for  $a = 1, 2, \dots, m$  and  $i = 1, 2, \dots, n$  where,

$$G_{ai} = \vartheta_{(i+a)} - \vartheta_{(i)}, i = 1, 2, \dots, n-a, \quad (6)$$

$$G_{ai} = 2\pi - \vartheta_{(ai)} + \vartheta_{(i+a)-n} \text{ for } i = (n+1) - a, (n+2) - a, \dots, n \quad . \quad (7)$$

A patch of  $m$  outliers will be identified using (at least)  $a$ -step spacings, specifically two steps, three steps and four steps which correspond to detecting a patch of two outliers, three outliers and four outliers, respectively.

### 2.3. The cut-off points for RCDU and Ga statistics

In this analysis, the two outlier detection tests, RCDU statistic and Ga statistic, will be used to identify outliers in the sample of VM distribution. The samples are obtained from the observations of  $n$  from VM. ( $\mu = \alpha, \kappa$ ). The sample will then be created using different sample sizes of  $n$ , and also the value of the concentration parameter,  $\kappa$ . The values of sample sizes,  $n$  and the concentration of parameters,  $\kappa$  are referred to the previous study by Mohamed *et al.* (2016). This is to make sure that RCDU statistic's cut-off point and Ga statistic's cut-off point values are significant and suit every combination of sample sizes,  $n$  and concentration parameter,  $\kappa$ . Note that, the cut-off point values for Ga statistic will be referred to Mohamed *et al.* (2016) meanwhile the cut-off points for RCDU statistic will be referred to Badarisam *et al.* (2020). The interpolation calculation will be employed to provide a more precise value of cut-off locations.

### 3. Performance Tests of RCDU statistic and G2 statistic, G3 statistic and G4 statistic

The samples are obtained from the observations of  $(n-m)$  from VM ( $\mu = \alpha, \kappa$ ) (free from outliers) with another  $m$  observations ( $m$ -outliers) of VM ( $\mu = \alpha + \lambda\pi, \kappa$ ), where  $\lambda$  is the contamination degree and value of  $\lambda$  is between 0 and 1 (see Abuzaid *et. al* (2009) & Mohamed *et al.* (2016)). Next, the comparison between the performances of RCDU statistic and  $G_2$  statistic,  $G_3$  statistic,  $G_4$  statistic will be conducted with a few combinations of concentration parameter,  $\kappa$  and sample sizes,  $n$  to detect a patch of two outliers, three outliers and four outliers.

The performances,  $PCD$  defined as the proportion of correct detections, are calculated based on the efficiency of the Ga statistic and RCDU statistic in detecting the correct positioning of outliers and exceeding the cut-off points established by Mohamed *et al.* (2016) and Badarisam *et al.* (2020), respectively. Mohamed *et al.* (2016) stated key strength of the proposed Ga statistic is its flexibility in detection. As noted in Section 4.4 of their paper, a patch of  $m$  outliers can be successfully detected using any version of the Ga statistic where the  $a$ -step is greater than or equal to the true number of outliers,  $m$  ( $m \leq a$ ).

#### 3.1. A patch of two outliers

Figure 1 displays the performance of RCDU and  $G_2$  statistics based on fixed concentration parameters of  $\kappa = 5.29$  and sample sizes of  $n = 20$ . It illustrates that the RCDU statistic performs slightly better in the  $PCD$  to detect a patch of two outliers compared to the  $G_2$  statistic at a contamination level,  $\lambda$  between 0 and 1. Once the contamination level is greater than 0.75, the performance of both statistics is almost the same as the value for RCDU statistic and  $G_2$  statistic in the  $PCD$  is nearly to one. Similar results are observed for the performance of RCDU statistic and  $G_2$  statistic with fixed  $n = 50$  and  $\kappa = 10.27$ . Figure 2 shows that the RCDU statistic achieves a slightly better  $PCD$  for identifying a patch of two outliers compared to the  $G_2$  statistic, specifically at a contamination level between 0 and 0.75. Once the contamination level is greater than 0.75, the performance of both statistics is the same as the value for RCDU statistic and  $G_2$  statistic in the  $PCD$  is equal to one.

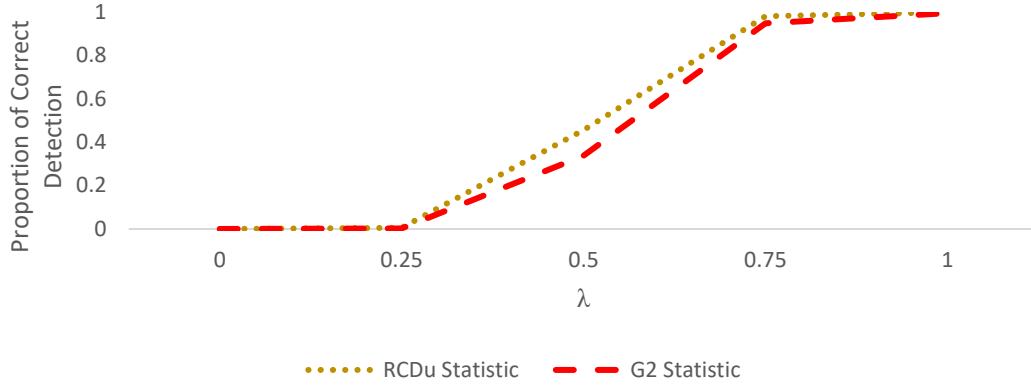


Figure 1: Performance of RCDU statistic and  $G_2$  statistic in  $PCD$  of a patch of two outliers when  $\kappa=5.29$  and  $n=20$

For a fixed  $\kappa = 5.29$  across all considered sample sizes ( $n = 20, 50$  and  $100$ ), the RCDU statistic demonstrates superior performance to the  $G_2$  statistic in identifying a patch of two outliers. A similar pattern is observed when the sample size is fixed at  $n = 50$  and the concentration parameter varies ( $\kappa = 5.29, 6.53, 8.61$ ), where RCDU consistently outperforms  $G_2$  in detecting the same outlier configuration.

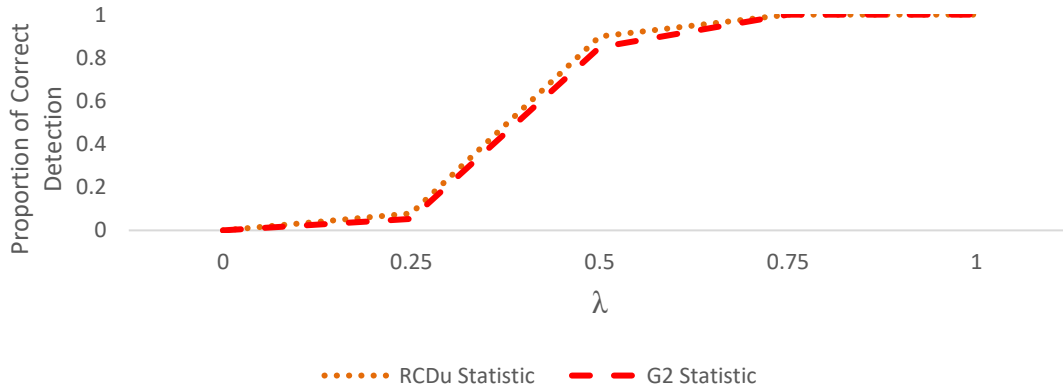


Figure 2: Performance of RCDU statistic and  $G_2$  statistic in  $PCD$  of a patch of two outliers when  $\kappa=10.27$  and  $n=50$

### 3.2. A patch of three outliers

This subsection examines the ability of the RCDU and  $G_3$  statistics to detect a patch of three outliers under a fixed  $\kappa = 5.29$  and  $n = 20$ . As illustrated in Figure 3, the RCDU statistic exhibits higher power in terms of  $PCD$  for identifying a patch of three outliers than the  $G_3$  statistic across contamination levels  $\lambda \in [0, 1]$ . However, when the contamination level exceeds  $0.75$ , the performances of the two statistics converge, with the  $PCD$  values for both RCDU and  $G_3$  approaching a value of one.

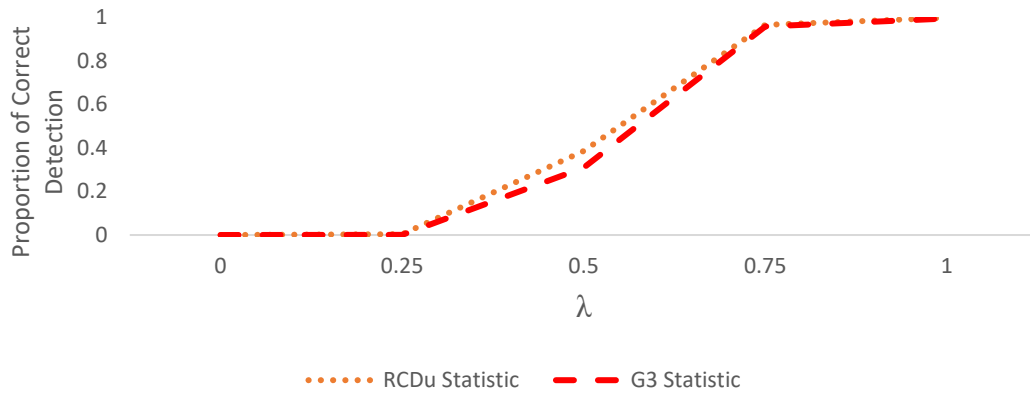


Figure 3: Performance of RCDU statistic and  $G_3$  statistic in  $PCD$  of a patch of three outliers when fixed  $\kappa=5.29$  and  $n=20$

For a fixed  $n = 50$  and  $\kappa = 10.27$ , the comparative performance of the RCDU and  $G_3$  statistics is presented in Figure 4. The results indicate that RCDU attains a marginally higher proportion of correct detection ( $PCD$ ) than  $G_3$  in identifying a patch of three outliers. When the contamination level exceeds 0.75, the performance of both statistics becomes indistinguishable, with  $PCD$  values approaching unity. Nonetheless, under contaminated settings, RCDU maintains a slight advantage over  $G_3$  in detecting a patch of three outliers. A similar trend is observed for a fixed concentration parameter  $\kappa = 5.29$  across all considered sample sizes  $n = 20, 50$  and  $100$ , where RCDU consistently outperforms  $G_3$ . In addition, when the sample size is fixed at  $n = 50$  and the concentration parameter varies  $\kappa = 5.29, 6.53, 8.61$  and  $10.27$ , RCDU continues to exhibit superior performance in terms of  $PCD$  for detecting a patch of three outliers.

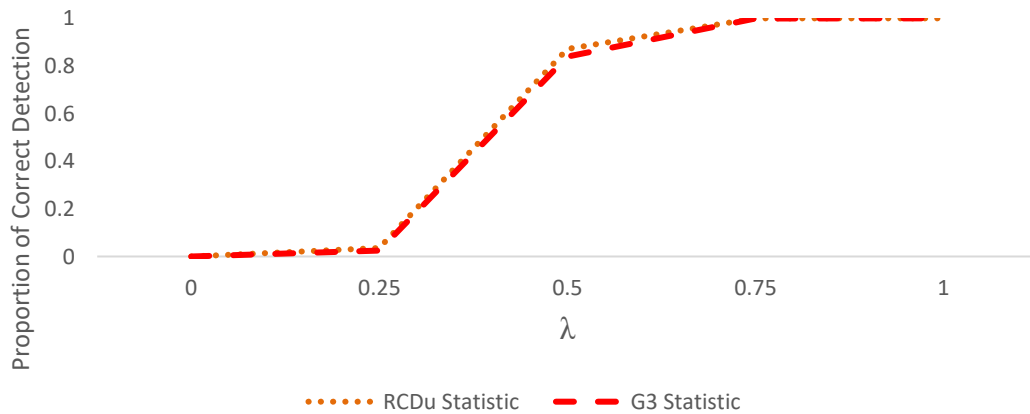


Figure 4: Performance of RCDU statistic and  $G_3$  statistic in  $PCD$  of a patch of three outliers when fixed  $\kappa=10.27$  and  $n=50$

### 3.3. A patch of four outliers

Figure 5 shows the performance of RCDU statistic and  $G_4$  statistic with fixed concentration parameters of  $\kappa = 5.29$  and sample sizes of  $n = 20$ . The figure demonstrates that the RCDU statistic and the  $G_4$  statistic perform almost similar in the  $PCD$  to detect a patch of four outliers

at a contamination level,  $\lambda$  between 0 and 1. However, when contamination level is at 0.75, the  $G_4$  statistic is performed better than the RCDU statistic in terms of  $PCD$  but once the contamination level is greater than 0.75, the performance of both statistics is almost the same as the value for RCDU statistic and  $G_4$  statistic in the  $PCD$  is nearly to one. This confirms that, in the presence of outliers, the RCDU statistic and the  $G_4$  statistic perform almost the similarly in detecting a patch of four outliers.

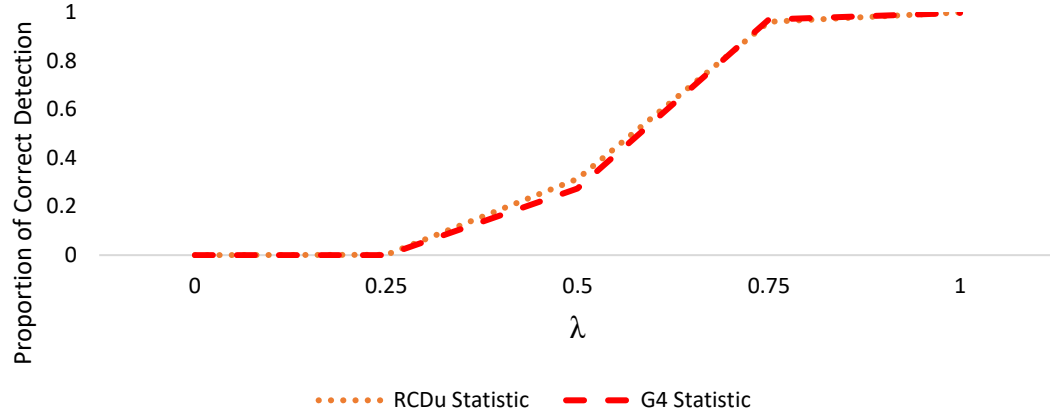


Figure 5: Performance of RCDU statistic and  $G_4$  statistic in  $PCD$  of a patch of four outliers when fixed  $\kappa=5.29$  and  $n=20$

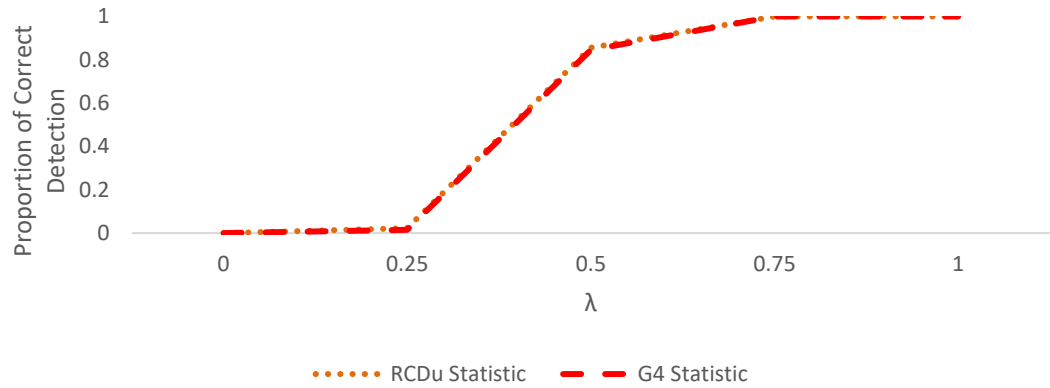


Figure 6: Performance of RCDU statistic and  $G_4$  statistic in  $PCD$  of a patch of four outliers when  $\kappa=10.27$  and  $n=50$

As for the performance of RCDU statistic and  $G_4$  statistic with fixed sample size of  $n = 50$  and  $\kappa = 10.27$ , Figure 6 shows that the RCDU statistic achieve a similar performance as the  $G_4$  statistic in the  $PCD$  to detect a patch of four outliers at all contamination levels. However, in terms of the performance of the combination of fixed  $\kappa = 5.29$  with all the sample sizes,  $n = 20, 50$  and  $100$ , the RCDU statistic is performed slightly better than the  $G_4$  statistic in detecting a patch of four outliers. The similar performances of  $PCD$  are observed in detecting a patch of

four outliers for the combination between fixed sample size,  $n = 50$  and different concentration parameters,  $\kappa = 5.29, 6.53, 8.61$  and  $10.27$ .

#### 4. Illustrative Example

This section examines a secondary wind-direction dataset to demonstrate the application of the RCDU statistic together with the  $G_a$  statistic;  $G_2$  statistic,  $G_3$  statistic and  $G_4$  statistic outlier detection tests defined earlier. The use of a real dataset is motivated by previous results showing comparable performance between the RCDU and  $G_a$  statistic in detecting patches of two outliers under large sample sizes and higher concentration parameters. In this section, both approaches are applied to wind-direction data to assess their effectiveness in identifying potential outliers.

Wind direction was measured using High-Frequency (HF) radar and Anchored Wave Buoy techniques, which are briefly introduced alongside a description of the dataset. The data were collected along the Holderness (Humberside) coast of the North Sea, United Kingdom, and were originally reported by Hussin (1997). The dataset comprises 129 observations recorded over 22.7 days and includes measurements of time (in days) and wind direction (in radians) obtained from both HF radar and Anchored Wave Buoy systems. Residuals from the Downs and Mardia circular regression model, as reported by Mahmood *et al.* (2017b) and Rambli *et al.* (2015), are used for subsequent analysis.

Figure 7 presents circular histograms for both measurement techniques, indicating similar distributional characteristics, while Figure 8 shows a conventional scatter plot of wind direction. All observations are expressed in radians within the interval  $[0, 2\pi)$ . Both figures suggests the presence of four potential outliers: two located in the upper-left region of the plot and two occurring between approximately two and four radians. The two points in the upper-left region, however, are considered consistent with the remaining observations, as their proximity to neighboring points reflects the closed-range nature of circular variables. Figure 9 displays the circular plot of residuals for the wind-direction data. Together, Figures 8 and 9 indicate that observations 38 and 111 deviate markedly from the overall pattern and are therefore identified as suspected outliers, as they are not aligned with the majority of the observations.

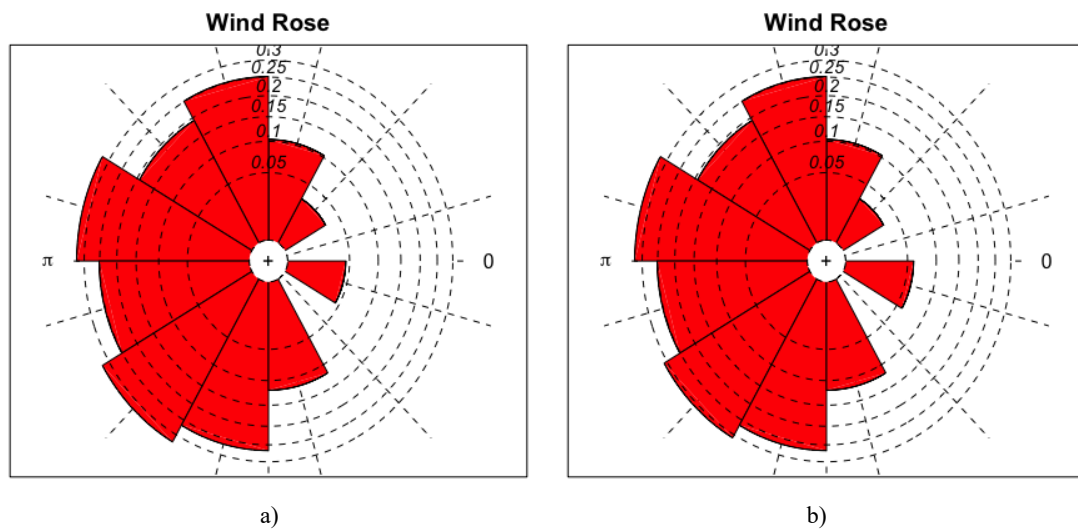


Figure 7: Circular histograms for wind direction data, a) HF (High Frequency) and b) Anchored Wave Buoy



Next, further testing is carried out by using RCDU statistic and Ga statistic to prove that both observations 38 and 111 are classified as outliers or not. As for Ga statistic,  $G_2$  statistic will be chosen as wind dataset consists of two outliers and new estimated of concentration parameter,  $\kappa$  from distribution of VM is obtained. In this particular test, circular residuals of wind data are being used thus, the new concentration parameter is  $\kappa = 9.40$ . The new cut-off points for RCDU statistics and  $G_2$  statistics at 5% significance level are 1.255 and 0.456 respectively. After that, the values for both RCDU statistics and  $G_2$  statistics are obtained which are 1.8085, 3.1107, 1.1928 and 2.3597 respectively which exceed the cut-off points thus, this proves that observations 38 and 111 are classified as outliers.

Both RCDU statistic and  $G_2$  statistic have successfully identified observations 38 and 111 as outliers. Hence, it is advisable for meteorologists to conduct further research on these observations to determine whether they are outliers due to human error or measurement inaccuracies, or if they are attributable to natural phenomena such as rain, thunderstorms, or climate changes.

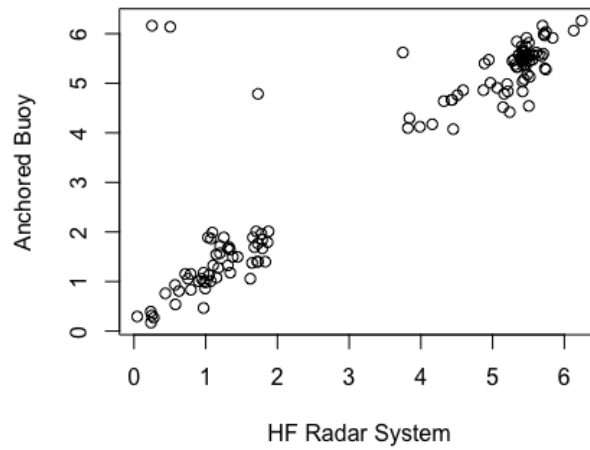


Figure 8: Linear plot for both measurements

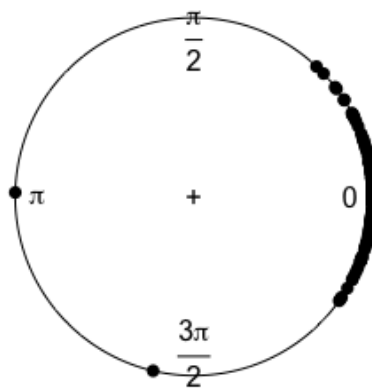


Figure 9: Circular plot for wind residual

## 5. Conclusion

This study compares the effectiveness of two discordancy tests, the RCDU statistic and  $G_a$  statistic;  $G_2$  statistic,  $G_3$  statistic,  $G_4$  statistic, in detecting patches of two, three, and four outliers in circular data following the VM distribution. Overall, the RCDU statistic slightly outperforms the  $G_a$  statistic ( $G_2$  and  $G_3$ ) when applied to samples from the VM distribution, for all tested sample sizes ( $n$ ) and concentration parameters ( $\kappa$ ). In the case of  $G_4$ , the RCDU and  $G_a$  statistics show nearly equivalent performance on VM samples, for large sample size and large concentration parameter. The finding indicates that the RCDU is more capable in detecting outliers when the dataset contains fewer outliers, as the median utilized is robust against the impact of a limited number of outliers. However, if there are more outliers, the  $G_a$  statistic can still do quite well because having outliers in a patch makes the neighboring distance better at finding outliers than if there are fewer outliers. These conclusions are supported by applying both statistics to a real wind dataset, which indicates that the wind data follow VM distribution and identifies observations 38 and 111 as outliers. The comparison of performance between these two statistics can be further investigated by using the percentage of outliers present in the data as future work.

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