

COUPLE STRESS HYBRID NANOFLUID FLOW OVER A SHRINKING RIGA PLATE WITH ELECTROMAGNETIC AND SUCTION EFFECT

(Aliran Tekanan Bergandingan Nanobendalir Hibrid Melepasi Plat Riga Mengecut dengan Kesan Elektromagnet dan Sedutan)

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ABSTRACT

This study investigates the steady two-dimensional boundary layer flow of a couple stress hybrid nanofluid past a shrinking Riga plate. The flow is controlled by an externally powered Riga plate capable of producing Lorentz forces, strengthening flow stabilization. Unlike prior works that focused individually on couple stress fluids, hybrid nanofluids, or Riga plates, this work integrates all three within the framework of a shrinking surface. This unique combination offers new insights into boundary layer control, particularly in advanced microfluidic and cooling applications, where precise regulation of both momentum and heat transfer is crucial. The hybrid nanofluid considered in this study consists of aluminium alloys (AA7072) and (AA7075) nanoparticles dispersed in methanol (CH_3OH). The governing system of partial differential equations is converted to ordinary differential equations using similarity transformations, which are then solved numerically using MATLAB's *bvp4c* solver. The results reveal that the inclusion of hybrid nanoparticles enhances heat transfer and improves thermal retention compared to the base fluid. Increasing suction significantly stabilizes the boundary layer, suppressing velocity profiles and reducing thermal boundary layer thickness. Conversely, higher values of the Riga plate parameter reduce velocity gradients while causing a slight increase in the thermal boundary layer thickness, indicating a trade-off between flow retardation and heat retention.

Keywords: hybrid nanofluid; couple stress; Riga plate; suction; shrinking

ABSTRAK

Kajian ini menyiasat aliran mantap dua dimensi lapisan sempadan serta ciri-ciri pemindahan haba bagi tekanan bergandingan nanobendalir hibrid merentasi plat Riga mengecut. Aliran ini dikawal oleh plat Riga yang mampu menghasilkan daya Lorentz bagi mengukuhkan penstabilan aliran. Tidak seperti kerja terdahulu yang memfokuskan secara individu pada bendalir tekanan bergandingan, nanobendalir hibrid atau plat Riga, kerja ini menyepadukan ketiga-tiganya dalam rangka permukaan yang mengecut. Gabungan unik ini menawarkan pandangan baharu tentang kawalan lapisan sempadan, terutamanya dalam aplikasi mikrobendalir dan penyejukan termaju, di mana pengawalseliaan tepat bagi kedua-dua momentum dan pemindahan haba adalah penting. Bendalir nano hibrid yang dipertimbangkan dalam kajian ini terdiri daripada aloi aluminium (AA7072) dan (AA7075) nanozarah yang tersebar dalam metanol (CH_3OH). Sistem persamaan pembezaan separa ditukar kepada persamaan pembezaan biasa melalui penjelmaan keserupaan, dan kemudiannya diselesaikan secara berangka menggunakan penyelesaian *bvp4c* dalam perisian MATLAB. Keputusan mendedahkan bahawa kemasukan nanopartikel hibrid meningkatkan pemindahan dan pengekalan haba berbanding dengan bendalir asas. Peningkatan parameter sedutan dilihat dengan ketara menstabilkan lapisan sempadan, meningkatkan profil halaju dan mengurangkan ketebalan lapisan sempadan terma. Sebaliknya, nilai parameter plat Riga yang lebih tinggi mengurangkan kecerunan halaju sambil menyebabkan sedikit peningkatan dalam ketebalan lapisan sempadan haba, menunjukkan perimbangan antara penangguhan aliran dan pengekalan haba.

Kata kunci: nanobendalir hibrid; tekanan bergandingan; plat Riga; sedutan; pengecutan

1. Introduction

The improvement of fluid flow and heat transfer has been of great importance in engineering and industrial applications for a long time, especially for cooling systems, biomedical systems, microfluidic devices, and energy conversion systems (Hafeez *et al.* 2025; Abbas *et al.* 2024; Mubeena *et al.* 2024). Increased demand for miniaturized and more efficient systems impacts the adequacy of classical coolant fluids and fluid dynamic models, which struggle to address the thermal management problems. This calls for better models of coolant fluids, non-Newtonian fluids, nanofluids, thermal performance enhancers, and electromagnetic boundary layer control mechanisms. One such advanced model is the couple stress fluid introduced by Stokes (1984), which generalizes the classical Navier–Stokes equations by including the effects of couple stresses arising from the rotation of fluid microelements. This model is particularly useful in modeling the flow of microstructural fluids such as lubricants, polymeric suspensions, and biological fluids like synovial fluid and blood. In comparison to Newtonian fluids, couple stress fluids can withstand twisting motion as well as additional resistance to shear, which makes them more appropriate for non-classical physics modeling of flow systems. Several researchers, such as Devani *et al.* (2024), Zainal *et al.* (2025), Mokhtar *et al.* (2025) have studied couple stress fluids in different boundary layer configurations and reported their effect on skin friction and velocity profile modulation.

The effectiveness of the Riga plates in controlling the flow has been studied quite extensively. For example, Ali *et al.* (2023) analyzed MHD nanofluid flow dynamics over a Riga plate and remarked that the Lorentz force suppressed flow reversal and reduced the shear stress at the boundaries. Zahoor *et al.* (2025) studied flow past a Riga plate with elastic viscoelastic fluids and noted that the electromagnetic forcing significantly changed flow and heat transfer behavior. Nonetheless, most studies done so far concentrate on stretching surfaces and use a simple model of non-Newtonian fluids or a Newtonian model. All these advanced features: couple stress fluids, hybrid nanofluids, shrinking plates, and Lorentz force acting through a Riga plate have not been captured together to a reasonable depth. While Mahabaleshwar *et al.* (2022), Mohanavel *et al.* (2025), and Rajamani and Reddy (2022) attempted to incorporate couple stress effects into the models of nanofluid and hybrid nanofluids over stretching or shrinking sheets, very few have tried to consider them simultaneously in the framework of a shrinking Riga plate. This particular arrangement is not only a complex mathematical problem, but also offers relevance for real life systems that need active thermal control under strict geometric confinement, such as microelectromechanical systems (MEMS), lab on chip platforms, and flexible electronics.

The innovative use of a Riga plate, which generates a Lorentz force along the flow direction, offers a unique electromagnetic approach to controlling and stabilizing such flows without mechanical intervention. Previous investigations have considered some of the individual aspects of this study, such as couple stress fluids (Mahabaleshwar *et al.* 2022), hybrid nanofluids (Rajamani & Reddy 2022), or Riga plates (Ali *et al.* 2023; Zahoor *et al.* 2025). Waini *et al.* (2023) examined couple stress fluids with hybrid nanoparticles but only in the case of stretching surfaces. However, to the best of our knowledge, no study has yet attempted to combine couple stress fluid, hybrid nanofluid, shrinking surface, and Riga plate within a single framework. This integration distinguishes the present work and establishes its originality in the literature. Hence, this study aims to fill that gap by formulating and solving the boundary layer equations for a couple stress hybrid nanofluid (AA7072-AA7075/CH₃OH) over a shrinking

Riga plate. By employing similarity transformations and solving the reduced system numerically via MATLAB's bvp4c solver, this research explores the influence of key physical parameters, including couple stress, nanoparticle volume fractions, and Lorentz force. The findings offer new insights into the design of efficient and actively controlled thermal systems, particularly in microscale applications where precise flow and heat transfer regulation are critical.

2. Mathematical Model

Figure 1 illustrates the schematic model of the boundary layer flow of couple stress hybrid nanofluid (AA7072-AA7075/CH₃OH) past a shrinking Riga plate. In this case, u represents the velocity component in the x – direction, while represents velocity component in the y – direction. The velocity as the plate surface stretches or shrinks is referred to as $U_w(x) = ax$ (a is a constant), the velocity of wall mass flux is V_w and the couple stress fluid is denoted as ξ . The variable surface temperature is represented by $T_w = T_\infty + bx^2$ (b is a constant) with the surrounding temperature T_∞ . In this study, the electromagnetic parameter associated with the Riga plate is taken as constant. This assumption is consistent with earlier works (Ali *et al.* 2023; Zahoor *et al.* 2025) and corresponds physically to maintaining a uniform magnetization and constant electrode current density along the Riga plate. Such a configuration is realistic in controlled laboratory or microfluidic setups where steady electromagnetic forcing is imposed.

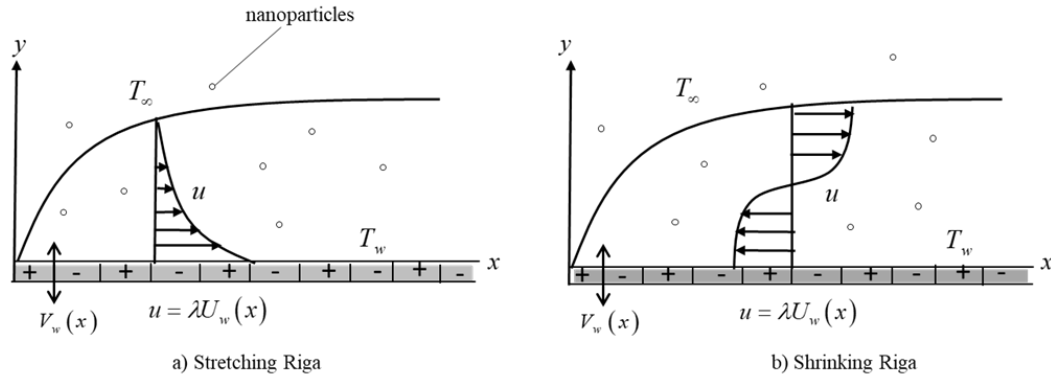


Figure 1: The schematic model of stretching/shrinking Riga plate

The governing equation for the respective case is formulated by considering the boundary layer and the Boussinesq approximation to reduce the complexity of mathematical models. The non-dimensional variables are applied to transform the model's equation into dimensionless form before it is ready for numerical computation. By applying similarity transformations, the system of governing PDEs is reduced to a set of ordinary differential equations (ODEs). These ODEs, along with their corresponding boundary conditions, naturally form a boundary value problem (BVP). This reformulation is essential as it allows the use of MATLAB's bvp4c algorithm, which is specifically designed to solve BVPs with high accuracy and stability. The explicit connection between the transformed ODEs and the BVP ensures that the numerical results obtained reflect the correct physical behavior of the system.

In this study, the flow is assumed to be steady, two-dimensional, and laminar, with nanoparticles uniformly dispersed in the base fluid without agglomeration. The electromagnetic

parameter is taken as constant to represent a steady Lorentz force, while thermal radiation and viscous dissipation are neglected. These assumptions simplify the analysis while still representing realistic conditions in microscale fluid systems. In general, the governing mathematical models of couple stress hybrid nanofluid flow past a vertical plate are given as follows (Khan *et al.* 2013; Dhlamini *et al.* 2021):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}} \frac{\partial^2 u}{\partial y^2} - \frac{\xi}{\rho_{hnf}} \frac{\partial^4 u}{\partial y^4} + \frac{\pi j_0 M}{8 \rho_{hnf}} \exp\left(-\frac{\pi}{a_1} y\right), \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \frac{\partial^2 T}{\partial y^2}, \quad (3)$$

along with the boundary conditions

$$\begin{aligned} u &= \lambda U_w(x), \quad v = V_w, \quad T = T_w(x) = T_\infty + ax^2 \quad \text{at } y = 0, \\ u &\rightarrow 0, \quad \frac{\partial u}{\partial x} \rightarrow 0, \quad \frac{\partial^2 u}{\partial y^2} \rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty. \end{aligned} \quad (4)$$

Next, the following variables of similarity were adopted (Turkyilmazoglu 2014; Waini *et al.* 2023);

$$\begin{aligned} \psi &= (av_f)^{\frac{1}{2}} xf(\eta), \quad u = axf'(\eta), \quad v = -(ax)^{\frac{1}{2}} f(\eta), \\ \eta &= y(a/v_f)^{\frac{1}{2}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \end{aligned} \quad (5)$$

hence,

$$V_w = -(av_f)^{\frac{1}{2}} S, \quad (6)$$

where S is the mass flux parameter with $S < 0$ for injection and $S > 0$ for suction. Here, $f(\eta)$ represents the dimensionless stream function, where $()'$ denoted the differentiation with respect to the similarity variables. In another note, ν_f is the kinematic viscosity of fluid, $M = M_x$ and j_0 represents the variable form of the magnet's magnetisation and the current density in the electrodes, respectively. Next, with the help of similarity transformations presented in the relations (5) and (6), the boundary value problem is defined by Eqs. (2)-(3), and boundary conditions (4) are systematically transformed into the following ordinary differential equations as follows

$$-\xi f'''' + \frac{\mu_{hnf}}{\rho_{hnf}} \frac{\mu_f}{\rho_f} f''' + ff'' - f'^2 + \frac{Z}{\rho_{hnf} / \rho_f} \exp^{-d\eta} = 0, \quad (7)$$

$$\frac{1}{\text{Pr}} \left(\frac{k_{hmf}/k_f}{(\rho C_p)_{hmf}/(\rho C_p)_f} \right) \theta'' + f \theta' - 2 f' \theta = 0, \quad (8)$$

$$\begin{aligned} f(0) &= S, f'(0) = \lambda, \theta(0) = 1, \\ f'(\eta) &\rightarrow 0, f''(\eta) \rightarrow 0, f'''(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \end{aligned} \quad (9)$$

where the Prandtl number is $\text{Pr} = \nu_f / \alpha_f$ and the parameter which describes stretching or shrinking is given as λ . The couple stress parameter is presented as $\xi = \xi_0 a / \rho_{hmf} \nu_f^2$, the Riga plate (electromagnetic) parameter and the width parameter are denoted as $Z = \pi j_0 M_0 / 8 a^2 \rho_{hmf}$ and $d = \frac{\pi}{a_1} \sqrt{\frac{\nu_f}{a}}$, respectively. Additionally, Table 1 demonstrates the characteristic properties of hybrid nanofluid and Table 2 presents the nanoparticle's properties and the base fluid where ϕ_1 is AA7072 and ϕ_2 denotes the AA7075 nanoparticle.

 Table 1: Characteristic properties of hybrid nanofluid (Raza *et al.* 2016; Takabi & Salehi 2014)

Characteristics	Hybrid nanofluid
Dynamic viscosity,	$\mu_{hmf} = \mu_f / (1 - \phi_{hmf})^{2.5}$
Thermal capacity,	$(\rho C_p)_{hmf} = (1 - \phi_{hmf})(\rho C_p)_f + \phi_1 (\rho C_p)_{n1} + \phi_2 (\rho C_p)_{n2}$
Density,	$\rho_{hmf} = (1 - \phi_{hmf}) \rho_f + \phi_1 \rho_{n1} + \phi_2 \rho_{n2}$
Thermal conductivity,	$k_{hmf} = \left[\frac{\left(\frac{\phi_1 k_{n1} + \phi_2 k_{n2}}{\phi_{hmf}} \right) + 2k_f + 2(\phi_1 k_{n1} + \phi_2 k_{n2}) - 2\phi_{hmf} k_f}{\left(\frac{\phi_1 k_{n1} + \phi_2 k_{n2}}{\phi_{hmf}} \right) + 2k_f - (\phi_1 k_{n1} + \phi_2 k_{n2}) + \phi_{hmf} k_f} \right] \times k_f$
Thermal expansion coefficient,	$(\rho \beta)_{hmf} = (1 - \phi_2) [(1 - \phi_1)(\rho \beta)_f + \phi_1 (\rho \beta)_{n1} + \phi_2 (\rho \beta)_{n2}]$

 Table 2: The nanoparticles and base fluid properties (Tlili *et al.* 2020; Swalmeh *et al.* 2019; Sharma *et al.* 2023)

Characteristics	C_p (J/kgK)	k (W / mK)	ρ (kg/m ³)	$\beta \times 10^{-6}$ (1/K)	Pr
AA7072	893	222	2720	23.6	
AA7075	960	173	2810	23.5	
CH ₃ OH	2545	0.2035	792	149	7.38

The quantities of interest in this case are the skin friction coefficient $C_f = \frac{\mu_{hmf}}{\rho_f U_w^2} \left(\frac{\partial u}{\partial y} \right)_{y=0}$ and the

local Nusselt number $Nu_x = \frac{x k_{hmf}}{k_f (T_w - T_\infty)} \left(-\frac{\partial T}{\partial y} \right)_{y=0}$. Employing the above-mentioned information, we finally derive

$$\text{Re}_x^{1/2} C_f = \frac{\mu_{hnf}}{\mu_f} f''(0), \quad \text{Re}_x^{-1/2} Nu_x = -\frac{k_{hnf}}{k_f} \theta'(0), \quad (10)$$

where $\text{Re}_x = U_w x / \nu_f$.

3. Results and Discussion

This section analyzes the effects of different parameters on the thermal and flow characteristics of a couple stress hybrid nanofluid flowing over a shrinking Riga plate. The study focuses on the influence of type of fluids (ϕ_1, ϕ_2), suction parameters (S), and Riga plate (electromagnetic) parameter (Z) on the velocity $f'(\eta)$ and temperature profile $\theta(\eta)$ distributions. Table 3 shows the comparison of the results dependability of Waini *et al.* (2023) and the current findings are in good agreement with previous work. Since the model results correlate well with previously published research findings, we believe that the proposed mathematical model is appropriate for the actual dynamic fluid flow problem under consideration in this study. Meanwhile, Table 4 presents the current results of $\text{Re}_x^{1/2} Cf$ for various couple stress parameters (ξ).

Table 3: Results generation of $\text{Re}_x^{1/2} Cf$ for various ϕ when $Z = d = 0, \xi = 0.01, \lambda = S = 1, \text{Pr} = 7.38$

ϕ_1	$\phi_2 = 0$		$\phi_2 = 0.005$	
	Waini <i>et al.</i> (2023)	Present	Waini <i>et al.</i> (2023)	Present
0	−9.392542	−9.392546	−9.513503	−9.513505
0.005	−9.510239	−9.510237	−9.634587	−9.634586

Table 4: Results of $\text{Re}_x^{1/2} Cf$ for various ξ when $Z = 0.1, d = 0.01, S = 2, \phi_1 = \phi_2 = 0.01$, and $\text{Pr} = 7.38$

λ	$\xi = 0.01$	$\xi = 0.02$	$\xi = 0.03$
−1.0	2.04800791	1.967355564	1.913811982
−1.10	2.264990425	2.157254739	2.087697117
−1.200	2.479973318	2.325726323	2.258083694
−1.3000	2.6929587310	2.528508845	2.4249895790
−1.4000	2.9039488130	2.7098800570	2.5884321980
−1.50000	3.1129457150	2.8884247710	2.7484285610

The impact of the nanoparticles volume fraction on (ϕ_1, ϕ_2) the velocity profile $f'(\eta)$ over a shrinking Riga plate is shown in Figure 2. Here, the shrinking surface condition is denoted by $\lambda = -1.1$ and three different cases are noted: a pure base fluid ($\phi_1 = \phi_2 = 0.00$) a single-component nanofluid ($\phi_1 = 0.00, \phi_2 = 0.01$) and a hybrid nanofluid ($\phi_1 = \phi_2 = 0.01$). As ϕ_1 and ϕ_2

increases, $f'(\eta)$ improves, which indicates that the boundary layer becomes thinner and the magnitude of the negative velocity gradient increases. Although the velocity and temperature profiles across the three cases appear visually similar, the hybrid nanofluid consistently demonstrates enhanced heat transfer characteristics and modified flow behavior compared to single nanofluids. These improvements, while subtle in appearance, become more evident under stronger suction and Riga plate parameters. This highlights the practical value of hybrid nanofluids in improving thermal regulation, even when the overall profile shapes are close.

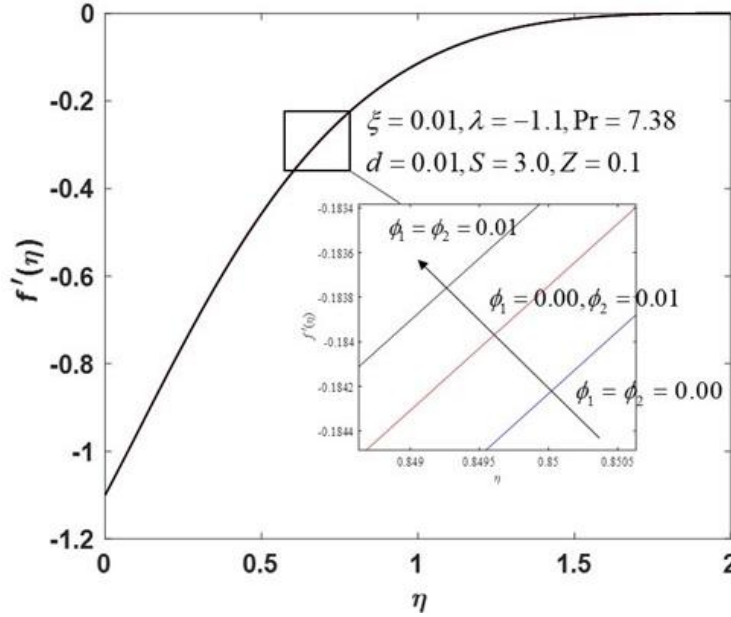


Figure 2: Velocity profile $f'(\eta)$ with several ϕ_1 and ϕ_2

Physically, this means that the fluid deceleration is more pronounced closer to the surface, which is the case as more nanoparticles are included. Concerning a shrinking Riga plate, the electromagnetic force acts to stabilize the otherwise unstable boundary layer flow, and the effect of nanoparticles helps enhance this stabilizing effect. Thus, higher ϕ_1 and ϕ_2 values result in sharper velocity profiles, indicating improved boundary layer control due to the combined effect of the Lorentz force and enhanced fluid properties.

Meanwhile, Figure 3 portrays the temperature profile $\theta(\eta)$ for various ϕ_1 and ϕ_2 over a shrinking Riga plate. It is observed that as the nanoparticle volume fractions increase, moving from the base fluid case ($\phi_1 = \phi_2 = 0.00$) to single nanoparticle addition ($\phi_1 = 0.00, \phi_2 = 0.01$), and finally to hybrid nanofluid ($\phi_1 = \phi_2 = 0.01$) the temperature profiles shift upward. This upward shift shows that thermal retention at the surface has improved because of the better conductivity of nanofluids and hybrid nanofluids in comparison to base fluids. The magnified view thus serves to emphasize that the inclusion of nanoparticles affects the thermophysical properties, supporting the idea that higher concentrations of particles increase the thickness of the boundary layer, which improves the heat transfer efficiency of the flow system when an electromagnetic force from the Riga plate is applied.

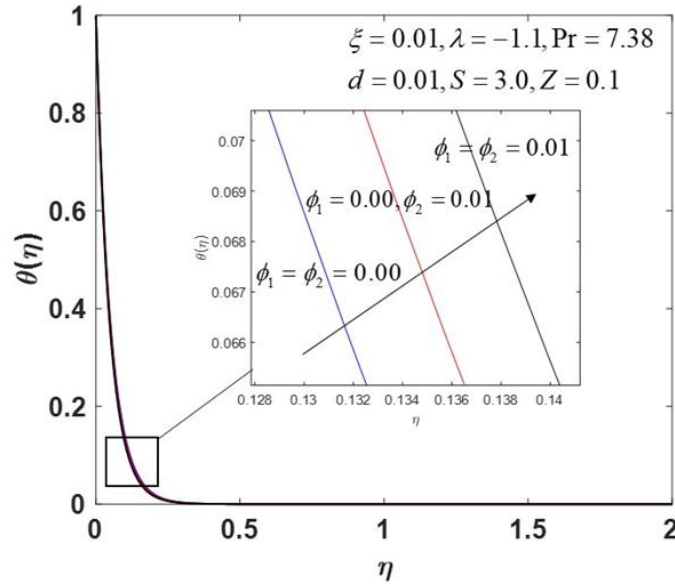


Figure 3: Temperature profile $\theta(\eta)$ with several ϕ_1 and ϕ_2

The correlation between $f'(\eta)$ and the suction parameter (S) is represented in Figure 4. It is evident that increasing the suction parameter from $S=1.0, 2.0, 3.0$ results in a downward shift in the velocity profile, indicating a suppression of the velocity field throughout the boundary layer. This trend suggests that stronger suction draws more fluid toward the plate surface, thereby reducing the flow velocity away from the wall. Physically, suction acts to thin the boundary layer and stabilize the flow, particularly important in shrinking surface problems where flow separation and reverse flow are common challenges. The reduction of $f'(\eta)$ with an increment to S implies a reduction in the momentum thickness and a more compressed boundary layer, signifying the dominant role of suction in enhancing boundary layer control in the presence of electromagnetic actuation from the Riga plate.

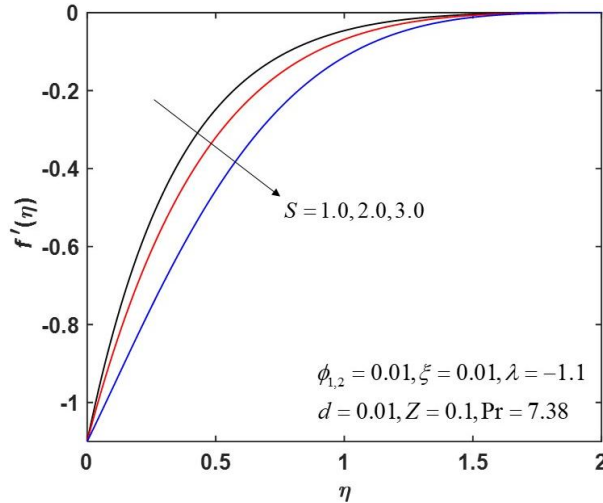


Figure 4: Velocity profile $f'(\eta)$ with several S

Figure 5 illustrates the variation of the dimensionless $\theta(\eta)$ in response to different values of the suction parameter $S = 1.0, 2.0, 3.0$. It is evident from the figure that the $\theta(\eta)$ decreases more rapidly with increasing values of S . Specifically, as S increases from 1.0 to 3.0, the thermal boundary layer becomes thinner, and the thermal penetration into the fluid is significantly reduced. This behavior can be explained by the fact that suction at the wall increases the extraction of fluid particles adjacent to the surface, which, in turn, diminishes the boundary layer thermal stratification.

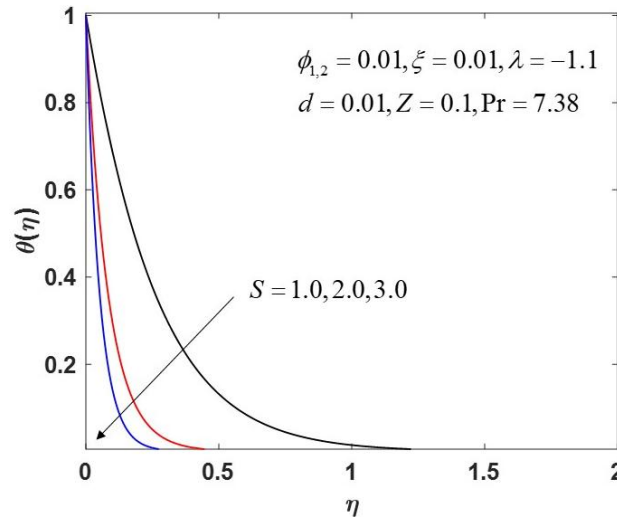


Figure 5: Temperature profile $\theta(\eta)$ with several S

The $f'(\eta)$ shown in Figure 6 illustrates the effect of varying the Riga plate parameter Z , as $Z = 0.1, 0.2, 0.3$. Contrary to the expected enhancement in flow acceleration, the velocity profiles shift downward as Z increases, indicating a reduction in fluid velocity within the boundary layer. This behavior suggests that, in the presence of a strong shrinking surface ($\lambda = -1.1$), the electromagnetic force introduced by the Riga plate (represented by Z) has a retarding effect on the flow rather than an accelerating one. One explanation could be that the Lorentz force, depending on its direction and the interaction with the contracting flow field, may be able to behave as a resistive force for a certain parameter set. When Z increases, the boundary layer thickens slightly which results in more pronounced fluid deceleration, thus leading to lower velocity gradient close to the wall. This result emphasizes that the impact of the Riga plate is extremely critical and sensitive to the combinations of parameters and may in some cases, render assistance toward mitigating flow instead of increasing it.

Figure 7 presents the effect of Z , on the $\theta(\eta)$ and the results indicate that as Z increases, the temperature profile slightly increases near the wall, as highlighted in the magnified inset. Such behavior suggests that the Lorentz force produced by the Riga plate has a negligible contribution to thermal retardation causing an increase in the thermal boundary layer thickness. This, in turn, can be explained due to the damping effect of the electromagnetic force acting on the velocity field, decreasing convection and increasing local temperature. Even though the change is minor, it supports the trend that there is less fluid velocity for increasing Z , which reduces the cooling effect of advection and increases thermal retention. So, the Riga plate parameter increases the thermal profile slightly to demonstrate the delicate balance between electromagnetic control and heat transfer in boundary layer flow.

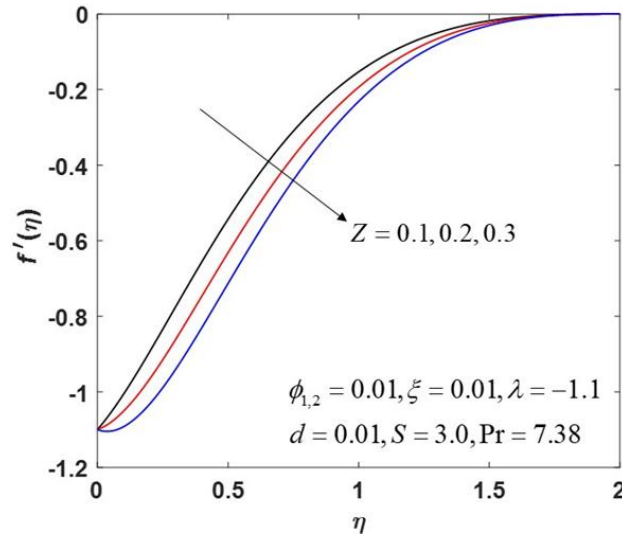


Figure 6: Velocity profile $f'(\eta)$ with several Z

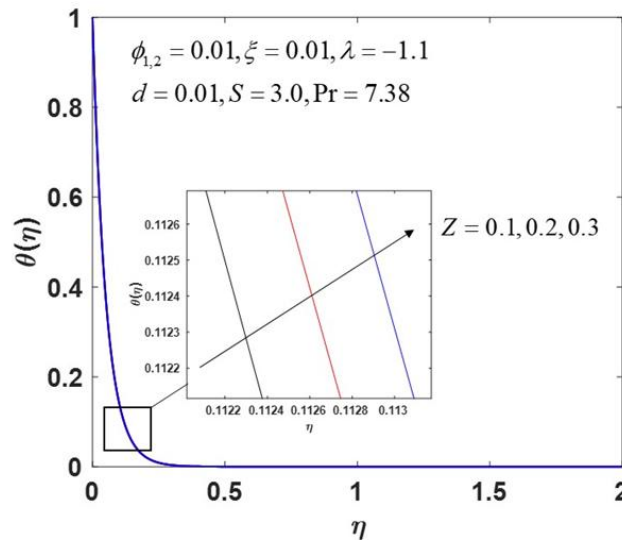


Figure 7: Temperature profile $\theta(\eta)$ with several Z

4. Conclusion

In summary, this study contributes to the literature in several significant ways, particularly by integrating couple stress fluid, hybrid nanofluid, shrinking surface, and Riga plate effects into a unified boundary layer model. The inclusion of nanoparticles enhances both momentum and thermal transport characteristics due to increased effective viscosity and thermal conductivity, compared to both single nanofluids and the base fluid. Meanwhile, suction is identified as a dominant parameter in stabilizing shrinking surface flows, leading to thinner velocity and temperature boundary layers. Interestingly, the effect of the Riga plate parameter reveals a nuanced interaction with the flow field. While it is commonly expected to accelerate the fluid via electromagnetic forces, the results indicate a retarding effect on the velocity under strong shrinking conditions, suggesting that the Lorentz force may act as a resistive agent depending on the parameter regime. Correspondingly, a slight rise in the temperature profile is observed

with increasing Riga plate parameter, attributed to diminished convective cooling due to flow deceleration. Overall, the study underscores the complex but controllable interplay between nanoparticle inclusion, suction, and electromagnetic actuation in optimizing flow and heat transfer in Riga plate boundary layer systems.

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