

## A SIMULATION STUDY ON BIAS EFFECTS IN PROBABILITY DISTRIBUTION FITTING FOR RAINFALL EXTREMES

(Kajian Simulasi Terhadap Kesan Bias dalam Penyesuaian Taburan Kebarangkalian untuk  
Hujan Ekstrem)

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### ABSTRACT

Accurate modeling of extreme rainfall is essential for hydrological planning and disaster mitigation in Malaysia. However, biases in rainfall data, whether due to measurement errors or model outputs, can distort the fit of probability distributions used in extreme value analysis. This study simulates synthetic extreme rainfall data under four systematic bias scenarios: additive, multiplicative, combined, and non-linear. Each dataset was generated via Monte Carlo simulation to mimic the annual characteristics of extreme rainfall. Four probability distributions, which are Gamma, Weibull, Burr Type XII, and Kappa, were fitted to the biased and bias-corrected datasets, with parameters estimated using the Maximum Likelihood Estimation (MLE) method. Quantile Mapping (QM) was employed to correct the biases prior to fitting. Model performance was assessed using Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Nash-Sutcliffe Efficiency (NSE), Kling-Gupta Efficiency (KGE), and Goodness-of-Fit (GoF) tests. Accordingly, the findings reveal that the Kappa distribution consistently performs best under additive and combined biases. By contrast, the Burr Type XII distribution outperforms others under non-linear scenarios. While Weibull yields low fitting errors for multiplicative biases, it fails to pass statistical fit tests. The gamma distribution performs poorly across all scenarios. Overall, these results highlight the importance of selecting appropriate distribution models following bias correction, with Kappa and Burr identified as robust choices for modeling extreme rainfall under biased conditions in Malaysia.

**Keywords:** extreme rainfall; bias correction; quantile mapping; simulation study; probability distribution

### ABSTRAK

Pemodelan hujan ekstrem yang tepat adalah penting bagi perancangan hidrologi dan mitigasi bencana di Malaysia. Namun, bias dalam data hujan sama ada akibat ralat pengukuran atau output model boleh memesongkan penyesuaian taburan kebarangkalian dalam analisis nilai ekstrem. Kajian ini mensimulasikan data hujan ekstrem sintetik melalui simulasi Monte Carlo di bawah empat senario bias sistematik: aditif, multiplikatif, gabungan dan tidak selanjar. Empat taburan kebarangkalian iaitu Gamma, Weibull, Burr Jenis XII dan Kappa dipadankan kepada data yang terjejas dan telah dibetulkan bias, dengan anggaran parameter dilakukan menggunakan kaedah Anggaran Kebolehjadian Maksimum (MLE). Pembetulan bias dilaksanakan menggunakan kaedah Pemetaan Kuantil (QM). Prestasi model dinilai menggunakan Ralat Punca Min Kuasa Dua (RMSE), Ralat Mutlak Purata (MAE), Kecekapan Nash-Sutcliffe (NSE), Kecekapan Kling-Gupta (KGE) dan ujian kesesuaian statistik. Dapatan menunjukkan taburan Kappa berprestasi terbaik untuk bias aditif dan gabungan, manakala Burr Jenis XII adalah terbaik dalam senario bias tidak selanjar. Weibull mencatat ralat rendah untuk bias multiplikatif tetapi gagal dalam ujian kesesuaian statistik. Taburan Gamma menunjukkan prestasi lemah dalam semua senario. Keputusan ini menegaskan kepentingan pemilihan model taburan yang sesuai selepas pembetulan bias, dengan Kappa dan Burr dikenalpasti sebagai pilihan teguh untuk pemodelan hujan ekstrem di bawah data yang terkesan bias di Malaysia.

**Kata kunci:** hujan ekstrem; pembetulan bias; pemetaan kuantil; kajian simulasi; taburan kebarangkalian

## 1. Introduction

Extreme rainfall in Malaysia and other tropical countries is dangerous for infrastructure, crops, and safety, as intense monsoons and rainstorms are very frequent in these regions. Often, such sudden events lead to flash floods that have cost the country billions of ringgit in the past (Tan *et al.* 2021). For these reasons, robust probabilistic models of heavy rain are needed to reduce flood risks and inform infrastructure planning (Tan *et al.* 2021). Notably, probability distribution fitting is very helpful, enabling the calculation of return periods for high rainfall magnitudes. In this field, mapping daily rainfall annual maxima is performed mainly using the Generalized Extreme Value (GEV) distribution. Many studies demonstrate that, among all distribution families, the Fréchet form of the GEV distribution works best for analyzing global rainfall extremes (Papalexiou & Koutsoyiannis 2013). To achieve good results, Malaysian experts test different probability distributions for extreme rainfall to determine which models perform best (Zalina *et al.* 2002). In particular, the purpose is to examine how different probability distributions perform when fitting rainfall extremes in Malaysia, focusing on additive, multiplicative, combining, and non-linear biases. Even though researchers have studied rainfall distribution extensively, skewed rainfall data can lead to errors in estimating extreme events. Due to weakly parameterized schemes, approximate models, and data from small regions of the globe, climate models exhibit biases in their results. In the absence of correcting these biases, the distributions of extreme rainfall may not present accurate patterns, leading to incorrect calculations of return levels. For this reason, we need to study how systematic bias in rainfall information can affect the analysis of extreme events when probability distributions are used. Furthermore, the performance of the Gamma, Kappa, Weibull, and Burr Type XII distributions, after the application of synthetic biases, and the improvement of the fit caused by bias correction via Quantile Mapping (QM), is not apparent.

When investigating extreme rainfall in Malaysia, it is crucial to understand how systematic bias influences rainfall-extreme modeling and whether bias-correction methods can effectively mitigate these effects. In this study, three key questions are considered. First, how does the presence of systematic bias in rainfall data affect the fitting of probability distributions for extreme rainfall analysis? Second, to what extent can QM reduce such biases and restore the statistical characteristics of the original rainfall distribution? Third, which probability distribution among Gamma, Weibull, Burr Type XII, and Kappa provides the best representation of extreme rainfall after bias correction? Therefore, the objectives of this study are to examine the impact of different types of systematic bias on probability distribution fitting. This includes evaluating the effectiveness of QM in correcting biased rainfall data and comparing the performance of several candidate distributions in modeling bias-corrected extreme rainfall. It is hypothesized that systematic bias degrades the accuracy of distribution fitting. At the same time, applying QM improves the agreement between the corrected rainfall data and the original unbiased distribution. The following section reviews relevant literature on QM and extreme rainfall modeling. Section 3 describes the simulation framework and bias correction methodology. Section 4 presents the results and discussion, and Section 5 concludes the study.

## 2. Review on Quantile Mapping and Extreme Rainfall Modeling

The problem of climate change and its consequences for extreme weather, particularly rainfall, is a global concern, particularly in tropical countries such as Malaysia. Both the estimation and projection of extreme rainfall are essential for managing water resources, reducing disasters, and planning infrastructure. Nevertheless, climate model outputs, while indispensable for future

projections, often exhibit systematic biases when compared to observed data. Accordingly, QM has been used to address this gap. As such, this review delves into the global and Malaysian applications of QM, its specific utility for extreme rainfall, and the common evaluation metrics used to assess its performance.

### **2.1. Quantile mapping**

QM is a mere distributional mapping used to transform the Cumulative Distribution Function (CDF) of a climate model's data to conform to the CDF of the real world (Gudmundsson *et al.* 2012; Teutschbein & Seibert 2012). Usually, the QM selects the model's simulated values and adjusts them to match the actual distribution observed in a reference dataset covering the same period. Modelers widely prefer QM as it corrects bias across the entire distribution, rather than just the usual mean and variance, which is vital for studying rare extremes. Worldwide, QM methods are applied to different climate factors (temperature, precipitation, and related hydrology) gathered from both GCMs and RCMs (Thiemeßl *et al.* 2011). It has been proven that using this approach reduces biases in weather simulations and better represents daily weather, major events, and seasonal changes. For example, Thiemeßl *et al.* (2011) reported that QM-based RCM precipitation forecasts for Europe were more accurate in statistical comparisons than forecasts made directly by the RCM. According to Gudmundsson *et al.* (2012), as in our case, QM significantly reduces and corrects climate model errors in hydrology, a widely used application. Still, issues persist, mostly about whether the bias will persist as climate changes and how well extremes can be predicted when they fall outside the observed range (Maraun *et al.* 2017; Berg *et al.* 2024). Thus, QM is essential in climate studies, since it provides reliable ways to make climate models available for local and regional use.

The frequent flooding in Malaysia due to extreme rainfall events means that QM and other bias-correction methods should be applied to climate model results. Various studies in Malaysia have used QM to improve the accuracy of regional weather forecasts and satellite-based estimates. Notably, most research in Malaysia aims to improve the accuracy of daily rainfall predictions, which are needed for water management and flood forecasting. For instance, in the study by Ngai *et al.* (2020), the QM bias was removed from the daily rainfall data collected across Malaysia in the APHRODITE observed dataset. Moreover, results demonstrated that QM improved rainfall statistics by correcting biases in frequency, mean intensities, and the 90th percentile values produced by CORDEX-SEA regional climate models. The authors discovered that QM is useful for correcting rainfall data errors, thereby improving the presentation of mean, variance, and the probability of wet days. On the same note, the effectiveness of QM in bias-correcting satellite-based precipitation products, which are crucial in regions with limited rain gauge networks, has been investigated in other local studies. This includes notable reductions in Root Mean Square Error (RMSE) and improved hydrological behavior observed. In a machine learning-based bias correction and conventional approaches study of Peninsular Malaysia, it was highlighted that the accuracy of rainfall inputs is important for monitoring floods and droughts (Iqbal *et al.* 2022). Such applications demonstrate the value of QM in delivering more confident climate information for local impact investigations, aiding the formulation of evidence-based decisions on climate change adaptation measures in Malaysia. The monsoonal nature of the Malaysian climate, with wet and dry seasons, suggests that seasonal biases must be considered, and QM should, in general, be able to correct them by working on empirical distributions.

## 2.2. Distributional fitting for extreme rainfall

Extreme rainfall events are of fundamental importance to many applications, and distributional QM methods are particularly well-suited to this task. This contrasts with simpler methods of bias correction, which shift means or variances, though not the whole distribution, and thus cannot correctly bias-correct the tails where extreme events lie. Given that QM can replicate the entire CDF, it can be applied to adjust the bias in the frequency and severity of extreme precipitation. In extreme-oriented literature, it is often stated that, despite the tendency of QM to result in an improvement in the simulation of precipitation extremes, issues can arise in the aspect of unstable behavior in the very high quantiles due to the scarcity of data in the extreme tails (Tani & Gobiet 2019; Berg *et al.* 2024). Extended or hybrid QM versions, with the tails grounded on a combination of non-parametric empirical mapping and Extreme Value Theory (EVT), have been proposed by some researchers as a solution to the problem (Tani & Gobiet 2019). Meanwhile, Rajulapati *et al.* (2023) proposed Semi-Parametric QM (SPQM) in North America, which uses empirical probabilities for simulations and a parametric model (Generalized Exponential of Type-1 distribution) at observations. Concerning the replication of the observed statistics, wet and dry spells, and extreme quantiles, SPQM did well. The innovations are particularly timely, as precise correction for tails becomes necessary when fitting extreme-value distributions such as the GEV, Kappa, and Burr Type XII to long return periods. This is usually performed by parameterizing extreme-value distributions at the extremes. Additionally, the choice of theoretical distribution used in the fitting is highly significant, as different distributions capture different characteristics of extreme events. For example, the GEV distribution is theoretically sound for the block maxima. It is usually used in the analysis of annual maximum rainfall (Coles 2001). On a similar note, the gamma distribution has been widely used to correct rainfall data bias (Piani *et al.* 2009; Li *et al.* 2010). Other distributions widely applied in hydrological frequency analysis include Gumbel, Gamma, Exponential, Kappa, Burr Type XII, and Generalized Lindley, among others, which offer diverse flexibility in modeling rainfall distributions, including their tails (Heo *et al.* 2019). Building on this, the other group of studies relies on mixture models, e.g., Bernoulli-Exponential, Bernoulli-Log-normal, Bernoulli-Gamma, and Bernoulli-Weibull, where it is assumed that the Bernoulli distribution can be used to estimate the probability of precipitation occurrence, and the rest to estimate precipitation intensities (Gudmundsson *et al.* 2012). Following this, the efficiency of QM rests on the capacity of the selected distributions to approximate the empirical distribution of the biased and observed data, at the upper quantiles.

## 2.3. Evaluation metrics

The thorough examination of bias correction strategies and the suitability of probability distributions is critical to ensure the credibility of climate projections and their subsequent hydrological effects. Such a combination of statistical performance measures and Goodness-of-Fit (GoF) tests is typically used to provide a comprehensive picture. Common measures of error include the RMSE, Mean Absolute Error (MAE), Nash-Sutcliffe Efficiency (NSE), and Kling-Gupta Efficiency (KGE) (Moriassi *et al.* 2007). The squaring mechanism of RMSE is more sensitive to larger errors. Thus, it is more sensitive to extreme deviations. It is useful when one is interested in measuring high-impact events. In comparison, MAE is a less indirect statistic of the mean absolute error, which is not further multiplied by the standard deviation of the errors or the sample size, and consequently, a more self-explanatory and intuitive statistic of model performance (Willmott & Matsuura 2005). To assess overall predictive power, a common measure in hydrology is the NSE, which quantifies the GoF between simulated and observed data (Nash & Sutcliffe 1970). NSE may, however, be overly sensitive to peak flows

and insensitive to low flows. As a reaction to this, the KGE was developed, which decomposes the GoF into correlation, bias, and variability components, and is a relatively balanced and diagnostic measure of model performance (Gupta *et al.* 2009). Finally, GoF tests are procedures that provide formal statistical evidence of how well a fitted distribution models the observed data. The statistic used to measure the maximum absolute difference between the theoretical and empirical CDFs is the Kolmogorov-Smirnov (K-S) test (Wilks 2011). It also includes the mean-squared difference between the empirical and hypothetical CDFs. Therefore, the Cramér-von Mises (CvM) test is more opposed to differences on the entire range, and hence may be more powerful than K-S when an overall distributional fit is to be assessed. This is especially true when used in extreme-value applications (Stephens 1974; Laio 2004). Such an aggregate application of the different metrics and tests provides an effective means to assess the bias-correction methodology's ability to recover the true statistical characteristics of extreme rainfall events.

### **3. Methodology**

#### **3.1. Monte Carlo simulation of GEV synthetic extreme rainfall series**

The Monte Carlo simulation approach will assume the generation of synthetic sequences of extreme rainfall under the GEV distribution. The simple distribution is the GEV distribution, which will be consistently applied in the simulation of annual maximum rainfall since it is a theoretically derived distribution of block maxima in extreme value analysis (Coles 2001). To explore how the amount of available data influences the effects of bias and the ability to correct it, the simulation will be conducted with different sample sizes of each synthetic series,  $n = 50, 100, 500, 1,000$  years. Furthermore, the simulation experiments were conducted using several synthetic record lengths, ranging from short records (approximately 30-50 years) that resemble typical rainfall observation periods to longer sequences (100-1,000 years). Notably, the inclusion of longer records allows the examination of the stability and asymptotic behavior of the bias correction and distribution-fitting procedures under different levels of sampling variability.

GEV distributions are used to simulate daily rainfall, and the annual maximum is taken from them. In all sample sizes and bias cases, a Monte Carlo simulation will be performed with 1,000 independent repetitions to ensure robust statistical inference. The simulations are made explicit for the monsoon season and consecutive rainfall days in Malaysia by providing a better representation of Malaysian extreme rainfall properties using a first-order Markov chain. Additive, multiplicative, combining, and non-linear biases are also simulated, and the entire analysis pipeline is used: distributional QM, performance metrics, and GoF testing. Moreover, the GEV distribution parameters used in the simulation were selected to represent typical characteristics of annual maximum rainfall observed in tropical regions, such as Malaysia. At the same time, previous studies on extreme rainfall modeling in Malaysia and similar climatic regions guided the parameter ranges. In particular, the objective is not to reproduce a specific rainfall station. Rather, it is to generate synthetic rainfall series with realistic statistical properties for evaluating the impact of systematic bias and bias-correction procedures.

#### **3.2. Monsoon periods**

Malaysia has distinct monsoon seasons that significantly affect rainfall patterns and extremes. It is also reported that the Northeast Monsoon is more robust and influential in its rain-bearing characteristics than the Southwest Monsoon (Jabatan Meteorologi Malaysia n.d.). Northeast Monsoon, November to March; Southwest Monsoon, beginning late May to September, are of

variable intensities, and have spatial distributions of rainfall. According to the literature, the average dates of the Southwest Monsoon onset and withdrawal were May 23 and September 27 (Hadi & Idris 2024). In comparison, the mean dates of the Northeast Monsoon arrival and departure were determined to be November 1 to March 15 (Wan *et al.* 2024). To capture such seasonality in the simulation, the GEV distribution parameters  $(\mu, \sigma, \xi)$  will be varied, depending on representative values of these parameters in each season, instead of applying one set of parameters throughout the year. Following this, extreme events are more pronounced and extensive during the Northeast Monsoon, which is reflected in larger values of location ( $\mu$ ), scale ( $\sigma$ ), and shape ( $\xi$ ) parameters than those of the Southwest Monsoon. All simulated observations are apportioned to the monsoon period.

### 3.3. Wet day generation

A first-order Markov chain will be incorporated into the model to generate wet and dry days in the simulated series, better representing the temporal characteristics of daily rainfall. This can explain the continued occurrence of rainfall characteristics typical of tropical climates in Malaysia (Wilks 2011). As such, this wet-or-dry sequence, calculated daily, will be used to produce daily rainfall accumulations, from which annual maximums will be selected. The first-order Markov chain is the classification of the probability of a given state (wet or dry) at time  $t$  based on the state at time  $t - 1$  only as the pertinent information. It assumes that the future state is the only function of the present state. The two states are a wet day,  $W$ , in which the rainfall is measurable, and a dry day  $D$  in which the rainfall is not measurable in case it rains daily. Four transition probabilities (Weiss 1964) determine the behavior of the chain:

$P_{WW}$ : Conditional probability of a wet day, when the preceding day was wet,

$$P(W_t | W_{t-1}). \quad (1)$$

$P_{WD}$ : Conditional probability of a dry day, when the preceding day was wet,

$$P(D_t | W_{t-1}) = 1 - P_{WW}. \quad (2)$$

$P_{DW}$ : Conditional probability of a wet day, when the preceding day was dry,

$$P(W_t | D_{t-1}). \quad (3)$$

$P_{DD}$ : Conditional probability of a dry day, when the preceding day was dry,

$$P(D_t | D_{t-1}) = 1 - P_{DW}. \quad (4)$$

The transition matrix is as follows,

$$P = \begin{bmatrix} P(D \rightarrow D) & P(D \rightarrow W) \\ P(W \rightarrow D) & P(W \rightarrow W) \end{bmatrix} = \begin{bmatrix} 1 - P_{DW} & P_{DW} \\ 1 - P_{WW} & P_{WW} \end{bmatrix}. \quad (5)$$

From the stationary distribution conditions,

$$\pi_D = \pi_D(1 - P_{DW}) + \pi_W(1 - P_{WW}). \quad (6)$$

In which  $\pi_D$  is the long-run (stationary) probability of a dry day and  $\pi_W$  is the long-run (stationary) probability of a wet day. The solution of  $\pi_W$  offers the initialization of the Markov Chain (Wilks 2011) as follows:

$$\pi_W = \frac{P_{DW}}{1 - P_{WW} + P_{DW}}. \quad (7)$$

A different set of transition probabilities ( $P_{DW}, P_{WW}$ ) are given to different monsoon seasons to account for the seasonal differences in rainfall persistence within each Monte Carlo realization. Wet days shall be coded as 1, and dry days shall be coded as 0. The transition probabilities for the first-order Markov chain were selected to reflect realistic wet-dry persistence patterns typical of tropical rainfall climates. Similar stochastic rainfall generation approaches have been widely applied in hydrological modeling studies (Wilks 2011). The chosen probabilities are intended to mimic general rainfall persistence during monsoon seasons in Malaysia rather than to represent a specific station.

### 3.4. Bias formulation

To formally assess the performance of bias correction methods, some form of systematic biases will be introduced to the ownership of the generated true GEV synthetic annual maximum rainfall series,  $X_{obs}$ . These bias formulations are directly taken from Tian *et al.* (2013), which classify systematic errors in rainfall measurements into additive and multiplicative models. In the case of this simulation study, we shall consider only the systematic aspect of bias by ignoring the random error term ( $\epsilon_i$ ). Under each Monte Carlo iteration, a series of  $n$  actual annual maximum rainfall values,  $X_{obs} = \{X_{obs,1}, X_{obs,2}, \dots, X_{obs,n}\}$  will be simulated. The selected bias parameter combinations represent typical underestimation and overestimation scenarios commonly observed in climate model outputs and rainfall measurements. While a wider range of parameter combinations could be considered, the selected scenarios provide representative cases that clearly demonstrate the effects of additive, multiplicative, and non-linear biases on distribution fitting.

#### 3.4.1. Linear bias model

The model is founded on an additive error model in Tian *et al.* (2013), having the following form:

$$Y_i = a + bX_i + \epsilon_i. \quad (8)$$

By extracting the systematic component and rewriting the notation to suit our purposes:

$$X_{bias,i} = a + bX_{obs,i}. \quad (9)$$

This model represents a bias in which the biased value is linearly related to the true value. Moreover, where  $a$  is an offset term, which is analogous to a fixed additive bias, and  $b$  is a scale coefficient, which is analogous to a multiplicative or scaling factor. This linear bias model is highly flexible for simulating various linear systematic errors.

(1) Additive Bias Simulation: This is attained by fixing  $b = 1$  and changing the intercept  $a$ .  $a \in \{-30, -15, +15, +30\}$  would model consistent underestimation or overestimation of all rainfall amounts, as often highlighted to result from instrument calibration problems (Tian *et al.* 2013).

(2) **Multiplicative Bias Simulation:** It is simulated by setting  $a = 0$  and by changing the slope  $b$ .  $b \in \{0.6, 0.8, 1.2, 1.4\}$  is proportional underestimation or overestimation. It is common in remote sensing products where the error is proportional to the intensity of rainfall (McMillan *et al.* 2011).

(3) **Additive and Multiplicative Bias Simulation:** This is performed by simultaneously changing  $a$  and  $b$  for each case as above. This situation indicates the presence of complex systematic errors, which are often reported in climate model results, where both the mean and variance are distorted.

#### 3.4.2. Non-linear bias model

This model was founded on the multiplicative error model proposed by Tian *et al.* (2013), presented as:

$$Y_i = a \cdot X_i^b \cdot e^{\epsilon_i}. \quad (10)$$

The systematic component of this formulation defines a non-linear systematic error whose bias is proportional to the true value, following a power law. The model is especially useful for modeling biases skewed towards various components of rainfall distribution, which is usually important for properly modeling extreme values (Tian *et al.* 2013). The non-linear relation is then used to obtain the biased value of rainfall  $X_{bias,i}$  of the  $i$ -th data point as follows:

$$X_{bias,i} = a \cdot (X_{obs,i})^b, \quad (11)$$

where  $a$  is a scaling coefficient,  $b$  is an exponent, which defines non-linear scaling of the bias. This power-law bias model is non-linear. It can be used to simulate biases in which the magnitude of the error is disproportionately related to the true rainfall value. When  $b = 1$ , the model reduces to a multiplicative bias ( $X_{bias,i} = aX_{obs,i}$ ). When  $b > 1$ , the model brings in an accelerating exaggeration of increasing values of rainfall. This may represent a situation in which extremes are progressively overestimated. When  $b < 1$ , the model brings in a dampening or compression effect on greater values of rainfall. This can simulate situations in which the extremes are systematically underestimated, or in which their variability is lower.

#### 3.5. Bias correction quantile mapping

QM, also known as distribution mapping, is a statistical post-processing method used to correct systematic biases in simulated climate model results. It matches probability distributions of the modeled and observed data by shifting each model value using quantiles of the observed data distribution. The correction to the bias value is given by:

$$X_{corr,i} = F_{obs}^{-1}(F_{mod}(X_{bias,i})). \quad (12)$$

$F_{mod}(X_{bias,i})$  is the CDF of the biased data,  $F_{obs}^{-1}()$  is the quantile function of the observed data. The quantile of the biased value  $X_{bias,i}$  is derived under the model CDF. The bias-corrected value  $X_{corr,i}$  is the quantile function corresponding to the observed distribution. This ensures distributional consistency between corrected model data and observations.

### 3.6. Distributional mapping

Distributional mapping is used to adjust for systematic bias in simulated rainfall extremes, aligning modeled and observed rainfall distributions using theoretical probability distributions. It is a method that uses the modeled values, weighted by their cumulative probability under the model distribution. Concurrently, it transforms them to the quantiles of the observed distribution using the inverse of the observed CDF. The method is particularly well adapted to hydrological applications involving extreme values, such as rainfall, where the form and tail behavior of the distribution are of importance (Gudmundsson *et al.* 2012; Teutschbein & Seibert 2012).

#### 3.6.1. Gamma distribution

The gamma distribution is frequently used to model positively skewed data. It can therefore be applied to rainfall, which is non-negative and, in most cases, skewed. It is highly accommodating with two parameters. It is considered appropriate for rainfall events, as reported in earlier studies (Piani *et al.* 2009; Teutschbein & Seibert 2012). The Probability Density Function (PDF) and the cumulative density function are as follows:

$$f(x) = \frac{x^{\alpha-1} e^{(-\frac{x}{\theta})}}{\Gamma(\alpha) \theta^\alpha}, \quad x \geq 0; \alpha, \theta > 0, \quad (13)$$

$$F(x) = \frac{\gamma(\alpha, \frac{x}{\theta})}{\Gamma(\alpha)}, \quad x \geq 0; \alpha, \theta > 0, \quad (14)$$

where  $x$  represents the rainfall data.  $\alpha$  is the shape parameter that determines skewness and peak. The dispersion of the distribution is determined by  $\theta$  as the scale parameter.  $\Gamma(\alpha)$  as the gamma function and  $\gamma(\alpha, \frac{x}{\theta})$  as the incomplete lower gamma function. The special case of  $\alpha = 1$  reduces to an exponential distribution. An increase in  $\alpha$  skews the mode to the right, and the distribution becomes more symmetric. An increase in  $\theta$  widens the distribution and explains more variation.

#### 3.6.2. Kappa distribution

It is highlighted that the Kappa distribution is highly useful for bias adjustment of rainfall data, especially compared with common distributions. In a previous study, it was reported that the Kappa distribution was superior for modeling the statistical characteristics of measured rainfall, especially the extreme values, and often outperformed well-known distributions, including the Gamma (Heo *et al.* 2019; Strong *et al.* 2025). The CDF and PDF of the Kappa distribution were deduced by Hosking (1994) as follows:

$$F(x) = \left\{ 1 - h \left[ 1 - \frac{k(x-\xi)}{\alpha} \right]^{\frac{1}{k}} \right\}^{\frac{1}{h}}, \quad (15)$$

$$f(x) = \frac{1}{\alpha} \left[ 1 - \frac{k(x-\xi)}{\alpha} \right]^{\frac{1}{k}-1} \left\{ 1 - h \left[ 1 - \frac{k(x-\xi)}{\alpha} \right]^{\frac{1}{k}} \right\}^{\frac{1}{h}-1}, \quad (16)$$

where the location, scale, first shape, and second shape parameters are denoted as  $\xi, \alpha, k, h$ , respectively.  $\xi$  translates the distribution horizontally,  $\alpha$  affects the spread or dispersion of the distribution,  $k$  affects skewness and kurtosis, whilst the  $h$  parameter affects tail behavior and

heaviness of the distribution. Heavier tails are obtained by increasing  $h$ . The combination of  $k$  and  $h$  offers enormous flexibility in distributional form, and the combination can fit highly skewed rainfall data. Various combinations may result in different levels of unimodality, skewness, and tail attributes. GEV distribution is obtained by putting  $h = 0$  in the Kappa distribution. It offers the Generalized Pareto distribution when  $h = 1$ . For  $h = 1$  and  $k = 0$ , it yields an exponential distribution. Furthermore, the limits are as follows:

$$\xi + \frac{\alpha(1+h^{-k})}{k} \leq x \leq \xi + \frac{\alpha}{k} \text{ if } h > 0, k > 0, \quad (17)$$

$$\xi + \alpha \log h \leq x < \infty \text{ if } h > 0, k = 0, \quad (18)$$

$$\xi + \frac{\alpha(1+h^{-k})}{k} \leq x < \infty \text{ if } h > 0, k < 0, \quad (19)$$

$$-\infty < x < \xi + \frac{\alpha}{k} \text{ if } h \leq 0, k > 0, \quad (20)$$

$$-\infty < x < -\infty \text{ if } h \leq 0, k = 0, \quad (21)$$

$$\xi + \frac{\alpha}{k} \leq x < \infty \text{ if } h \leq 0, k < 0. \quad (22)$$

### 3.6.3. Weibull distribution

The Weibull distribution is commonly used in reliability engineering for time-to-failure data analysis and has many applications in hydrology, particularly for modeling maximum rainfall events and wind speeds (Wilks 2011). Correspondingly, the Bernoulli-Weibull mixture has recently been applied to model daily precipitation, in which the magnitude of wet days is modeled as a Weibull component (Gudmundsson *et al.* 2012). The Weibull distribution CDF and PDF are presented below:

$$f(x) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}, \quad x \geq 0; k > 0; \lambda > 0, \quad (23)$$

$$F(x) = 1 - e^{-\left(\frac{x}{\lambda}\right)^k}, \quad x \geq 0; k > 0; \lambda > 0, \quad (24)$$

where  $\lambda$  is a scale parameter, by which the distribution width is determined. Accordingly, bigger  $\lambda$  provides a rightward shift of the distribution. Meanwhile,  $k$  is a shape parameter that stipulates the skew of the distribution. In addition,  $\lambda$  offers different scales to the distribution horizontally. Differing  $k$  alters the general form of the PDF, from a strongly decreasing curve to a biased bell-curve.

### 3.6.4. Burr type XII distribution

Like the Kappa distribution, the Burr Type XII distribution, especially the two-parameter version, has been investigated for its suitability for QM in precipitation bias correction. The comparative analysis conducted by Heo *et al.* (2019) observed that the Kappa distribution tended to perform better than the Burr Type XII distribution in their case study involving South Korea. Additionally, the occurrence of Burr Type XII distribution in this analysis indicates its potential and applicability as a flexible distribution that could be employed in modeling rainfall

and its extremes. The Burr Type XII distribution is characterized as follows with respect to its PDF and CDF.

$$f(x) = \frac{kc}{\alpha} \left(\frac{x}{\alpha}\right)^{c-1} \left(1 + \left(\frac{x}{\alpha}\right)^c\right)^{-(k+1)}, \quad x > 0; \alpha > 0; c > 0; k > 0, \quad (25)$$

$$F(x) = 1 - \left(1 + \left(\frac{x}{\alpha}\right)^c\right)^{-k}, \quad x > 0; \alpha > 0; c > 0; k > 0, \quad (26)$$

where  $\alpha$  is the scale parameter that changes the size of the underlying distributions. A bigger  $\alpha$  stretches the distribution along the  $x$ -axis. The first shape parameter  $c$ , when  $c \leq 1$ , makes the PDF have an L-shape. Otherwise, when  $c > 1$ , the PDF is unimodal. The second shape parameter  $k$ , making the distribution heavier or lighter on the upper tail. When  $k$  is larger, then the tail is lighter. By contrast, when  $k$  is smaller, then the tail is heavy, permitting larger probabilities of observing extreme values. The parameter  $c$ , combined with  $k$  enables the Burr Type XII distribution to have a large variety of skewness and kurtosis.

### **3.7. Maximum likelihood estimation**

In distributional mapping, parameter estimation of the distribution is a crucial factor in the accuracy of bias correction. All the probability distributions, Gamma, Kappa, Weibull, and Burr Type XII, are also fitted to the observed and biased rainfall data with the Maximum Likelihood Estimation (MLE). MLE is a statistical procedure that chooses parameter values that maximize the likelihood of the observed data under a given probability distribution (Myung 2003). Let us assume that  $X = (x_1, x_2, \dots, x_n)$  is a sample of size  $n$  with independent and identically distributed (*i. i. d.*) random variables that are governed by a probability distribution with PDF  $f(x_i; \theta)$ , where the unknown parameters are gathered in the vector  $\theta$ . When the distribution is Gamma, then  $\theta = (\alpha, \theta)$ . The probability of the observed data points is the likelihood function  $L(\theta; x_1, x_2, \dots, x_n)$ . Thus, the product of the single PDFs given independently and identically distributed (*i. i. d.*):

$$L(\theta; x_1, x_2, \dots, x_n) = \prod_{i=1}^n f(x_i; \theta). \quad (27)$$

More conveniently, to calculate things, we often use the log-likelihood function,  $\ln(L(\theta; x_1, x_2, \dots, x_n))$ , since the log transforms products into sums, which are easier to differentiate:

$$\ln(L(\theta; x_1, x_2, \dots, x_n)) = \sum_{i=1}^n \ln(f(x_i; \theta)). \quad (28)$$

$\theta$  maximizes this log-likelihood function, then the values of  $\theta$  maximizing this log-likelihood are the MLE of the corresponding parameters,  $\hat{\theta}$ . Partial derivatives of the log-likelihood function are usually computed with respect to each parameter; the derivatives are equated to zero, and the system of equations thus obtained is solved:

$$\frac{\partial \ln L(\theta)}{\partial \theta} = 0. \quad (29)$$

### **3.8. Performance evaluation metrics**

To fully evaluate the consequences of the bias and the improvements in the fitted probability distributions resulting from bias correction, an extensive set of statistical performance measures

will be applied. To gauge the discrepancy between the bias-corrected and true values in each Monte Carlo simulation, the following error measures are estimated.

### 3.8.1. Root mean square error

RMSE is a usual measure of the mean squared difference of observed and predicted values. It is prone to large errors since squaring differences disproportionately magnifies them.

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (X_{corr,i} - X_{obs,i})^2}{N}}. \quad (30)$$

### 3.8.2. Mean absolute error

MAE is used to measure the mean absolute error of the differences between absolute predictions and observations. Compared to RMSE, it places equal emphasis on each error, regardless of its size. MAE is less sensitive to outliers than RMSE and provides a robust estimate of average error.

$$MAE = \frac{\sum_{i=1}^N |X_{corr,i} - X_{obs,i}|}{N}. \quad (31)$$

### 3.8.3. Nash-Sutcliffe efficiency

NSE is a well-recognized and commonly used statistic for the predictive capability of hydrological models. It implies the goodness of fit of the plot of observed data versus simulated data to the straight line. Note that an NSE value of 1 indicates an ideal situation in which simulated and observed rainfall data are identical. When the NSE equals 0, the model simulations agree with the mean of the observed rainfall data. Negative values imply that the average of the observed rainfall data predicts better than the model.

$$NSE = 1 - \frac{\sum_{i=1}^N (X_{corr,i} - X_{obs,i})^2}{\sum_{i=1}^N (X_{corr,i} - \bar{X}_{obs})^2}. \quad (32)$$

### 3.8.4. Kling-Gupta efficiency

KGE is a more efficient diagnostic measure that seeks to address NSE's weaknesses by explicitly breaking down the GoF into three terms: correlation, bias, and variability. KGE of 1 that signifies a perfect fit. It addresses NSE shortcomings by incorporating several performance aspects.

$$KGE = 1 - \sqrt{(r - 1)^2 + (\beta - 1)^2 + (\gamma - 1)^2}, \quad (33)$$

where  $X_{corr,i}$  is the bias-corrected data,  $X_{obs,i}$  is the observed data,  $N$  is the number of data points,  $\bar{X}_{obs}$  is the mean observed data,  $r$  is the Pearson Correlation coefficient between  $X_{corr,i}$  and  $X_{obs,i}$ ,  $\beta = \frac{\mu_{X_{corr,i}}}{\mu_{X_{obs,i}}}$  is the bias ratio, and  $\gamma = \frac{\sigma_{X_{corr,i}}}{\sigma_{X_{obs,i}}}$  is the variability ratio.

### 3.9. Goodness-of-fit tests

Along with the comparison of the general performance of the fitted distributions, of interest is a formal test of how well the fitted distributions represent the underlying empirical distribution of the generated rainfall data. Accordingly, the GoF tests provide evidence to either accept or reject the hypothesis that a sample is drawn from a known theoretical distribution, i.e., Gamma, Kappa, Weibull, and Burr Type XII. In the research, the K-S and CvM tests will be used. The Empirical Cumulative Distribution Function (ECDF) of the data and the theoretical CDF of the fitted distribution are compared in these tests.

#### 3.9.1. Kolmogorov-Smirnov test

The K-S test is applied to simulated data to assess the plausibility that the data were generated from the fitted theoretical probability distribution. The statistic of a one-sample test is the K-S distance between the ECDF  $F_n(x)$  of the sample and the approximate theoretical distribution  $F(x)$ .  $\sup_x$  is the supremum of the set of distances, and  $n$  is the sample length.

$$D_n = \sup_x |F_n(x) - F(x)|. \quad (34)$$

Table 1: Decision Criteria at Alpha = 0.05

|                                   |                                                                             |
|-----------------------------------|-----------------------------------------------------------------------------|
| Null Hypothesis ( $H_0$ ):        | These observations come under the theoretical distribution.                 |
| Alternative Hypothesis ( $H_1$ ): | The observations do not come under the prescribed theoretical distribution. |

#### 3.9.2. Cramér-von Mises test

Another non-parametric GoF test is the CvM test, which compares the ECDF of a sample with a theoretical CDF. Nevertheless, rather than focusing on the largest difference, the CvM test considers the mean of the squared differences across the entire distribution. It is hence more sensitive to departures across a distribution rather than just at a point. The one-sample CvM test statistic is provided below, where  $F(x_{(i)})$  is the theoretical CDF at the  $i$ -th ordered observation  $x_{(i)}$  and  $n$  is the sample size:

$$W = \frac{1}{12n} + \sum_{i=1}^n \left( \frac{2i-1}{2n} - F(x_{(i)}) \right)^2. \quad (35)$$

Table 2: Decision Criteria at Alpha = 0.05

|                                   |                                                                             |
|-----------------------------------|-----------------------------------------------------------------------------|
| Null Hypothesis ( $H_0$ ):        | These observations come under the theoretical distribution.                 |
| Alternative Hypothesis ( $H_1$ ): | The observations do not come under the prescribed theoretical distribution. |

## 4. Results and Discussion

### 4.1. Additive bias

At small sample sizes of 50, the Burr distribution recorded lower RMSE and MAE across all bias levels and thus surpassed others in correcting both under- and overestimation. Even when faced with a degree of bias, there were few errors in the Burr and Kappa distributions at sample sizes of 100. With  $n = 500$  and 1,000 overall performances levelled out, Kappa was the most

efficient, with the lowest RMSE and MAE among other distributions, no matter what the degree of bias was. Following this, Burr also maintained high accuracy, with low MAE and RMSE, further affirming its trustworthiness as a bias adjustment measure. However, gamma had the worst RMSE and MAE across all levels of bias and sample sizes. These results demonstrate that Gamma's shape and scale may not be appropriate for correcting additive structural biases when precipitation intensities are extreme. All distributions demonstrated consistent NSE and KGE values  $> 0.90$ , though with some notable results. Across sample sizes and bias magnitudes, the Kappa and Burr distributions consistently yielded the highest NSE and KGE values. The Gamma distribution had an extremely high NSE, a strong indicator of agreement in variance, though only mediocre KGE performance, and poorer performance in modeled correlation, mean, and variability. At the same time, while the Weibull distribution was consistently efficient, it had lower KGE than Kappa and Burr when the sample size was small.

Table 3: RMSE and MAE of additive bias

| No. of years | Metric | Distribution | Bias<br>−30   | Bias<br>−15   | Bias<br>+15   | Bias<br>+30   |
|--------------|--------|--------------|---------------|---------------|---------------|---------------|
| 50           | RMSE   | Gamma        | 2.1054        | 1.7898        | 1.2202        | 1.4693        |
|              |        | Weibull      | 1.4975        | 0.6768        | 0.5668        | 1.0633        |
|              |        | Burr         | <b>0.4850</b> | <b>0.2344</b> | <b>0.2052</b> | <b>0.4074</b> |
|              |        | Kappa        | 1.5043        | 1.2428        | 1.4708        | 1.2467        |
|              | MAE    | Gamma        | 1.4640        | 1.3031        | 0.8701        | 1.0084        |
|              |        | Weibull      | 1.0883        | 0.4957        | 0.4207        | 0.7916        |
|              |        | Burr         | <b>0.2820</b> | <b>0.1350</b> | <b>0.1155</b> | <b>0.2243</b> |
|              |        | Kappa        | 0.4783        | 0.3785        | 0.4564        | 0.4151        |
| 100          | RMSE   | Gamma        | 2.6234        | 2.0647        | 1.6214        | 1.7198        |
|              |        | Weibull      | 1.5154        | 0.6941        | 0.5912        | 1.0916        |
|              |        | Burr         | 0.3595        | <b>0.2762</b> | <b>0.1800</b> | <b>0.3040</b> |
|              |        | Kappa        | <b>0.2158</b> | 0.5428        | 0.5115        | 0.3652        |
|              | MAE    | Gamma        | 1.8135        | 1.4423        | 1.1102        | 1.1496        |
|              |        | Weibull      | 1.1765        | 0.5408        | 0.4610        | 0.8588        |
|              |        | Burr         | 0.2113        | <b>0.1343</b> | <b>0.1077</b> | 0.1690        |
|              |        | Kappa        | <b>0.0546</b> | 0.1387        | 0.1301        | <b>0.0943</b> |
| 500          | RMSE   | Gamma        | 3.2284        | 2.0938        | 1.6983        | 1.7795        |
|              |        | Weibull      | 1.2065        | 0.5542        | 0.4729        | 0.8835        |
|              |        | Burr         | 0.4558        | 0.1994        | 0.1758        | 0.3357        |
|              |        | Kappa        | <b>0.1088</b> | <b>0.0992</b> | <b>0.1000</b> | <b>0.0958</b> |
|              | MAE    | Gamma        | 2.4140        | 1.5446        | 1.2484        | 1.2957        |
|              |        | Weibull      | 0.9344        | 0.4275        | 0.3652        | 0.6831        |
|              |        | Burr         | 0.3570        | 0.1547        | 0.1323        | 0.2462        |
|              |        | Kappa        | <b>0.0130</b> | <b>0.0114</b> | <b>0.0113</b> | <b>0.0113</b> |
| 1000         | RMSE   | Gamma        | 2.4342        | 3.3672        | 2.0331        | 2.1495        |
|              |        | Weibull      | 1.3434        | 0.6142        | 0.5236        | 0.9780        |
|              |        | Burr         | 0.4815        | 0.1746        | 0.1594        | 0.3078        |
|              |        | Kappa        | <b>0.0152</b> | <b>0.0159</b> | <b>0.0189</b> | <b>0.0176</b> |
|              | MAE    | Gamma        | 1.7254        | 2.4633        | 1.4599        | 1.5105        |
|              |        | Weibull      | 1.0383        | 0.4758        | 0.4083        | 0.7652        |
|              |        | Burr         | 0.3194        | 0.1299        | 0.1114        | 0.2054        |
|              |        | Kappa        | <b>0.0011</b> | <b>0.0012</b> | <b>0.0013</b> | <b>0.0013</b> |

Table 4: NSE and KGE of additive bias

| No. of years | Metric | Distribution | Bias<br>−30   | Bias<br>−15   | Bias<br>+15   | Bias<br>+30   |
|--------------|--------|--------------|---------------|---------------|---------------|---------------|
| 50           | NSE    | Gamma        | 0.9923        | 0.9936        | 0.9966        | 0.9965        |
|              |        | Weibull      | 0.9980        | 0.9996        | 0.9997        | 0.9989        |
|              |        | Burr         | <b>0.9997</b> | <b>0.9999</b> | <b>1.0000</b> | <b>0.9997</b> |
|              |        | Kappa        | 0.9893        | 0.9920        | 0.9904        | 0.9909        |
|              | KGE    | Gamma        | 0.9499        | 0.9531        | 0.9688        | 0.9643        |
|              |        | Weibull      | 0.9659        | 0.9840        | 0.9864        | 0.9743        |
|              |        | Burr         | <b>0.9920</b> | <b>0.9961</b> | <b>0.9966</b> | <b>0.9934</b> |
|              |        | Kappa        | 0.9814        | 0.9849        | 0.9824        | 0.9848        |
| 100          | NSE    | Gamma        | 0.9908        | 0.9930        | 0.9958        | 0.9960        |
|              |        | Weibull      | 0.9983        | <b>0.9997</b> | 0.9997        | 0.9991        |
|              |        | Burr         | <b>0.9997</b> | 0.9970        | <b>0.9998</b> | <b>0.9999</b> |
|              |        | Kappa        | 0.9991        | 0.9973        | 0.9976        | 0.9983        |
|              | KGE    | Gamma        | 0.9401        | 0.9502        | 0.9618        | 0.9610        |
|              |        | Weibull      | 0.9744        | 0.9886        | 0.9899        | 0.9807        |
|              |        | Burr         | 0.9945        | <b>0.9958</b> | <b>0.9972</b> | <b>0.9955</b> |
|              |        | Kappa        | <b>0.9973</b> | 0.9931        | 0.9935        | 0.9953        |
| 500          | NSE    | Gamma        | 0.9865        | 0.9922        | 0.9954        | 0.9958        |
|              |        | Weibull      | 0.9991        | 0.9998        | 0.9999        | 0.9995        |
|              |        | Burr         | 0.9998        | 1.0000        | <b>1.0000</b> | 0.9999        |
|              |        | Kappa        | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> |
|              | KGE    | Gamma        | 0.9270        | 0.9515        | 0.9611        | 0.9616        |
|              |        | Weibull      | 0.9851        | 0.9935        | 0.9943        | 0.9890        |
|              |        | Burr         | 0.9942        | 0.9978        | 0.9984        | 0.9972        |
|              |        | Kappa        | <b>0.9995</b> | <b>0.9996</b> | <b>0.9996</b> | <b>0.9996</b> |
| 1000         | NSE    | Gamma        | 0.9914        | 0.9861        | 0.9932        | 0.9944        |
|              |        | Weibull      | 0.9990        | 0.9998        | 0.9998        | 0.9995        |
|              |        | Burr         | 0.9968        | <b>1.0000</b> | <b>1.0000</b> | 0.9999        |
|              |        | Kappa        | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> |
|              | KGE    | Gamma        | 0.9547        | 0.9248        | 0.9564        | 0.9573        |
|              |        | Weibull      | 0.9860        | 0.9936        | 0.9944        | 0.9896        |
|              |        | Burr         | 0.9941        | 0.9986        | 0.9991        | 0.9986        |
|              |        | Kappa        | <b>1.0000</b> | <b>1.0000</b> | <b>0.9999</b> | <b>0.9999</b> |

#### 4.2. Multiplicative bias

To evaluate the effectiveness of the distributional QM method under multiplicative bias conditions, simulated biases of 0.6, 0.8, 1.2, and 1.4 were applied to rainfall extremes, thereby encompassing both underestimation ( $< 1.0$ ) and overestimation ( $> 1.0$ ). For RMSE, the Weibull distribution yielded excellent results, with errors ranging from 0.01 to 0.02 across all bias variables and sample sizes. On the same note, Weibull recorded the fewest errors in the MAE cases for 50 and 100 years. The Kappa distribution performed competitively overall, especially in terms of MAE. In 500-year and 1,000-year records, Kappa was nearly as good as Weibull for some bias factors, especially at 500 years when bias = 1.2, with a MAE of 0.0113, and at 1,000 years when bias = 0.6, with a MAE of 0.0011. Furthermore, the Burr distribution remained consistent across all biases and sample sizes except at Bias = 0.8 for longer years. Gamma continually turned out to be the worst. RMSE and MAE values for Gamma also revealed clear results: it had quite substantial errors, indicating its failure to reduce multiplying biases relative to the other distributions assessed. Weibull, Burr, and Kappa distributions yielded very high NSE scores (1.0000) in most cases, with most of the data and bias factors. In particular, Weibull exhibited a constant NSE of 1.0000 across all periods and bias factors.

Similarly, Burr and Kappa were very close to 1.0000, or even the closest possible value of Kappa at 50 years, with bias = 0.6. The gamma distribution was lower for NSE values. Over the 1,000 years, the NSE ranged from 0.9975 to 0.9959, with a modest decrease in efficiency and a higher bias. Weibull, Burr, and Kappa distributions perform well on the KGE measure and often yield values close to 1.0000. As such, Weibull came out to be 0.9998 across practically all bias and sample-size scenarios. Kappa's KGE improved significantly when periods were sufficiently large. While the Burr distribution remained consistent, its KGE value was lower when bias = 0.8 over longer periods. The Gamma distribution consistently performed worse as bias increased.

Table 5: RMSE and MAE of multiplicative bias

| No. of years | Metric | Distribution | Bias 0.6      | Bias 0.8      | Bias 1.2      | Bias 1.4      |
|--------------|--------|--------------|---------------|---------------|---------------|---------------|
| 50           | RMSE   | Gamma        | 0.8999        | 0.9448        | 1.6714        | 1.8965        |
|              |        | Weibull      | <b>0.0144</b> | <b>0.0138</b> | <b>0.0152</b> | <b>0.0182</b> |
|              |        | Burr         | 0.0230        | 0.0482        | 0.0320        | 0.0610        |
|              |        | Kappa        | 1.5043        | 1.2428        | 1.4708        | 1.2467        |
|              | MAE    | Gamma        | 0.6917        | 0.7210        | 1.2710        | 1.4187        |
|              |        | Weibull      | <b>0.0125</b> | <b>0.0122</b> | <b>0.0133</b> | <b>0.0159</b> |
|              |        | Burr         | 0.0147        | 0.0333        | 0.0196        | 0.0348        |
|              |        | Kappa        | 0.4783        | 0.3785        | 0.4564        | 0.4151        |
| 100          | RMSE   | Gamma        | 1.1551        | 1.3120        | 2.2475        | 2.3094        |
|              |        | Weibull      | <b>0.0137</b> | <b>0.0142</b> | <b>0.0162</b> | <b>0.0151</b> |
|              |        | Burr         | 0.0245        | 0.1342        | 0.0240        | 0.0381        |
|              |        | Kappa        | 0.2158        | 0.5428        | 0.5115        | 0.3652        |
|              | MAE    | Gamma        | 0.8515        | 0.9485        | 1.6356        | 1.6530        |
|              |        | Weibull      | <b>0.0122</b> | <b>0.0126</b> | <b>0.0142</b> | <b>0.0131</b> |
|              |        | Burr         | 0.0158        | 0.0936        | 0.0153        | 0.0235        |
|              |        | Kappa        | 0.0546        | 0.1387        | 0.1301        | 0.0943        |
| 500          | RMSE   | Gamma        | 1.1009        | 1.4188        | 2.1740        | 2.4867        |
|              |        | Weibull      | <b>0.0128</b> | <b>0.0128</b> | <b>0.0133</b> | <b>0.0130</b> |
|              |        | Burr         | 0.0205        | 0.1930        | 0.0228        | 0.0445        |
|              |        | Kappa        | 0.1088        | 0.0992        | 0.1000        | 0.0958        |
|              | MAE    | Gamma        | 0.8425        | 1.0742        | 1.6351        | 1.8733        |
|              |        | Weibull      | <b>0.0115</b> | 0.0116        | 0.0121        | 0.0118        |
|              |        | Burr         | 0.0143        | 0.1310        | 0.0156        | 0.0301        |
|              |        | Kappa        | 0.0130        | <b>0.0114</b> | <b>0.0113</b> | <b>0.0113</b> |
| 1000         | RMSE   | Gamma        | 1.3105        | 1.4514        | 3.3750        | 4.6834        |
|              |        | Weibull      | <b>0.0136</b> | <b>0.0142</b> | <b>0.0133</b> | <b>0.0149</b> |
|              |        | Burr         | 0.0316        | 0.2161        | 0.0227        | 0.0481        |
|              |        | Kappa        | 0.0152        | 0.0159        | 0.0189        | 0.0176        |
|              | MAE    | Gamma        | 0.9796        | 1.0796        | 2.4942        | 3.3918        |
|              |        | Weibull      | 0.0123        | 0.0128        | 0.0118        | 0.0134        |
|              |        | Burr         | 0.0216        | 0.1288        | 0.0155        | 0.0324        |
|              |        | Kappa        | <b>0.0011</b> | <b>0.0012</b> | <b>0.0013</b> | <b>0.0013</b> |

Table 6: NSE and KGE of multiplicative bias

| No. of years | Metric | Distribution | Bias 0.6      | Bias 0.8      | Bias 1.2      | Bias 1.4      |
|--------------|--------|--------------|---------------|---------------|---------------|---------------|
| 50           | NSE    | Gamma        | 0.9971        | 0.9970        | 0.9932        | 0.9913        |
|              |        | Weibull      | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> |
|              |        | Burr         | <b>1.0000</b> | 0.9999        | <b>1.0000</b> | 0.9998        |
|              |        | Kappa        | 0.9893        | 0.9920        | 0.9904        | 0.9909        |
|              | KGE    | Gamma        | 0.9749        | 0.9738        | 0.9531        | 0.9480        |
|              |        | Weibull      | <b>0.9998</b> | <b>0.9998</b> | <b>0.9998</b> | <b>0.9998</b> |
|              |        | Burr         | 0.9996        | 0.9991        | 0.9995        | 0.9990        |
|              |        | Kappa        | 0.9814        | 0.9849        | 0.9824        | 0.9848        |
| 100          | NSE    | Gamma        | 0.9965        | 0.9961        | 0.9916        | 0.9900        |
|              |        | Weibull      | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> |
|              |        | Burr         | <b>1.0000</b> | 0.9997        | <b>1.0000</b> | 0.9999        |
|              |        | Kappa        | 0.9991        | 0.9973        | 0.9976        | 0.9983        |
|              | KGE    | Gamma        | 0.9699        | 0.9669        | 0.9421        | 0.9410        |
|              |        | Weibull      | <b>0.9998</b> | <b>0.9998</b> | <b>0.9998</b> | <b>0.9998</b> |
|              |        | Burr         | 0.9996        | 0.9982        | 0.9996        | 0.9994        |
|              |        | Kappa        | 0.9973        | 0.9931        | 0.9935        | 0.9953        |
| 500          | NSE    | Gamma        | 0.9970        | 0.9959        | 0.9914        | 0.9890        |
|              |        | Weibull      | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> |
|              |        | Burr         | <b>1.0000</b> | 0.9995        | <b>1.0000</b> | 0.9999        |
|              |        | Kappa        | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> |
|              | KGE    | Gamma        | 0.9730        | 0.9654        | 0.9467        | 0.9382        |
|              |        | Weibull      | <b>0.9998</b> | <b>0.9998</b> | <b>0.9998</b> | <b>0.9998</b> |
|              |        | Burr         | 0.9997        | 0.9977        | 0.9996        | 0.9993        |
|              |        | Kappa        | 0.9995        | 0.9996        | 0.9996        | 0.9996        |
| 1000         | NSE    | Gamma        | 0.9959        | 0.9951        | 0.9849        | 0.9775        |
|              |        | Weibull      | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> |
|              |        | Burr         | <b>1.0000</b> | 0.9986        | <b>1.0000</b> | 0.9999        |
|              |        | Kappa        | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> | <b>1.0000</b> |
|              | KGE    | Gamma        | 0.9706        | 0.9673        | 0.9210        | 0.8926        |
|              |        | Weibull      | 0.9998        | 0.9998        | 0.9998        | 0.9998        |
|              |        | Burr         | 0.9996        | 0.9974        | 0.9996        | 0.9993        |
|              |        | Kappa        | <b>1.0000</b> | <b>1.0000</b> | <b>0.9999</b> | <b>0.9999</b> |

#### 4.3. Additive and multiplicative bias

Given QM's effectiveness in correcting both multiplicative and additive biases in rainfall data, the Kappa and Burr distributions yielded the lowest RMSE and MAE across various biases. While the Weibull distribution performed well for purely multiplicative biases, its performance immediately decreased when an additive bias was present. The Gamma distribution still had the highest RMSE and MAE, further emphasizing its limited ability to correct for combining biased designs. In the first bias scenario, Kappa elicited the lowest RMSE across all distributions when the sample size was large. The values were as low as 0.1088 for 500 years and even lower at 0.0152 for 1,000 years. The Burr with the second ranking had an RMSE of 0.3240 over 50 years. Weibull produced an RMSE greater than those of Burr and Kappa. Such findings indicate that, although applicable to modeling multiplicative biases, the Weibull distribution becomes inaccurate for additive facets. Specifically, MAE was the lowest in the Kappa, with a MAE of 0.0546 at 100 years and 0.0011 at 1,000 years. In essence, Burr was always competitive, with an MAE of 0.1841 at 50 years; the distribution performed especially well at shorter records. MAE obtained by Weibull and Gamma were significantly and considerably larger than those obtained by Burr and Kappa.

Table 7: RMSE and MAE of additive and multiplicative bias

| No. of years | Metric | Distribution | Bias<br>$1.4 * X_{obs} - 30$ | Bias<br>$0.6 * X_{obs} + 30$ |
|--------------|--------|--------------|------------------------------|------------------------------|
| 50           | RMSE   | Gamma        | 2.1734                       | 1.6651                       |
|              |        | Weibull      | 1.0160                       | 1.6297                       |
|              |        | Burr         | <b>0.3240</b>                | <b>0.6144</b>                |
|              |        | Kappa        | 1.5043                       | 1.2428                       |
|              | MAE    | Gamma        | 1.5508                       | 1.1288                       |
|              |        | Weibull      | 0.7375                       | 1.2073                       |
|              |        | Burr         | <b>0.1841</b>                | <b>0.3384</b>                |
|              |        | Kappa        | 0.4783                       | 0.3785                       |
| 100          | RMSE   | Gamma        | 2.8595                       | 1.9556                       |
|              |        | Weibull      | 1.0226                       | 1.6594                       |
|              |        | Burr         | 0.2357                       | <b>0.5025</b>                |
|              |        | Kappa        | <b>0.2158</b>                | 0.5428                       |
|              | MAE    | Gamma        | 2.0031                       | 1.3184                       |
|              |        | Weibull      | 0.7980                       | 1.3234                       |
|              |        | Burr         | 0.1378                       | <b>0.2669</b>                |
|              |        | Kappa        | <b>0.0546</b>                | 0.1387                       |
| 500          | RMSE   | Gamma        | 2.8333                       | 1.7415                       |
|              |        | Weibull      | 0.8179                       | 1.3631                       |
|              |        | Burr         | 0.2965                       | 0.5114                       |
|              |        | Kappa        | <b>0.1088</b>                | <b>0.0992</b>                |
|              | MAE    | Gamma        | 2.0912                       | 1.2503                       |
|              |        | Weibull      | 0.6338                       | 1.0503                       |
|              |        | Burr         | 0.2327                       | 0.3644                       |
|              |        | Kappa        | <b>0.0130</b>                | <b>0.0114</b>                |
| 1000         | RMSE   | Gamma        | 3.0219                       | 2.1040                       |
|              |        | Weibull      | 0.9082                       | 1.4988                       |
|              |        | Burr         | 0.2546                       | 0.5041                       |
|              |        | Kappa        | <b>0.0152</b>                | <b>0.0159</b>                |
|              | MAE    | Gamma        | 2.1610                       | 1.4505                       |
|              |        | Weibull      | 0.7034                       | 1.1730                       |
|              |        | Burr         | 0.1930                       | 0.3178                       |
|              |        | Kappa        | <b>0.0011</b>                | <b>0.0012</b>                |

For the second bias scenario, Burr had the minimum RMSE at 50 and 100 years. Kappa had the lowest RMSE when the data size was larger, with 0.0159 at 1,000 years compared to 1.2428 at 50 years. Weibull RMSE was larger than that of Burr and Kappa, whereas Gamma yielded the highest RMSE. Burr also produced the lowest MAE, as illustrated by 0.3384 at 50 years and 0.2669 at 100 years. Kappa had the lowest MAE over long records, with 0.0012 over 1,000 years. Weibull and Gamma also presented significantly large MAE values. NSE for both Burr and Kappa had high values of 1.0000 or values very close to it across all bias and data length conditions. Under the first bias scenario, Burr yielded 0.9999 at 50 years and 0.9996 in the second one. With the span of data increased to 500 and 1,000 years, the NSE values were always 0.9999 or 1.0000 for both the Burr and Kappa distributions. Kappa had the highest KGE among the distributions studied when the sample size is large. Nonetheless, Burr recorded KGE values of 0.9947 and 0.9899 for the first and second bias scenarios at 50 years, respectively. Weibull and Gamma had lower KGE values than the previous two distributions across all bias scenarios and sample sizes.

Table 8: NSE and KGE of additive and multiplicative bias

| No. of years | Metric | Distribution | Bias<br>$1.4 * X_{obs} - 30$ | Bias<br>$0.6 * X_{obs} + 30$ |
|--------------|--------|--------------|------------------------------|------------------------------|
| 50           | NSE    | Gamma        | 0.9891                       | 0.9960                       |
|              |        | Weibull      | 0.9991                       | 0.9975                       |
|              |        | Burr         | <b>0.9999</b>                | <b>0.9996</b>                |
|              |        | Kappa        | 0.9893                       | 0.9920                       |
|              | KGE    | Gamma        | 0.9458                       | 0.9600                       |
|              |        | Weibull      | 0.9768                       | 0.9602                       |
|              |        | Burr         | <b>0.9947</b>                | <b>0.9899</b>                |
|              |        | Kappa        | 0.9814                       | 0.9849                       |
| 100          | NSE    | Gamma        | 0.9875                       | 0.9955                       |
|              |        | Weibull      | 0.9992                       | 0.9979                       |
|              |        | Burr         | <b>1.0000</b>                | <b>0.9998</b>                |
|              |        | Kappa        | 0.9991                       | 0.9973                       |
|              | KGE    | Gamma        | 0.9338                       | 0.9578                       |
|              |        | Weibull      | 0.9826                       | 0.9721                       |
|              |        | Burr         | 0.9964                       | 0.9926                       |
|              |        | Kappa        | <b>0.9973</b>                | <b>0.9931</b>                |
| 500          | NSE    | Gamma        | 0.9863                       | 0.9962                       |
|              |        | Weibull      | 0.9996                       | 0.9988                       |
|              |        | Burr         | 0.9999                       | 0.9998                       |
|              |        | Kappa        | <b>1.0000</b>                | <b>1.0000</b>                |
|              | KGE    | Gamma        | 0.9362                       | 0.9675                       |
|              |        | Weibull      | 0.9898                       | 0.9835                       |
|              |        | Burr         | 0.9964                       | 0.9965                       |
|              |        | Kappa        | <b>0.9995</b>                | <b>0.9996</b>                |
| 1000         | NSE    | Gamma        | 0.9821                       | 0.9952                       |
|              |        | Weibull      | 0.9995                       | 0.9988                       |
|              |        | Burr         | <b>1.0000</b>                | 0.9998                       |
|              |        | Kappa        | <b>1.0000</b>                | <b>1.0000</b>                |
|              | KGE    | Gamma        | 0.9374                       | 0.9636                       |
|              |        | Weibull      | 0.9905                       | 0.9842                       |
|              |        | Burr         | 0.9977                       | 0.9980                       |
|              |        | Kappa        | <b>1.0000</b>                | <b>1.0000</b>                |

#### 4.4. Non-Linear bias

The underlying ability to reconstruct distorted rainfall observations using a non-linear bias was assessed in this section. Two different bias patterns were simulated: the first bias amplifies higher values, and the second bias dampens them, especially at higher values. Weibull provided the best results in terms of RMSE and MAE. Therefore, it is highly resistant to complex non-linear distortions. Burr and Kappa were moderate, causing rather small errors in comparison to Gamma, though often more than the ones caused by Weibull. In the first bias scenario, Weibull achieved reasonably small RMSE values across all data lengths. Burr, on the other hand, had significantly greater RMSEs of 0.1332 at 50 years and 0.0301 at 1000 years than the Weibull. However, it recorded lower Kappa RMSEs than 1.9667 at 50 years and 0.1049 at 1,000 years. Weibull recorded the lowest MAE, 0.0146 at 50 years and 0.0140 at 1,000 years. Burr had relatively large MAE values, as did 0.0814 at 50 years and 0.0204 at 1,000 years. By contrast, Kappa recorded MAE values of 0.6459 and 0.0517 after 50 and 1,000 years, respectively. Gamma had the highest RMSE and MAE. The same conclusion can apply to the second bias scenario, except that Gamma RMSE and MAE decreased drastically compared to the first bias scenario.

Table 9: RMSE and MAE of non-linear bias

| No. of years | Metric | Distribution | Bias<br>$1.1 * X_{obs}^{1.2}$ | Bias<br>$0.9 * X_{obs}^{0.8}$ |
|--------------|--------|--------------|-------------------------------|-------------------------------|
| 50           | RMSE   | Gamma        | 5.5448                        | 1.2340                        |
|              |        | Weibull      | <b>0.0164</b>                 | <b>0.0130</b>                 |
|              |        | Burr         | 0.1332                        | 0.0331                        |
|              |        | Kappa        | 1.9667                        | 1.6095                        |
|              | MAE    | Gamma        | 3.8262                        | 0.8307                        |
|              |        | Weibull      | <b>0.0146</b>                 | <b>0.0115</b>                 |
|              |        | Burr         | 0.0814                        | 0.0236                        |
|              |        | Kappa        | 0.6459                        | 0.5194                        |
| 100          | RMSE   | Gamma        | 5.9403                        | 1.5837                        |
|              |        | Weibull      | <b>0.0163</b>                 | <b>0.0130</b>                 |
|              |        | Burr         | 0.0483                        | 0.0235                        |
|              |        | Kappa        | 0.5775                        | 0.9386                        |
|              | MAE    | Gamma        | 3.9431                        | 1.0313                        |
|              |        | Weibull      | <b>0.0145</b>                 | <b>0.0115</b>                 |
|              |        | Burr         | 0.0283                        | 0.0152                        |
|              |        | Kappa        | 0.1882                        | 0.2813                        |
| 500          | RMSE   | Gamma        | 7.2097                        | 1.3935                        |
|              |        | Weibull      | <b>0.0140</b>                 | <b>0.0123</b>                 |
|              |        | Burr         | 0.0279                        | 0.0237                        |
|              |        | Kappa        | 0.1722                        | 0.1690                        |
|              | MAE    | Gamma        | 5.1593                        | 0.9771                        |
|              |        | Weibull      | <b>0.0127</b>                 | <b>0.0111</b>                 |
|              |        | Burr         | 0.0192                        | 0.0167                        |
|              |        | Kappa        | 0.0566                        | 0.0545                        |
| 1000         | RMSE   | Gamma        | 6.0584                        | 1.5076                        |
|              |        | Weibull      | <b>0.0156</b>                 | <b>0.0127</b>                 |
|              |        | Burr         | 0.0301                        | 0.0373                        |
|              |        | Kappa        | 0.1049                        | 0.0994                        |
|              | MAE    | Gamma        | 4.2101                        | 1.0358                        |
|              |        | Weibull      | <b>0.0140</b>                 | <b>0.0114</b>                 |
|              |        | Burr         | 0.0204                        | 0.0255                        |
|              |        | Kappa        | 0.0517                        | 0.0487                        |

Weibull, Burr, and Kappa consistently had very high NSEs and often achieved values of 1.0000, except at small sample sizes, in both non-linearity bias cases. Gamma always scored lowest on NSE under the first bias scenario. However, it scored higher under the second bias scenario. Although such values may seem high in isolation, they are significantly lower than the almost-perfect scores of the other distributions, indicating a significant lack of effectiveness in capturing the variance in the non-linear biases. Meanwhile, Weibull, as well as Burr and Kappa, performed well in the KGE measure, scoring 0.9998 for Weibull and, in most cases, around 0.999 for Burr and Kappa, especially for sample sizes lasting beyond 50 years. The equivalence of Kappa at the comparison of 1,000 years was 0.9989 in both cases, and Burr had 0.9995. Gamma always had the lowest KGE values. Over 50 and 1,000 years, its KGE ranged from 0.8634 to 0.9700 and 0.8664 to 0.9723, respectively, lower than those of the other distributions. This ultimately confirms its lack of efficacy in restoring the observed statistical properties in the presence of non-linear biases.

Table 10: NSE and KGE of non-linear bias

| No. of years | Metric | Distribution | Bias<br>$1.1 * X_{obs}^{1.2}$ | Bias<br>$0.9 * X_{obs}^{0.8}$ |
|--------------|--------|--------------|-------------------------------|-------------------------------|
| 50           | NSE    | Gamma        | 0.8986                        | 0.9970                        |
|              |        | Weibull      | <b>1.0000</b>                 | <b>1.0000</b>                 |
|              |        | Burr         | 0.9990                        | <b>1.0000</b>                 |
|              |        | Kappa        | 0.9885                        | 0.9918                        |
|              | KGE    | Gamma        | 0.8634                        | 0.9700                        |
|              |        | Weibull      | <b>0.9998</b>                 | <b>0.9998</b>                 |
|              |        | Burr         | 0.9977                        | 0.9994                        |
|              |        | Kappa        | 0.9743                        | 0.9794                        |
| 100          | NSE    | Gamma        | 0.8866                        | 0.9960                        |
|              |        | Weibull      | <b>1.0000</b>                 | <b>1.0000</b>                 |
|              |        | Burr         | 0.9998                        | <b>1.0000</b>                 |
|              |        | Kappa        | 0.9988                        | 0.9971                        |
|              | KGE    | Gamma        | 0.8580                        | 0.9643                        |
|              |        | Weibull      | <b>0.9998</b>                 | <b>0.9998</b>                 |
|              |        | Burr         | 0.9992                        | 0.9996                        |
|              |        | Kappa        | 0.9919                        | 0.9872                        |
| 500          | NSE    | Gamma        | 0.8556                        | 0.9964                        |
|              |        | Weibull      | <b>1.0000</b>                 | <b>1.0000</b>                 |
|              |        | Burr         | <b>1.0000</b>                 | <b>1.0000</b>                 |
|              |        | Kappa        | <b>1.0000</b>                 | <b>1.0000</b>                 |
|              | KGE    | Gamma        | 0.8248                        | 0.9716                        |
|              |        | Weibull      | <b>0.9998</b>                 | <b>0.9998</b>                 |
|              |        | Burr         | 0.9995                        | 0.9996                        |
|              |        | Kappa        | 0.9989                        | 0.9989                        |
| 1000         | NSE    | Gamma        | 0.8765                        | 0.9956                        |
|              |        | Weibull      | <b>1.0000</b>                 | <b>1.0000</b>                 |
|              |        | Burr         | <b>1.0000</b>                 | <b>1.0000</b>                 |
|              |        | Kappa        | <b>1.0000</b>                 | <b>1.0000</b>                 |
|              | KGE    | Gamma        | 0.8664                        | 0.9723                        |
|              |        | Weibull      | <b>0.9998</b>                 | <b>0.9998</b>                 |
|              |        | Burr         | 0.9995                        | 0.9995                        |
|              |        | Kappa        | 0.9989                        | 0.9989                        |

#### 4.5. Goodness-of-fit tests

To evaluate whether the parametric distributions are sufficient to fix the bias of extreme rainfall statistically, synthetic data was simulated across  $n = 1,000$  years. The long simulation length, covering a wide range of interannual variability and the full extremes of behavior, as is typical in the Malaysian monsoonal climate, provided a very sound foundation for distributional fitting. Concurrently, the vast amount of data enabled stable estimates of performance indicators, especially for tail-sensitive applications such as return estimation.

Testing of distributional bias correction methods was also conducted using GoF tests, with the K-S and CvM tests applied to the corrected rainfall databases for four bias types: additive, multiplicative, combining, and non-linear. In each case, the proportion of GoF tests that did not reject the null hypothesis ( $\alpha > 0.05$ ) was computed across 1,000 simulations for the four distributions: Gamma, Weibull, Burr, and Kappa. Both Burr and Kappa distributions were superior in pass proportions to Gamma in all units of additive bias ( $-30, -15, +15, +30$ ). On the other hand, Gamma yielded significantly lower pass rates, ranging from 0.348 to 0.388. In contrast, Weibull failed all GoF tests, with both K-S and CvM returning 0 in all additive scenarios. The same trend was observed for multiplicative bias (0.6, 0.8, 1.2, 1.4): GoF pass

proportions tended to be greater than 0.82 and 0.88 for Burr and Kappa, respectively. The performance of Gamma was comparatively weak, with pass rates of 0.348 to 0.388, and Weibull again demonstrated no passes of the GoF, despite being very weak in accommodating scaled bias. When additive and multiplicative bias transformations were combined ( $1.4 * X_{obs} - 30$ ,  $0.6 * X_{obs} + 30$ ). The similar trend was observed. Both Burr and Kappa presented similarly high pass rates. In contrast, Gamma under combined biases did not pass the test, and Weibull gave a pass count of zero. Lastly, in case of non-linear bias cases ( $1.1 * X_{obs}^{1.2}$ ,  $0.9 * X_{obs}^{0.8}$ ), Burr and Kappa distributions once again gave results that sufficiently demonstrated consistency in GoF, which proved better in all comparisons to Gamma. Gamma demonstrated only moderate pass rates, and, again, Weibull performed poorly, with both the K-S and CvM proportions equal to 0 in all non-linear conditions.



Figure 1: Goodness of Fit test on linear and non-linear biases

Overall, the results in the paper suggest that the Burr and Kappa distributions are more flexible and robust, as they model the corrected rainfall distribution across a wide range of bias forms. Gamma, on the contrary, indicates regular underperformance, and Weibull is inapplicable in amending bias in the modeling of extreme rainfalls, irrespective of the nature or severity of the distortion. It should also be noted that the Burr Type XII and Kappa distributions contain more parameters than the Gamma and Weibull distributions, which provides additional flexibility in capturing complex rainfall distributions. This increased flexibility may partly explain their superior performance in several bias scenarios.

## 5. Conclusion

In this research, the work drastically compares the applicability of four probability distributions, namely Gamma, Weibull, Burr, and Kappa, in the correction of the simulated bias on extreme

rainfall using distributional QM. All the above are performed within four different bias structures: additive, multiplicative, and non-linear ones, using evaluation metrics, RMSE, MAE, NSE, KGE, and GoF tests, such as K-S and CvM. With additive bias, the Kappa distribution demonstrates better overall performance, reporting the lowest RMSE and MAE, and achieving high passes on both GoF tests. The fact that it is remarkably flexible in fitting skewness and tail behavior makes it especially good at using these bias-corrected extremes to produce the reconstruction. In the multiplicative bias case, the Weibull distribution has the lowest RMSE. However, under GoF tests, it fails, indicating an inadequate fit to the corrected distribution. Therefore, one will select the Kappa distribution, which has the second-best RMSE, the best MAE, and a statistically satisfactory GoF. When the bias is both additive and multiplicative, the Kappa distribution again produces the best results, both in terms of RMSE and MAE, and in its GoF test results. On a similar note, it is relatively resistant to highly biased networks, making it highly reliable for extreme rainfall adjustments. In the non-linear bias, while the Weibull distribution had the lowest RMSE and MAE, it was discarded since it failed all GoF tests. The Burr distribution is thus selected as it offers the second-best RMSE and MAE results, as well as consistently high GoF scores, making it more statistically robust for practical application. The selected distributions also perform well on NSE and KGE metrics.

Despite the robust results of the research, it has certain limitations. The analyses were conducted based on synthetically generated GEV rain sequences with generalized monsoonal characteristics of Malaysia. Though the design makes it easy to test for controlled bias, it does not fully capture the spatial variability and temporal non-stationarity observed at regional scales. Furthermore, the estimation of distribution parameters, especially those that are expansive, such as Kappa and Burr, is highly sensitive to sample size and starting values, which may impede real-world applications. Thus, future research should explore bias-correction techniques for true rainfall observations across various regions of Malaysia, which exhibit diverse hydroclimatic conditions. Furthermore, it is necessary to focus on monsoon and inter-monsoon transitions, as well as seasonal or regime-specific adjustments. It is convenient to compare parametric methods with non-parametric and machine-learning-based procedures. At the same time, special attention should be paid to multivariate methods that simultaneously approximate all characteristics of rainfall intensity, duration, and frequency. In addition, the findings reveal that among the considered distributions, the Kappa distribution may be the most consistent and accurate for mitigating biases in extreme rainfall data across a variety of bias scenarios. This is especially true for additive, multiplicative, and a combination of additive and multiplicative. In non-linear situations where the Kappa distribution cannot be applied, it is reasonable to use the Burr distribution.

In conclusion, this study suggests that flexible distributions, such as the Kappa and Burr Type XII distributions, are more robust for modeling extreme rainfall when systematic biases are present. Researchers working with climate model outputs or biased rainfall datasets should consider applying bias-correction techniques, such as QM, before fitting probability distributions. Building on this, the choice of distribution should account for tail behavior, as the upper tail strongly influences extreme rainfall events. In response, future studies may also consider incorporating the Anderson-Darling GoF test, which emphasizes the tails of the distribution and can provide additional insight into extreme rainfall events. Likewise, future work may also extend the current simulation framework to real rainfall datasets in Malaysia to evaluate the performance of bias-correction and distribution-fitting methods under real observational conditions.

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