

NUMERICAL AND RSM ANALYSIS OF HYBRID CNT NANOFLUID AND HEAT TRANSFER OVER A MOVING WEDGE UNDER HYDROMAGNETIC EFFECTS

(Analisis Berangka dan RSM bagi Aliran Nanobendalir CNT Hibrid dan Pemindahan Haba
Melepasi Baji Bergerak di Bawah Kesan Hidromagnetik)

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ABSTRACT

This study presents a mathematical and statistical analysis of hybrid carbon nanotube (CNT) nanofluid flow in the boundary layer across a wedge, emphasising the optimisation of heat transfer rates under the influence of hydromagnetic effects. The novelty of the study lies in the integrated use of Response Surface Methodology (RSM) to statistically maximise the thermal performance of this complex hybrid fluid system (single- and multi-walled CNTs in water), offering practical design parameters that extend beyond a numerical simulation. Using the bvp4c numerical method to solve the transformed ordinary differential equations (ODEs), we found that increasing the wedge and magnetic parameters significantly enhances both skin friction and heat transfer coefficients. Furthermore, the RSM analysis, optimised via the desirability function, successfully identified the best heat transfer conditions, concluding that the CNT volume fraction is the most influential factor and that the hybrid CNT nanofluid is superior to its mono-CNT counterpart for efficient heat transfer applications.

Keywords: boundary layer flow; heat transfer; hybrid carbon nanotube nanofluid; numerical method; response surface methodology; wedge

ABSTRAK

Kajian ini membentangkan satu analisis matematik dan statistik aliran nanobendalir nanotiub karbon (CNT) hibrid dalam lapisan sempadan melepasi baji, menekankan pengoptimuman kadar pemindahan haba di bawah pengaruh kesan hidromagnetik. Kebaharuan kajian terletak pada penggunaan bersepadu Kaedah Permukaan Sambutan (RSM) bagi memaksimumkan prestasi sistem haba bendalir kompleks hibrid (CNT ber dinding tunggal dan ber dinding berbilang dalam air) secara statistik, menawarkan reka bentuk parameter praktikal yang melangkaui simulasi berangka. Dengan menggunakan Kaedah Berangka bvp4c untuk menyelesaikan persamaan pembezaan biasa (PPB) yang diubah, kami telah mendapati bahawa peningkatan parameter baji dan magnetik meningkatkan kedua-dua pekali geseran kulit dan pemindahan haba. Selain itu, analisis RSM melalui Fungsi Keinginan, telah berjaya mengenal pasti keadaan pemindahan haba yang terbaik, dengan menyimpulkan bahawa pecahan isi padu CNT adalah faktor yang paling berpengaruh dan nanobendalir CNT hibrid adalah lebih baik daripada nanofluid mono-CNT untuk aplikasi pemindahan haba yang cekap.

Kata kunci: aliran lapisan sempadan; pemindahan haba; nanobendalir nanotiub karbon hibrid; kaedah berangka; kaedah permukaan sambutan; baji

1. Introduction

Many researchers have conducted experimental, numerical, and theoretical studies on hybrid nanofluids (HNF) due to their potential to enhance heat transfer efficiency. By combining two different nanoparticles to create HNF, the resulting fluid exhibits improved thermal properties, particularly in its capacity to conduct heat and electricity, compared to individual nanofluids.

As a result, various tools and devices across fields such as biomedical technology and cooling systems now make use of HNF. However, a review of recent research reveals an overwhelming abundance of material options. Zafar *et al.* (2023) noted that the majority of nanofluid models focus on metallic particles, primarily copper (Cu). This observation is supported by Nordin *et al.* (2024), who found that over 60% of hybrid nanofluid models consist of a combination of copper and aluminium oxide (Cu-Al₂O₃). To address this lack of diversity, the present study focuses on carbon nanotubes (CNTs). Tam *et al.* (2020), Thakur *et al.* (2022), Samat *et al.* (2024c) and Samat *et al.* (2025) have promoted the utilisation of CNTs, referencing their outstanding thermal, electrical, and mechanical characteristics. Moreover, Khoswan *et al.* (2024) demonstrated that incorporating CNTs with water produces high-quality nanofluids at a low cost. Despite these advantages, Nordin *et al.* (2024) reported that hybrid CNT models represent less than 4% of the existing literature, highlighting a significant gap in research.

In the realm of flow geometry, investigations into hybrid nanofluid flow over a movable wedge have built upon the traditional Blasius and Hiemenz problems, propelled by the requirements of the polymer extrusion and glass fibre sectors. Dinarvand *et al.* (2019) were the pioneers in examining a mathematical model of hybrid nanofluid flow over a movable wedge, a study subsequently expanded by Zainal *et al.* (2025). Nonetheless, these studies frequently neglect the intricate interplay of hydromagnetic (MHD) effects. Singh *et al.* (2023) characterise the hydromagnetic effect as the interaction between electrically conductive fluids and magnetic fields. Because CNTs are some of the best electrical conductors among nanoparticles, the interaction between the fluid and magnetic fields is important. Aladdin and Bachok (2021) and Samat *et al.* (2024b) showed that this property improves the flow and heat transfer properties. To rectify this, our model integrates the hydromagnetic effect into the hybrid CNT wedge flow, differentiating it from the research conducted by Zainal *et al.* (2025).

With the physical model established, critical challenges remain in the analytical methodology. Despite the significant volume of research on hybrid nanofluids (HNF), Minea (2020) and Zafar *et al.* (2023) identified limitations in current approaches. Minea (2020) highlighted the difficulties of experimental work in visualising physical profiles, particularly velocity and thermal distributions, and argued for the necessity of numerical approaches to overcome these illustration difficulties. However, while standard numerical approaches are sufficient for visualising flow profiles, Zafar *et al.* (2023) identified the determination of optimal numerical solutions as a remaining critical challenge. Addressing this methodological gap, the present study employs Response Surface Methodology (RSM) optimisation technique to enhance the numerical data with a specific focus on maximising the heat transfer response. In applying RSM, recent studies by Yahaya *et al.* (2023), Samat *et al.* (2024a), and Aldandani and Jan (2025) have focused on utilising nanoparticle volume fractions, geometric configurations, and physical parameters as the main controllable factors for optimising heat transfer.

To address the limitations of standard numerical approaches, this study employs RSM. According to Myers *et al.* (2016), RSM is significantly applicable in the design, development, and formulation of new products, as well as in enhancing existing product designs. Additionally, Myers *et al.* (2016) highlight that the aim of factor screening within RSM is to condense the list of candidate variables to a manageable few, thereby making subsequent experiments more efficient and requiring fewer runs or tests. This computational efficiency sets RSM apart from other optimisation techniques, such as genetic algorithms (GAs) or artificial neural networks (ANNs). Although GA and ANN are robust for handling large datasets, they often operate as black-box solvers, which can obscure physical insights. In contrast, RSM explicitly quantifies the interaction effects among key parameters (such as the magnetic field

and nanoparticle concentration), establishing it as the preferred method for deriving design sensitivities in this study.

The novelty of this study lies in its integrated framework, which unifies the physical complexity of hybrid CNT flows over a moving wedge under hydromagnetic influence with the RSM. This combination addresses a critical research gap identified by Nordin *et al.* (2024), who noted that the mathematical modelling of hybrid CNTs remains limited, with a few notable exceptions, such as Aladdin and Bachok (2021). To tackle this issue, the current mathematical formulation builds upon the boundary layer models established by Dinarvand *et al.* (2019) and Anuar *et al.* (2021). Specifically, the governing equations are derived from the nanofluid model proposed by Tiwari and Das (2007). However, as Babar and Ali (2019) assert, an accurate assessment of heat transport relies on appropriate property correlations; hence, the thermal conductivity of the hybrid CNTs is explicitly modelled using the correlations suggested by Xue (2005). The specific objectives of this research are

- (1) to formulate the mathematical model of hybrid CNT nanofluid flow over a movable wedge incorporating MHD effects;
- (2) to solve the governing equations numerically using the bvp4c function in MATLAB; and
- (3) to optimise the heat transfer coefficient and analyse parameter interactions using the desirability function method in RSM.

2. Model Framework

The model framework consists of the mathematical modelling formulations and optimisation procedures conducted by RSM. This study employs the Central Composite Design (CCD) as the methodological framework for RSM, with the objective of accurately measuring the curvature of the response surface. A second-order quadratic polynomial equation is formulated to mathematically relate the independent variables, namely, nanoparticle volume fraction parameter, ϕ , magnetic parameter, M , and wedge parameter, m , to the local Nusselt number, Nu_x . Additionally, the model's predictive accuracy is effectively validated using Analysis of Variance (ANOVA).

2.1. Mathematical modelling

The current model is constructed based on the following assumptions:

- The hybrid CNT nanofluid is prepared by combining SWCNT, MWCNT, and water;
- The flow is steady-state, free-convective, laminar, and incompressible;
- The fluid velocity is low enough that viscous dissipation and pressure work can be considered negligible;
- The model is organised within a two-dimensional (x, y) coordinate system;
- The model is developed based on the Tiwari and Das (2007) model;
- The wedge is situated around the constant values of wall temperature, T_w , and ambient temperature, T_∞ where $T_\infty < T_w$;
- In accordance with boundary layer theory (Schlichting & Gersten 2017), the wedge moves with a wall velocity $U_w = u_w(x) = ax^m$, which may align with or oppose the free stream velocity, defined as $U_\infty = u_\infty(x) = bx^m$. Here, x denotes the coordinate along the wedge surface with the dimension of length $[L]$. To satisfy dimensional consistency, a and b are positive constants having dimensions $[L^{1-m}t^{-1}]$, where t represents time. Since the flow is assumed to be steady, the velocity and temperature fields are invariant with respect to time t ; and

- According to Anuar *et al.* (2021), the dimensionless wedge parameter, m , is set within the range of 0 to 1.

The schematic model for the current study is illustrated in Figure 1. The term β in Figure 1 refers to the Hartree pressure gradient. According to Anuar *et al.* (2021), the Hartree pressure gradient parameter, β , is defined in terms of m , such that $m = \beta/(2 - \beta)$. To solve the model, the thermophysical correlations of hybrid CNT nanofluid, as shown in Table 1, and the thermophysical properties of SWCNT, MWCNT, and water at ambient temperature, as outlined in Table 2, are employed.

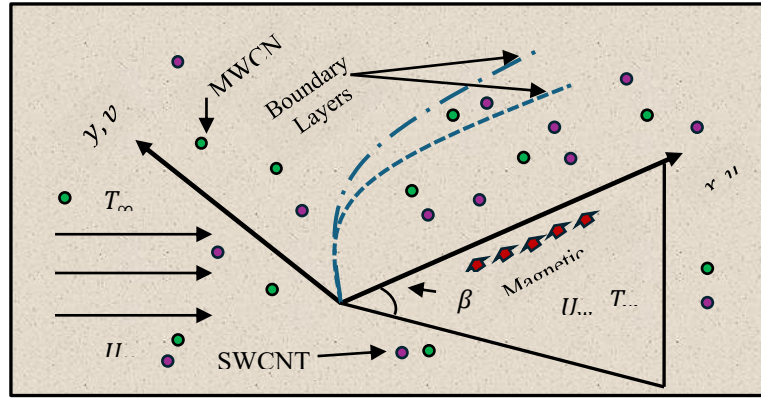


Figure 1: Schematic model

Next, by modifying the Anuar *et al.* (2021) model, the continuity, momentum, energy equations and boundary conditions can be expressed as follows:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation:

$$\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_{\infty} \frac{dU_{\infty}}{dx} + \frac{\mu_{hCNTnf}}{\rho_{hCNTnf}} \frac{\partial^2 u}{\partial y^2} + \frac{\sigma_{hCNTnf}}{\rho_{hCNTnf}} B_0^2 (U_{\infty} - u) \quad (2)$$

Energy equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa_{hCNTnf}}{(\rho C_p)_{hCNTnf}} \left(\frac{\partial^2 T}{\partial y^2} \right) \quad (3)$$

Boundary conditions:

$$\begin{aligned} u &= U_w, v = 0, T = T_w \text{ at } y = 0 \\ u &\rightarrow U_{\infty}, T \rightarrow T_{\infty} \text{ as } y \rightarrow \infty. \end{aligned} \quad (4)$$

The terms μ_{hCNTnf} , ρ_{hCNTnf} , σ_{hCNTnf} , κ_{hCNTnf} , and $(\rho C_p)_{hCNTnf}$ are defined as the dynamic viscosity, density, electrical conductivity, thermal conductivity, and specific heat capacity of hybrid CNT-water nanofluid, respectively. The term B_0 in Eq. (2) indicates the dimensionless magnetic field constant.

Table 1: Thermophysical properties of hybrid CNT-water nanofluid

Property	Correlation
Dynamic viscosity	$\mu_{hCNTnf} = \frac{\mu_f}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}}$
Density	$\rho_{hCNTnf} = (1 - \phi_2)[(1 - \phi_1)\rho_f + \phi_1\rho_1] + \phi_2\rho_2$
Heat capacity	$(\rho C_p)_{hCNTnf} = (1 - \phi_2)[(1 - \phi_1)(\rho C_p)_f + \phi_1(\rho C_p)_1] + \phi_2(\rho C_p)_2$
Thermal conductivity	$\frac{\kappa_{hCNTnf}}{\kappa_{CNTnf}} = \left(\frac{(1 - \phi_2) + 2\phi_2 \left(\frac{\kappa_2}{\kappa_2 - \kappa_{CNTnf}} \right) \ln \left(\frac{\kappa_2 + \kappa_{CNTnf}}{2\kappa_{CNTnf}} \right)}{(1 - \phi_2) + 2\phi_2 \left(\frac{\kappa_{CNTnf}}{\kappa_2 - \kappa_{CNTnf}} \right) \ln \left(\frac{\kappa_2 + \kappa_{CNTnf}}{2\kappa_{CNTnf}} \right)} \right)$ $\frac{\kappa_{CNTnf}}{\kappa_f} = \left(\frac{(1 - \phi_1) + 2\phi_1 \left(\frac{\kappa_1}{\kappa_1 - \kappa_f} \right) \ln \left(\frac{\kappa_1 + \kappa_f}{2\kappa_f} \right)}{(1 - \phi_1) + 2\phi_1 \left(\frac{\kappa_f}{\kappa_1 - \kappa_f} \right) \ln \left(\frac{\kappa_1 + \kappa_f}{2\kappa_f} \right)} \right)$
Electrical conductivity	$\frac{\sigma_{hCNTnf}}{\sigma_{CNTnf}} = \left(1 + \frac{3\phi_2 \left(\frac{\sigma_2}{\sigma_{CNTnf}} - 1 \right)}{\left(\left(\frac{\sigma_2}{\sigma_{CNTnf}} + 2 \right) - \left(\frac{\sigma_2}{\sigma_{CNTnf}} - 1 \right) \phi_2 \right)} \right)$ $\frac{\sigma_{CNTnf}}{\sigma_f} = \left(1 + \frac{3\phi_1 \left(\frac{\sigma_1}{\sigma_f} - 1 \right)}{\left(\left(\frac{\sigma_1}{\sigma_f} + 2 \right) - \left(\frac{\sigma_1}{\sigma_f} - 1 \right) \phi_1 \right)} \right)$

*The subscripts $hCNTnf$ = hybrid CNT nanofluid, $CNTnf$ = CNT nanofluid, f = water, 1 = SWCNT, and 2 = MWCNT.

Table 2: Physical properties of SWCNT, MWCNT, and water

Physical Property	Unit	Nanoparticles		Base Fluid Water
		SWCNT	MWCNT	
Density, ρ	kg/m ³	2,600	1,600	997
Specific heat, C_p	J/kg K	425	796	4,179
Thermal conductivity, κ	W/m K	6,600	3,000	0.613
Electrical conductivity, σ	S/m	1.0×10^8	3.5×10^6	5.0×10^{-2}

*Note: kg = kilogram, m = meter, J = Joule, K = Kelvin, S = Siemens

To reduce the complexity of pointing out Eqs. (1) to (4) in practical explanation, these equations are reduced using the similarity transformation method. The transformation can be done by utilising similarity variables (refer to Anuar *et al.* (2021)) as follows:

$$\eta = \left(\frac{(m+1)U_\infty}{2v_f x} \right)^{\frac{1}{2}} y, \psi = \left(\frac{2v_f x U_\infty}{m+1} \right)^{\frac{1}{2}} f(\eta), T = T_\infty + (T_w - T_\infty)\theta(\eta), \quad (5)$$

where η , ψ , and T represent the dimensionless boundary layer thickness, stream function, and temperature, respectively. Next, the velocity components (u, v) are expressed in dimensionless forms as follows:

$$u = \frac{\partial \psi}{\partial x}, \quad v = -\frac{\partial \psi}{\partial y}. \quad (6)$$

By substituting Eqs. (5) to (6) into Eqs. (1) to (4), the continuity as expressed in Eq. (1) is satisfied. Furthermore, Eqs. (2) to (4) are turned to

$$\left(\frac{A_1}{A_2}\right)f'''(\eta) + f(\eta)f''(\eta) + \left(\frac{A_3}{A_4}\right)\left(1 - (f'(\eta))^2\right) + M\left(\frac{A_5}{A_2A_4}\right)(1 - f'(\eta)) = 0 \quad (7)$$

$$\left(\frac{A_6}{A_7}\right)\theta''(\eta) + \text{Pr}f(\eta)\theta'(\eta) = 0 \quad (8)$$

$$f(0) = 0, f'(0) = \lambda, \theta(0) = 1, \quad f'(\eta) \rightarrow 1, \theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty. \quad (9)$$

The terms $A_i, i = 1, 2, \dots, 7$ can be denoted as follows:

$$A_1 = \frac{\mu_{hCNTnf}}{\mu_f}, A_2 = \frac{\rho_{hCNTnf}}{\rho_f}, A_3 = 2m, A_4 = m + 1, \quad (10)$$

$$A_5 = \frac{\sigma_{hCNTnf}}{\sigma_f}, A_6 = \frac{k_{hCNTnf}}{k_f}, A_7 = \frac{(\rho C_p)_{hCNTnf}}{(\rho C_p)_f}.$$

The dimensionless magnetic parameter, M , in Eq. (7) refers to $M = 2(\sigma_f/\rho_f)(B_0^2 x^{1-m}/U_\infty)$, where $x = x_0$ is a constant. M is defined in relation to the constant x_0 to prevent physical inconsistencies at the leading edge of the boundary layer flow (see Pantokratoras (2019)). Besides, the dimensionless moving parameter, λ , defined as the velocity ratio between U_w and U_∞ (where $\lambda = U_w/U_\infty$, according to Anuar *et al.* (2021)), can be categorised into the following cases:

- If $\lambda = 0$, the wedge remains static;
- If $\lambda > 0$, the wedge moves forward in a similar direction; and
- If $\lambda < 0$, the wedge moves in a contrary direction.

The clarification of different values of λ refers to the model established by Anuar *et al.* (2021).

To examine the dimensionless physical parameters, the skin friction coefficient, C_f and the local Nusselt number, Nu_x , (also utilised for computing the heat transfer coefficient) are defined as follows:

$$C_f = \frac{\tau_w}{\rho_f U_\infty^2}, \quad Nu_x = \kappa_f \frac{x q_w}{(T_w - T_\infty)}. \quad (11)$$

The shear stress, τ_w , and heat flux, q_w , at the surface can be expressed as follows:

$$\tau_w = \mu_{hCNTnf} \frac{\partial u}{\partial y}, q_w = -\kappa_{hCNTnf} \frac{\partial T}{\partial y} \text{ at } y = 0. \quad (12)$$

By utilising the similarity transformation method, Eqs. (11) and (12) are reduced in the following forms:

$$C_f \sqrt{Re_x} = A_1 \sqrt{\frac{A_4}{2}} f''(0), \frac{Nu_x}{\sqrt{Re_x}} = -A_6 \sqrt{\frac{A_4}{2}} \theta'(0). \quad (13)$$

2.2. RSM

To formulate the second-order regression models of heat transfer responses for both mono-, y_{CNT} , and hybrid CNT nanofluids, y_{hCNT} , three controllable independent parameters are selected. The selection of parameters is based on the most influential factor in the Tiwari and Das (2007) model (nanoparticle volume fraction parameter), the main physical effect (magnetic parameter), and the geometrical shape of the model (wedge parameter).

As noted by Myers *et al.* (2016), utilising three controllable factors is sufficient for creating a robust design of experiments, exceeding the minimum required to evaluate significant interaction effects. The general fitted models of y_{CNT} and y_{hCNT} are expressed below:

$$y_{CNT} = \beta_0 + \sum_{i=1}^3 \beta_i x_i + \sum_{i=1}^3 \sum_{j>1}^3 \beta_{ij} x_i x_j + \sum_{i=1}^3 \beta_{ii} x_i^2 + \varepsilon, \quad (14)$$

$$y_{hCNT} = \beta_0 + \sum_{i=1}^3 \beta_i x_i + \sum_{i=1}^3 \sum_{j>1}^3 \beta_{ij} x_i x_j + \sum_{i=1}^3 \beta_{ii} x_i^2 + \varepsilon, \quad (15)$$

where the β s are the regression coefficients. The terms x_1 , x_2 , and x_3 refer to nanoparticle volume fraction, magnetic, and wedge parameters, respectively. The statistical error ε in both Eqs. (14) and (15) is assumed to follow a normal distribution with a mean of zero and a variance of σ^2 (Myers *et al.* 2016).

Both models in Eqs. (14) and (15) are validated by the analysis of variance (ANOVA) test, where a p -value below 0.05 is adequate to reject the null hypothesis, indicating that the sources in the ANOVA results are statistically significant for both models. According to Verykoui and Nakas (2023), the p -value is utilised for the analysis of statistically significant models.

3. Results and Discussion

This section explores the numerical and optimisation outcomes for the hybrid CNT nanofluid flow and heat transfer over a moving wedge under the hydromagnetic effect.

3.1. Numerical experiment

The numerical solutions for this model are executed by using the bvp4c method in MATLAB. After effectively organising the MATLAB code, the solutions for the reduced skin friction, $f''(0)$, are tested to facilitate a comparison analysis with prior research (Anuar *et al.* 2021) and to validate the model's code. Due to the exceptional alignment of the code as displayed in Table 3, the solutions for the reduced skin friction, $f''(0)$, and the reduced heat transfer, $-\theta'(0)$, are obtained numerically by altering the parameters λ , M , and m .

Table 3: Various solutions for $f''(0)$ using different values of m with $Pr = 0.72$ and $\phi_1 = \phi_2 = \lambda = 0$

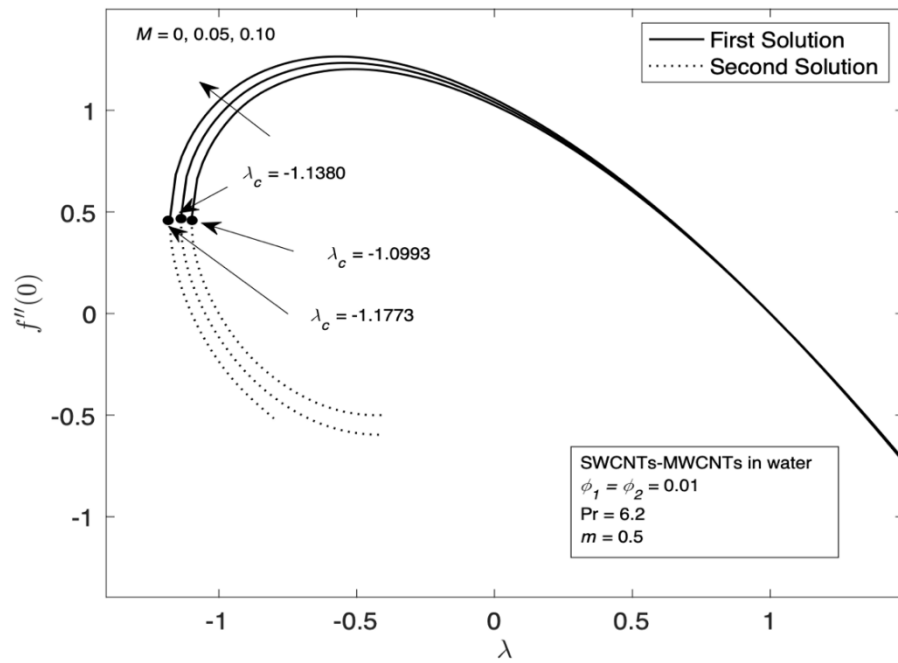
m	Anuar <i>et al.</i> (2021)	Current Model
0.00	0.4696	0.469600
0.09	0.6550	0.654979
0.20	0.8021	0.802126
0.33	0.9277	0.927654
0.50	1.0389	1.038900
1.00	1.2326	1.232588

The analysis of the solutions $f''(0)$ and $-\theta'(0)$ is performed to ascertain the presence of unique and non-unique solutions, as well as the behaviour of boundary layer separation resulting from variations in the parameters. The first investigation, as seen in Figure 2, aims to determine the variations $f''(0)$ and $-\theta'(0)$ resulting from adjustments in λ and M . The values of M are adjusted based on Mousavi *et al.* (2021). The variations in M denote changes in the strength of the external magnetic field, which has a direct impact on the Lorentz force exerted on the fluid. As shown in Figure 2, the other parameters remain constant.

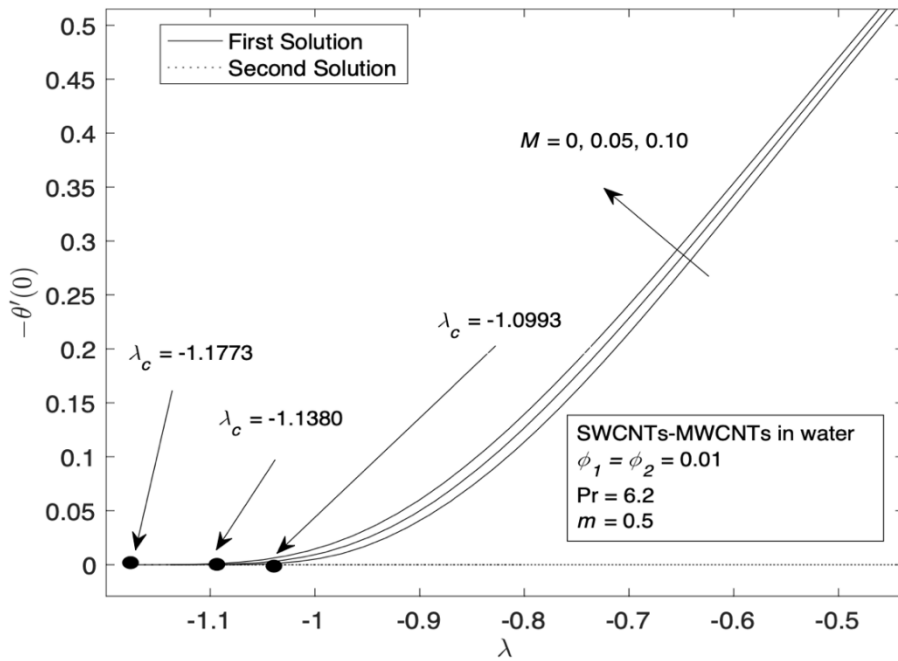
Figure 2 illustrates that the movement of the wedge within the range $\lambda > 0$ yields single solutions, whereas the opposite occurs when the wedge is within the range $\lambda_c \leq \lambda < 0$. Based on Figure 2, dual solutions are obtained numerically when the wedge moves contrary to the direction of the free stream, indicating a scenario of opposing flow where $\lambda < 0$. From a mathematical perspective, this indicates that there are two distinct roots present in the boundary layer equations for a particular range of parameters.

In addition, there are no solutions for $f''(0)$ and $-\theta'(0)$ if $\lambda_c > \lambda$, where λ_c denotes the intersection point between the first (horizontal line) and second (dotted line) solution. The existence of dual solutions is essential for predicting flow separation. The significance of the critical value (λ_c) is that it marks the precise point of boundary layer separation. Physically, this value represents the mathematical limit beyond which no solution exists, indicating the breakdown of laminar flow and the onset of turbulence.

In practical engineering applications, identifying λ_c is crucial for aerodynamic design, such as in aerofoils or turbine blades. By determining this limit, engineers can optimise surface velocities to maintain laminar flow, delay boundary layer separation, and reduce drag, thereby improving the device's energy efficiency. Based on Figure 2, the increase of M contributes to the delay in flow separation and widens the range of $f''(0)$ and $-\theta'(0)$. Moghimi *et al.* (2020) explain that the delay in flow transition is caused by the rise in the magnetic parameter due to the Lorentz force acting on the fluid flow.



(a)



(b)

Figure 2: Variation of (a) $f''(0)$ and (b) $-\theta'(0)$ due to change in λ and M

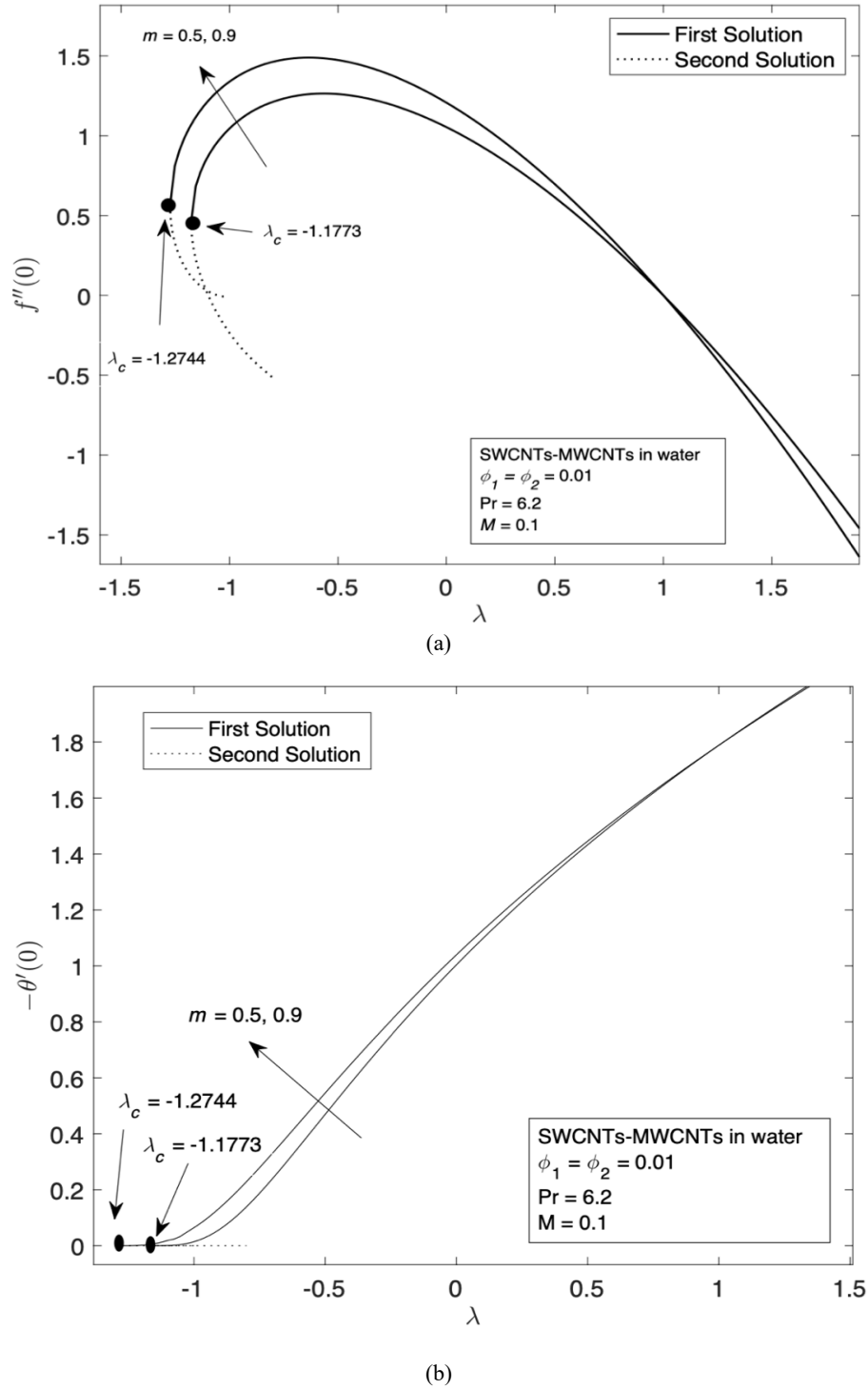


Figure 3: Variation of (a) $f''(0)$ and (b) $-\theta'(0)$ due to change in λ and m

The second investigation is performed to examine the variation of λ and m in relation to the solutions $f''(0)$ and $-\theta'(0)$. The values of λ and m are altered based on the Anuar *et al.* (2021) model. The changes in m signify distinct flow geometries and pressure gradients. According to Figure 3, the detection of single and coupled solutions follows the observations in Figure 2.

Furthermore, the increase in m in the range mentioned in Section 2.1 extends the boundary layer separation and emphasizes the range of $f''(0)$ and (b) $-\theta'(0)$. Anuar *et al.* (2021) assert that an increase in wedge angle results in a significant reduction in fluid velocity, hence slowing flow separation.

The convergence of the velocity and temperature profiles, as dictated by the boundary conditions outlined in Eq. (9), is demonstrated in Figures 4 and 5. By varying the values of M and m , both profiles in Figures 4 and 5 demonstrate that the profiles converge asymptotically to the boundary conditions established in Eq. (9). Moreover, the augmentation of M and m enhances the velocity profiles for the first solution, while the opposite effect occurs for the second solution. The first solution, which is summarised as stable and physically realised flow (see Nordin *et al.* 2024), has enhanced velocity profiles because the favourable effects of the wedge-induced pressure gradient (acceleration) and the magnetic force's stabilising influence dominate, effectively thinning the boundary layer and increasing the flow speed near the wall. Conversely, opposing tendencies are observed in the thermal profile for the first and second solutions as M and m increase. When comparing the boundary layer thickness, both Figures 4 and 5 show that the second solution has a greater thickness than the first solution.

The investigation in Figures 6 and 7 compares the performance of mono- and hybrid CNT nanofluids using various M and m values. The values of M and m are modified from 0 to 1, as seen Figures 6 and 7, respectively. The CNT nanoparticle volume fraction parameter is established at 0.01, as Wei *et al.* (2021) determined that this value optimally enhances the thermal, frictional, and economic performance of the model. Besides, the other parameters remain constant at $Pr = 6.2$, $m = 0.5$, $\lambda = 0.5$. Based on Figure 6, the improvement in M linearly raises the coefficients of skin friction and heat transmission. Increased M enhances the Lorentz force, which acts as a drag on the fluid flow, increasing the velocity gradient near the surface, thus linearly raising both the skin friction (momentum transfer) and the heat transmission (energy transfer). This greater resistance to flow causes more shear stress and more efficient transfer of momentum and heat away from the boundary.

The performance of mono- and hybrid CNT nanofluids is also conducted by varying m while keeping M , ϕ_1 , ϕ_2 , Pr , and λ constant. Both fluid types have a linear growth tendency as m rises, as displayed in Figure 7. The improvement in m generally relates to an increase in the pressure gradient along the wedge surface. This stronger, favourable pressure gradient accelerates the fluid near the surface, leading to a steeper velocity profile (higher shear rate) and a thinner momentum boundary layer, which directly results in a linear increase in the skin friction coefficient. Simultaneously, this faster fluid motion and thinner boundary layer enhance the rate of heat transfer between the wedge and the fluid, thus linearly raising the heat transmission coefficient.

In addition, Figures 6 and 7 show that hybrid CNT nanofluid has higher skin friction and heat transfer coefficients than mono CNT nanofluid. Based on these outcomes, mono-CNT is superior for enhancing aircraft design, since the fluid exhibits lower skin friction, resulting in less restricted fluid flow. Besides, mono-CNT nanofluid, displayed by a lower skin friction coefficient, is the preferred choice for applications where minimising viscous drag and pumping power losses are paramount, such as in micro-channel cooling systems or long-distance fluid transport in pipes, provided its heat transfer performance remains adequate for the given thermal load.

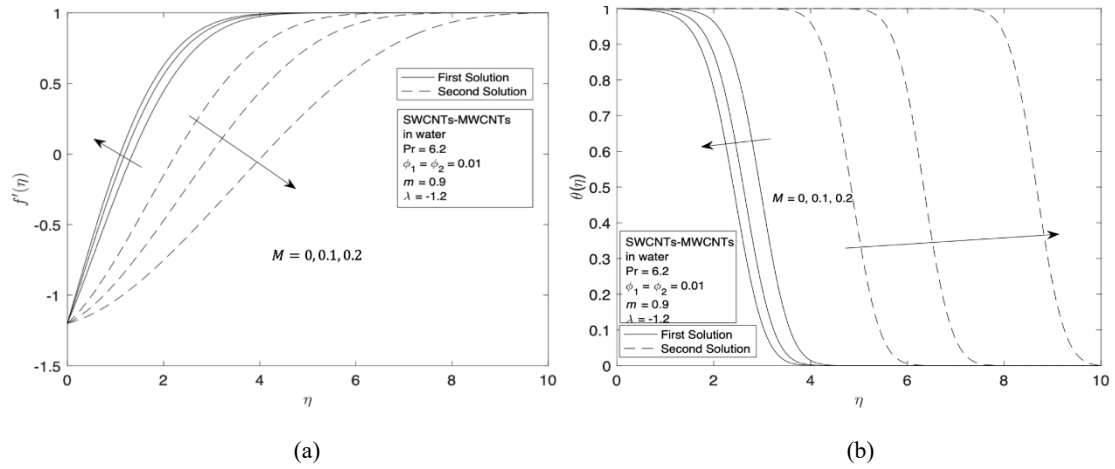


Figure 4: Variation of (a) velocity and (b) temperature profiles due to change in λ and M

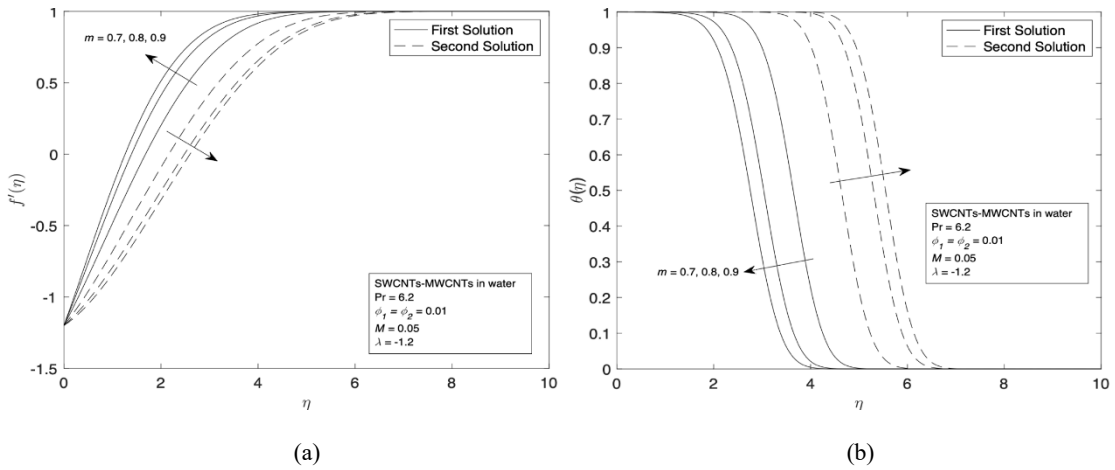


Figure 5: Variation of (a) velocity and (b) temperature profiles due to change in λ and m

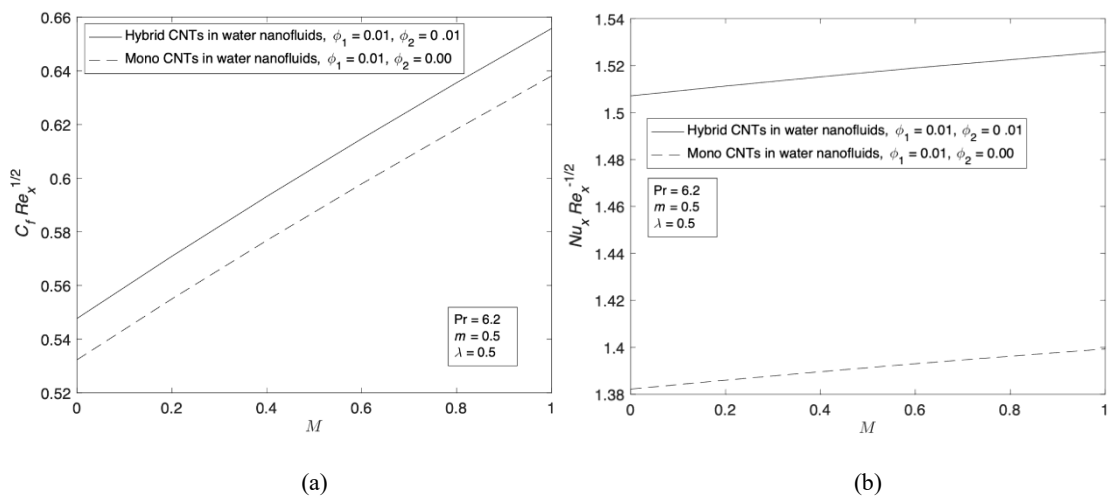
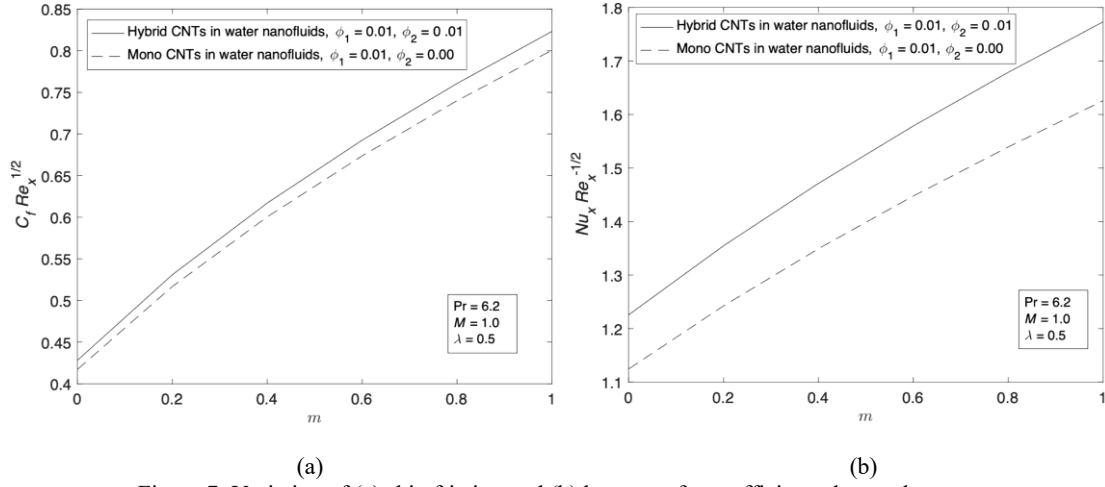


Figure 6: Variation of (a) skin friction and (b) heat transfer coefficients due to change in M


 Figure 7: Variation of (a) skin friction and (b) heat transfer coefficients due to change m

Nevertheless, hybrid CNT nanofluid is an effective choice for accelerated heat transfer, especially in cooling systems, due to its excellent heat transfer coefficient. Furthermore, the superior thermo-physical performance of hybrid CNT nanofluids, evidenced by their significantly higher skin friction and heat transfer coefficients relative to mono CNT nanofluids, designates them as the optimal working fluid for next-generation high-performance thermal systems. This enhanced capacity is particularly advantageous in applications such as compact, high-efficiency heat exchangers and advanced solar thermal collectors, where optimising the heat transfer rate is critical for improving the thermal system.

3.2. Optimisation of heat transfer response

To optimise the heat transfer response for both mono- and hybrid CNT nanofluids, the independent variables specified in Section 2.2 are structured in the design matrix with different limits, as presented in Table 4. The heat transfer response denotes the rate of convective heat transfer occurring at the interface between the solid wedge surface and both mono- and hybrid nanofluids. This metric essentially quantifies the system's cooling efficiency. A higher response value signifies that the nanofluid is more effectively extracting heat from the wedge surface. The range of all factors is reflected within the data presented in Section 3.1. Besides, the ranges of ϕ_{CNT} and ϕ_{hCNT} are based on the Wei *et al.* (2021) model, where their ranges are less than 0.05.

Table 4: Design matrix with different limits for mono- and hybrid CNT nanofluid

Factor		Limit		
Uncoded	Coded	Low −1	Medium 0	High +1
ϕ_{CNT}	x_1	0.01	0.02	0.03
ϕ_{hCNT}	x_2	0.10	0.20	0.30
m	x_3	0.50	0.70	0.90

To determine the number of runs, N , the following formulation is employed, as referenced in Myers *et al.* (2016). In this context, the number of factors, F , is 3, while the number of centres, C , is 6.

$$N = 2^F + 2F + C . \quad (16)$$

Therefore, 20 runs are utilised to construct the design of the experiment (DOE) for both mono- and hybrid CNT nanofluid, as displayed in Table 5. The data in Table 5 is generated using the first solution from Section 3.1, as Anuar *et al.* (2021) and Nordin *et al.* (2024) have affirmed that this solution is physically acceptable for assessing heat transfer responses in both models.

Table 5: Design of experiment (DOE) for y_{CNT} and y_{hCNT} models

Run	ϕ_{CNT} or ϕ_{hCNT}	Factor					Response	
		$[x_1]$	M	$[x_2]$	m	$[x_3]$	$[y_{CNT}]$	$[y_{hCNT}]$
1	0.01	[−1]	0.10	[−1]	0.50	[−1]	[1.4926]	[1.5092]
2	0.03	[+1]	0.10	[−1]	0.50	[−1]	[1.7208]	[1.7762]
3	0.01	[−1]	0.30	[+1]	0.50	[−1]	[1.4967]	[1.5133]
4	0.03	[+1]	0.30	[+1]	0.50	[−1]	[1.7255]	[1.7809]
5	0.01	[−1]	0.10	[−1]	0.90	[+1]	[1.6951]	[1.7140]
6	0.03	[+1]	0.10	[−1]	0.90	[+1]	[1.9547]	[2.0176]
7	0.01	[−1]	0.30	[+1]	0.90	[+1]	[1.6981]	[1.7170]
8	0.03	[+1]	0.30	[+1]	0.90	[+1]	[1.9582]	[2.0211]
9	0.01	[−1]	0.20	[0]	0.70	[0]	[1.5991]	[1.6169]
10	0.03	[+1]	0.20	[0]	0.70	[0]	[1.8439]	[1.9032]
11	0.02	[0]	0.10	[−1]	0.70	[0]	[1.7227]	[1.7603]
12	0.02	[0]	0.30	[+1]	0.70	[0]	[1.7264]	[1.7640]
13	0.02	[0]	0.20	[0]	0.50	[−1]	[1.6118]	[1.6469]
14	0.02	[0]	0.20	[0]	0.90	[+1]	[1.8298]	[1.8697]
15	0.02	[0]	0.20	[0]	0.70	[0]	[1.7246]	[1.7622]
16	0.02	[0]	0.20	[0]	0.70	[0]	[1.7246]	[1.7622]
17	0.02	[0]	0.20	[0]	0.70	[0]	[1.7246]	[1.7622]
18	0.02	[0]	0.20	[0]	0.70	[0]	[1.7246]	[1.7622]
19	0.02	[0]	0.20	[0]	0.70	[0]	[1.7246]	[1.7622]
20	0.02	[0]	0.20	[0]	0.70	[0]	[1.7246]	[1.7622]

*[] represents the coded values in Minitab.

As mentioned in Section 2.2, the DOE in Table 5 is validated by ANOVA. The ANOVA results for both the complete y_{CNT} and y_{hCNT} models are illustrated in Tables 6 and 7. From Tables 6 and 7, two sources (the square term, x_2^2 , and the 2-way interaction term, x_1x_2) contain p -values greater than 0.05. As a result, both terms are removed from the complete y_{CNT} and y_{hCNT} models. The exclusion of these terms generates the reduced y_{CNT} and y_{hCNT} models, as displayed in Tables 8 and 9. The data in Tables 8 and 9 are re-analysed utilising ANOVA to determine sources with p -values under 0.05. Tables 8 and 9 indicate that all sources present p -values below 0.05. This observation signifies that these sources are statistically significant to the reduced y_{CNT} and y_{hCNT} models.

Table 6: ANOVA result for the complete y_{CNT} model

Source	DF	Adj SS	Adj MS	F-value	p-value
Model	9	0.268382	0.029820	1500906.58	$< 1.00 \times 10^{-8}$
Linear	3	0.267702	0.089234	4491301.50	$< 1.00 \times 10^{-8}$
x_1	1	0.149154	0.149154	7507178.87	$< 1.00 \times 10^{-8}$
x_2	1	0.000036	0.000036	1808.70	$< 1.00 \times 10^{-8}$
x_3	1	0.118512	0.118512	5964916.94	$< 1.00 \times 10^{-8}$
Square	3	0.000190	0.000063	3182.48	$< 1.00 \times 10^{-8}$
x_1^2	1	0.000026	0.000026	1320.44	$< 1.00 \times 10^{-8}$
x_2^2	1	0.000000	0.000000	0.06	0.816
x_3^2	1	0.000039	0.000039	1971.36	$< 1.00 \times 10^{-8}$
2-Way Interaction	3	0.000491	0.000164	8235.75	$< 1.00 \times 10^{-8}$
x_1x_2	1	0.000000	0.000000	8.16	0.017
x_1x_3	1	0.000490	0.000490	24664.95	$< 1.00 \times 10^{-8}$
x_2x_3	1	0.000001	0.000001	34.15	$< 1.00 \times 10^{-8}$
Error	10	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$		
Lack-of-Fit	5	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$
Pure Error	5	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$		
Total	19	0.268382			

Table 7: ANOVA result for the complete y_{hCNT} model

Source	DF	Adj SS	Adj MS	F-value	p-value
Model	9	0.328798	0.036533	1463051.58	$< 1.00 \times 10^{-8}$
Linear	3	0.327981	0.109327	4378248.12	$< 1.00 \times 10^{-8}$
x_1	1	0.204090	0.204090	8173251.14	$< 1.00 \times 10^{-8}$
x_2	1	0.000036	0.000036	1445.71	$< 1.00 \times 10^{-8}$
x_3	1	0.123855	0.123855	4960047.51	$< 1.00 \times 10^{-8}$
Square	3	0.000148	0.000049	1979.23	$< 1.00 \times 10^{-8}$
x_1^2	1	0.000012	0.000012	498.37	$< 1.00 \times 10^{-8}$
x_2^2	1	0.000000	0.000000	0.08	0.781
x_3^2	1	0.000041	0.000041	1655.61	$< 1.00 \times 10^{-8}$
2-Way Interaction	3	0.000669	0.000223	8927.40	$< 1.00 \times 10^{-8}$
x_1x_2	1	0.000000	0.000000	6.06	0.034
x_1x_3	1	0.000668	0.000668	26749.66	$< 1.00 \times 10^{-8}$
x_2x_3	1	0.000001	0.000001	26.48	$< 1.00 \times 10^{-8}$
Error	10	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$		
Lack-of-Fit	5	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$
Pure Error	5	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$		
Total	19	0.328798			

Table 8: ANOVA result for the reduced y_{CNT} model

Source	DF	Adj SS	Adj MS	F-value	p-value
Model	8	0.268382	0.033548	1846831.94	$< 1.00 \times 10^{-8}$
Linear	3	0.267702	0.089234	4912396.42	$< 1.00 \times 10^{-8}$
x_1	1	0.149154	0.149154	8211036.06	$< 1.00 \times 10^{-8}$
x_2	1	0.000036	0.000036	1978.27	$< 1.00 \times 10^{-8}$
x_3	1	0.118512	0.118512	6524174.93	$< 1.00 \times 10^{-8}$
Square	2	0.000190	0.000095	5221.26	$< 1.00 \times 10^{-8}$
x_1^2	1	0.000031	0.000031	1688.87	$< 1.00 \times 10^{-8}$
x_3^2	1	0.000046	0.000046	2519.16	$< 1.00 \times 10^{-8}$
2-Way Interaction	3	0.000491	0.000164	9007.92	$< 1.00 \times 10^{-8}$
x_1x_2	1	0.000000	0.000000	8.92	0.012
x_1x_3	1	0.000490	0.000490	26977.49	$< 1.00 \times 10^{-8}$
x_2x_3	1	0.000001	0.000001	37.35	$< 1.00 \times 10^{-8}$
Error	11	0.000000	0.000000		
Lack-of-Fit	6	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$
Pure Error	5	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$		
Total	19	0.268382			

Table 9: ANOVA result for the reduced y_{hCNT} model

Source	DF	Adj SS	Adj MS	F-value	p-value
Model	8	0.328798	0.041100	1795815.90	$< 1.00 \times 10^{-8}$
Linear	3	0.327981	0.109327	4776942.61	$< 1.00 \times 10^{-8}$
x_1	1	0.204090	0.204090	8917528.33	$< 1.00 \times 10^{-8}$
x_2	1	0.000036	0.000036	1577.36	$< 1.00 \times 10^{-8}$
x_3	1	0.123855	0.123855	5411722.15	$< 1.00 \times 10^{-8}$
Square	2	0.000148	0.000074	3239.16	$< 1.00 \times 10^{-8}$
x_1^2	1	0.000015	0.000015	638.83	$< 1.00 \times 10^{-8}$
x_3^2	1	0.000048	0.000048	2113.07	$< 1.00 \times 10^{-8}$
2-Way Interaction	3	0.000669	0.000223	9740.35	$< 1.00 \times 10^{-8}$
x_1x_2	1	0.000000	0.000000	6.61	0.026
x_1x_3	1	0.000668	0.000668	29185.56	$< 1.00 \times 10^{-8}$
x_2x_3	1	0.000001	0.000001	28.89	
Error	11	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$		
Lack-of-Fit	6	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$
Pure Error	5	$< 1.00 \times 10^{-8}$	$< 1.00 \times 10^{-8}$		
Total	19	0.328798			

Given that all sources in Tables 8 and 9 are statistically significant, the final fitted second-order regression models for both y_{CNT} and y_{hCNT} , based on Eqs. (14) and (15), can be formulated as follows:

$$\begin{aligned} y_{CNT} = & 1.72459 + 0.122129x_1 + 0.001896x_2 + 0.108863x_3 \\ & + 0.000142x_1x_2 + 0.007827x_1x_3 - 0.000291x_2x_3 \\ & - 0.003096x_1^2 - 0.003782x_3^2 \end{aligned} \quad (17)$$

$$\begin{aligned} y_{hCNT} = & 1.76219 + 0.142860x_1 + 0.001900x_2 + 0.111290x_3 \\ & + 0.000138x_1x_2 + 0.009138x_1x_3 - 0.000287x_2x_3 \\ & - 0.002137x_1^2 - 0.003888x_3^2 . \end{aligned} \quad (18)$$

One advantage of employing RSM is the capacity to visually depict the interaction between input factors and responses through surface and contour plots. The results of examining interaction effects provide crucial guidance for optimising thermal engineering systems, which include modern heat exchangers and electronic cooling devices. These insights facilitate cost-benefit optimisation by highlighting advantageous relationships between parameters. For instance, if a significant interaction is identified between the nanoparticle volume fraction (ϕ) and the magnetic parameter (M), engineers can achieve a desired cooling rate (the Nusselt number) by adjusting the magnetic field intensity rather than increasing the concentration of costly nanoparticles.

Furthermore, the regression equation proposed in this study serves as a dependable forecasting tool for operational management. This equation allows for the real-time adjustment of input parameters, such as magnetic field intensity, in response to geometric changes. Thus, it helps maintain optimal heat transfer efficiency without the need for continuous physical testing.

The interactions between input factors and responses are illustrated in Figures 8 to 10. These plots are also employed to analyse the predominant factor influencing the heat transfer rates for both models when the factors are configured at their maximum levels (see Table 4). Before conducting the analysis, the factors x_3 , x_2 , and x_1 are assumed to be zero in Figures 8, 9, and 10, respectively. In Figure 8, it is noticeable that x_1 demonstrates a stronger impact than x_2 when both components increase to their maximum regulated values. The significant influence of x_1 is also apparent in comparison to x_3 (refer to Figure 9). Notwithstanding this, x_3 exhibits a greater influence than x_2 as both variables increase at their maximum regulated levels, as illustrated in Figure 8. Figures 8 to 10 also demonstrate that the increasing trends of x_1 , x_2 , and x_3 , up to their maximum limits as indicated in Table 4, positively affect both responses. Table 10 summarises the factors optimising both models. Table 10 indicates that the nanoparticle volume fraction parameter is the most significant parameter for optimising heat transfer responses in both nanofluid models. The outcome in Table 10 aligns with the findings of the Tiwari and Das (2007) model, wherein the nanoparticle volume fraction parameter is pivotal for enhancing the mathematical modelling of nanofluids.

The final phase of the optimisation process involves maximising the heat transfer responses for both the y_{CNT} and y_{hCNT} models by using the response optimiser in Minitab along with the desirability function method. Table 11 shows that the optimisation results for both the y_{CNT} and y_{hCNT} models reached the best desirability score, achieving a 100% perfect value. The current RSM analysis identified an optimised combination of governing parameters, yielding a maximum heat transfer coefficient of 2.0212 with 100% composite desirability for the hybrid

CNT nanofluid system. This optimised heat transfer rate demonstrates exceptional thermal efficiency in our proposed model. Quantitatively, this result represents an enhancement of more than 104% when compared to the Cu-Al₂O₃ hybrid nanofluid flow (Khashi'ie *et al.* 2022), which exhibited a coefficient range below 1.0. This stark difference confirms the superiority and practical relevance of the optimal parameter set identified specifically for the hybrid CNT nanofluid under combined hydromagnetic and wedge flow effects. Additionally, the heat transfer responses for both the y_{CNT} and y_{hCNT} models are optimised when every factor is at its maximum regulatory level, as shown in Table 4. Notably, the optimisation results presented in Table 11 are consistent with those in Section 3.1, where the heat transfer rate of hybrid CNT nanofluid exceeds that of mono-CNT nanofluid.

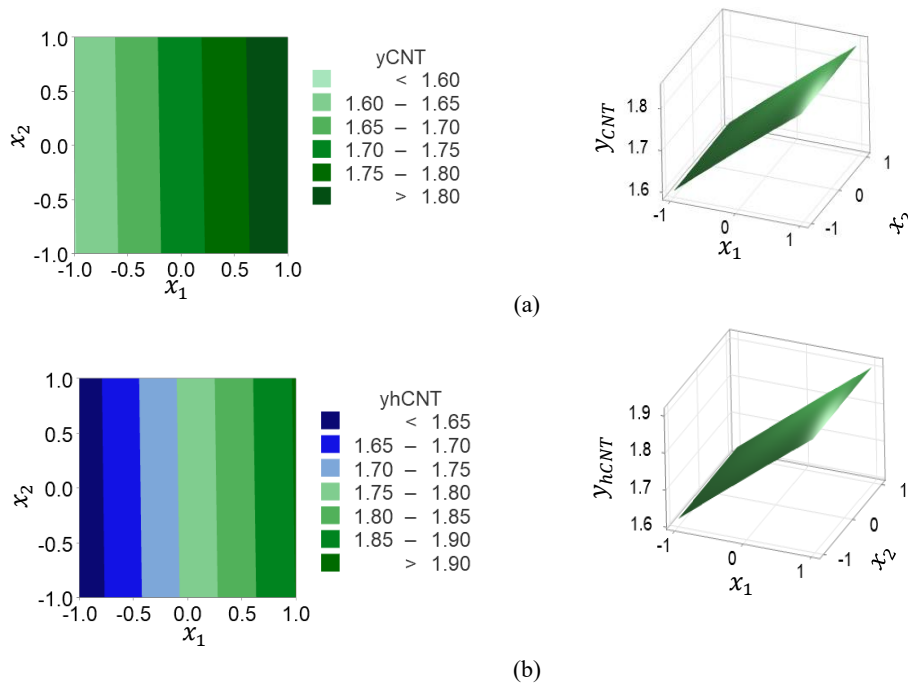


Figure 8: Contour and surface plots for the interaction of x_1 and x_2 for (a) y_{CNT} (b) y_{hCNT} models

Table 10: Summary results for determining the most influential factor

Interaction	Main factor to maximize the y_{CNT} and y_{hCNT} models
x_1 and x_2	x_1
x_1 and x_3	x_1
x_2 and x_3	x_3

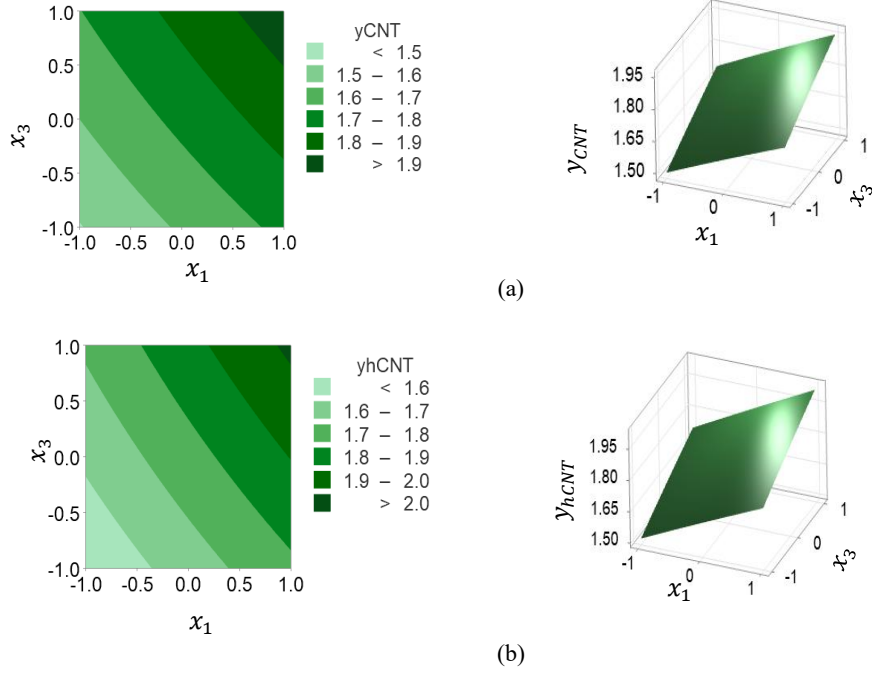


Figure 9: Contour and surface plots for the interaction of x_1 and x_3 for (a) y_{CNT} (b) y_{hCNT} models

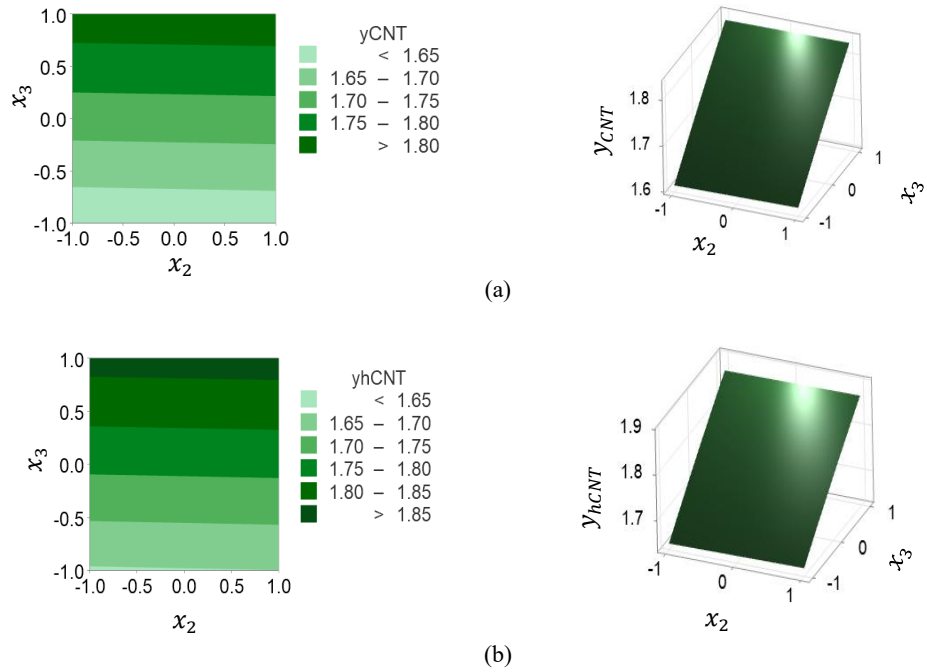


Figure 10: Contour and surface plots for the interaction of x_2 and x_3 for (a) y_{CNT} (b) y_{hCNT} models

Table 11: Optimisation results for the y_{CNT} and y_{hCNT} models

Solution	Composite Desirability	Independent Factor			Response	
		$[x_1]$	$[x_2]$	$[x_3]$	$[y_{CNT}]$	$[y_{hCNT}]$
1	1.00000	[+1]	[+1]	[+1]	[1.9583]	[2.0212]

*Note: The symbol [] indicates a coded value.

4. Conclusion

This study successfully conducted numerical and Response Surface Methodology (RSM) assessments of hybrid single- and multi-walled Carbon Nanotube (CNT) nanofluid flow and heat transfer over a movable wedge under the hydromagnetic effect. The novel contribution lies in utilising RSM to optimise and model this complex hybrid flow system, yielding critical insights into flow dynamics and thermal performance.

Regarding the flow field, the model revealed non-unique solutions for skin friction and heat transfer coefficients when the wedge moves in the opposite direction. Crucially, increasing the magnetic and wedge parameters proved beneficial, as this expanded the solution domain, postponed flow separation, and simultaneously increased both the skin friction and heat transfer coefficients.

In terms of performance, both numerical and RSM results consistently demonstrated that the hybrid CNT nanofluid significantly outperforms the mono-CNT nanofluid in terms of heat transfer rate. This enhancement, however, incurs a trade-off, as the hybrid fluid exhibits a larger skin friction coefficient, indicating that the mono-CNT fluid is superior for applications requiring minimised flow resistance. The RSM optimisation analysis further pinpointed the CNT volume fraction as the single most crucial factor affecting the heat transfer rate. Confirmation via the desirability function approach showed that maximising the controllable factors (CNT volume fraction, wedge, and magnetic parameters) leads to the greatest thermal performance improvement.

5. Future Scope

Future hybrid CNT nanofluid models should leverage the capabilities of artificial intelligence (AI) and machine learning approaches to develop new CNT nanofluid models. The promise of AI and machine learning methodologies can bring new perspectives in CNT nanofluid flow and heat transfer models over various surfaces and physical effects.

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Nomenclature

a, b	Constants
A	Coefficients
B_0	Magnetic field constant
C	Centre point
C_f	Skin friction coefficient
CNT	Carbon nanotube
f	Non-dimensional stream function
F	Factor number
i, j	Indices of the independent variables
k	Total number of independent variables (factors)
L	Length
m	Wedge parameter
M	Magnetic parameter
N	Trial number
Nu_x	Local Nusselt number
Pr	Prandtl number
Re_x	Local Reynolds number
t	Time
T	Constant temperature
u, v	Velocity components
U	Constant velocity
x, y	Cartesian coordinates
x_i^2	Square terms
$x_i x_j$	Interaction term between independent variables i and j
y_{CNT}	Heat transfer for CNT
y_{hCNT}	Heat transfer for hybrid CNT

Subscripts

w	Condition on wedge surface
∞	Ambient condition
hnf	Hybrid nanofluids
nf	Nanofluids
f	Base fluid (water)
$CNTnf$	Carbon nanotube nanofluid
$hCNTnf$	Hybrid carbon nanotube nanofluid
1	Single-walled carbon nanotube
2	Multi-walled carbon nanotube

Greek Symbols

α	Thermal diffusivity
μ	Dynamic viscosity
ρ	Density
β	Hartree pressure gradient
ψ	Non-dimensional stream function
η	Non-dimensional thickness
C_p	Specific heat for base fluid
σ	Electrical conductivity
κ	Thermal conductivity
A	Distance
ν	Kinematic viscosity
ϕ	CNTs volume fraction
λ	Moving parameter
λ_c	Critical value
ε	Error term
ν_f	Kinematic viscosity of the base fluid (water)
τ_w	Shear stress
q_w	Heat flux
θ	Non-dimensional temperature
β_0	Mean term
β_{ij}	Linear term
β_{ii}	Quadratic term

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