

A UNIFIED CRITIC-SIMILARITY HYBRID WEIGHTING AND AGGREGATION MODEL FOR MULTI-CRITERIA DECISION MAKING
(Model Bersepadu Hibrid Pemberatan dan Pengagregatan CRITIC-Kesamaan untuk Pembuatan Keputusan Pelbagai Kriteria)

NURUL FAQIHAN ZULKIFLI & NOR HANIMAH KAMIS*

ABSTRACT

Decision-making involving multiple competing criteria often relies heavily on subjective expert judgement, which may reduce the transparency, consistency, and reliability of the final decision. Many existing approaches lack an objective mechanism for determining criterion weights and provide limited flexibility in accommodating variations in expert preference structures. For instance, although Kamis *et al.* (2018) proposed a structured Group Decision-Making (GDM) framework, it does not explicitly incorporate objective weighting techniques or examine the influence of different similarity measures and aggregation operators on ranking stability. To address these limitations, this study proposes a hybrid Multi-Criteria Decision-Making (MCDM) framework that integrates the Criteria Importance Through Intercriteria Correlation (CRITIC) method for objective criteria weighting. The CRITIC method determines criteria weights based on contrast intensity and intercriteria correlation, thereby reducing subjective bias and reflecting the inherent characteristics of the decision data. Furthermore, three similarity measures, namely Cosine Similarity, Manhattan Similarity, and Additive Reciprocal Preference Similarity (ARPS), are incorporated into the GDM model to capture different aspects of expert preference similarity. The resulting similarity scores are aggregated using the Weighted Arithmetic Mean (WAM), Induced Ordered Weighted Averaging (IOWA), and Weighted Ordered Weighted Averaging (WOWA) operators. The proposed framework is demonstrated through a case study on hybrid energy system selection considering technological, economic, and environmental criteria. The results show consistent and stable alternative rankings across different similarity measures and aggregation operators, indicating improved robustness and reduced sensitivity to subjective judgement. Overall, the proposed framework provides a comprehensive and reliable decision-support model by integrating objective weighting, similarity-based preference modelling, and flexible aggregation techniques for complex multi-criteria decision-making problems.

Keywords: CRITIC; Multi-Criteria Decision-Making (MCDM); criteria weights; aggregation operators; similarity measures; hybrid energy systems

ABSTRAK

Kesukaran membuat keputusan yang melibatkan banyak kriteria yang bersaing kadangkala banyak bergantung kepada pertimbangan subjektif pakar, yang boleh menjejaskan ketelusan, konsistensi dan kebolehpercayaan keputusan akhir. Banyak pendekatan sedia ada tidak mempunyai mekanisme objektif untuk menentukan pemberat kriteria dan mempunyai fleksibiliti yang terhad dalam menangani perbezaan struktur keutamaan pakar. Sebagai contoh, Kamis *et al.* (2018) menyediakan rangka kerja pengagregatan yang berstruktur tetapi tidak menggabungkan teknik pemberatan objektif secara jelas atau mengkaji kesan ukuran keserupaan dan operator pengagregatan yang berbeza terhadap kestabilan pemeringkatan. Kekurangan ini menekankan keperluan kepada sistem sokongan keputusan yang lebih maju dan teguh. Kajian ini membentangkan satu rangka kerja MCDM hibrid yang menggabungkan pendekatan CRITIC untuk pemberatan kriteria secara objektif. Pendekatan CRITIC digunakan untuk mengira pemberat kriteria berdasarkan keamatan kontras dan korelasi antara kriteria,

sekali gus mengurangkan bias subjektif dan menyelaraskan pemberat dengan struktur asas data keputusan. Bagi memperluas model GDM oleh Kamis *et al.* (2018), tiga ukuran keserupaan iaitu “cosine similarity”, “Manhattan similarity”, dan “Additive reciprocal preference (ARPS)” ditambah bagi menangkap pelbagai elemen kedekatan keutamaan pakar. Skor keserupaan ini kemudiannya diagregatkan menggunakan kaedah “Weighted Arithmetic Mean (WAM)”, “Induced Ordered Weighted Averaging (IOWA)”, dan “Weighted Ordered Weighted Averaging (WOWA)”. Rangka kerja yang dicadangkan ditunjukkan melalui kajian kes pemilihan sistem tenaga hibrid terbaik berdasarkan kriteria teknologi, ekonomi dan alam sekitar. Keputusan perbandingan menunjukkan bahawa pendekatan yang dicadangkan menghasilkan pemeringkatan alternatif yang konsisten dan stabil merentasi pelbagai metrik keserupaan dan operator pengagregatan, sekali gus menunjukkan keteguhan yang lebih tinggi dan kepekaan yang lebih rendah terhadap pertimbangan subjektif. Sumbangan utama kajian ini ialah pengintegrasian secara sistematik pemberatan objektif, pemodelan keutamaan berasaskan keserupaan dan teknik pengagregatan yang fleksibel ke dalam satu rangka kerja pembuatan keputusan. Integrasi ini meningkatkan kebolehpertahanan dan kebolehsuaian keputusan, menjadikan model yang dicadangkan sesuai untuk konteks multi-kriteria yang kompleks dengan pelbagai sudut pandangan penilaian.

Kata kunci: CRITIC; pembuatan keputusan pelbagai kriteria; pemberat kriteria; operator pengagregatan; ukuran keserupaan; sistem tenaga hibrid

1. Introduction

Multi-Criteria Decision Making (MCDM) is a structured decision analysis technique used to evaluate, compare, and rank alternatives by considering multiple criteria. More broadly, the increasing complexity of quantitative decision and optimization problems has motivated the development of efficient computational approaches for solving large-scale mathematical problems (Alhawarat *et al.* 2021). There are several established evaluation methods in the MCDM domain: the Analytic Hierarchy Process (AHP) for pairwise comparison-based weighting; Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) for ranking alternatives relative to ideal solutions; Decision-Making Trial and Evaluation Laboratory (DEMATEL) for modelling causal relationships among criteria; and Criteria Importance Through Intercriteria Correlation (CRITIC) for deriving objective criteria weights from data characteristics. However, despite their widespread use, several popular MCDM approaches rely substantially on subjective expert judgement, particularly during the criterion weighting stages. Subjective techniques, such as the AHP, are highly susceptible to individual perceptions and cognitive biases, leading to conflicting outcomes when multiple experts participate. This issue becomes more pressing in complex decision-making situations where competing criteria and diverse expert viewpoints coexist. A major issue in Group Decision-Making (GDM) contexts is the absence of an objective, reproducible mechanism for deriving criterion weights that accounts for heterogeneity in expert preference structures.

Notably, the CRITIC method was preferred in this study to address the limitations of subjective weighting systems. Unlike preference-based techniques, CRITIC calculates criterion weights objectively by taking into account both contrast intensity and inter-criteria correlation extracted directly from the decision matrix (Diakoulaki *et al.* 1995). This permits more discriminating criteria to be prioritised with less redundancy. CRITIC lowers bias and improves consistency by focusing on statistical data qualities rather than expert judgement, which is especially useful in GDM situations with variable criteria.

Aside from criterion weighting, expert preferences in collective decision-making are rarely identical and often exhibit varying degrees of overlap. Using a single similarity metric may only capture one component of preference alignment, resulting in an inadequate portrayal of expert judgement connections. Thus, to solve this restriction, this study employs several similarity metrics, including cosine similarity (Cosine), Manhattan similarity (Manhattan), and Additive Reciprocal Preference Similarity (ARPS). Each measure captures distinct structural aspects of preference closeness, enabling a more comprehensive assessment of expert alignment without requiring consensus-based changes (Liu & Yang 2022). Similarly, the choice of aggregation operator significantly impacts the final decision. Linear aggregation approaches, such as the Weighted Arithmetic Mean (WAM), assign equal weight to all ordered inputs. Ordered-based operators, such as Induced Ordered Weighted Averaging (IOWA) and Weighted Ordered Weighted Averaging (WOWA), offer greater flexibility by integrating attitudinal character, influence ordering, and weight reallocation. Using several aggregation operators enables the proposed methodology to examine ranking stability under various aggregation philosophies, hence increasing confidence in the final selection. Since they collectively manage many aspects of GDM, the selected similarity measures and aggregation operators are suitable for the proposed hybrid MCDM approach for building construction Zhu *et al.* (2021). ARPS maintains reciprocal preference structures, Manhattan similarity assesses the degree of disagreement, and cosine similarity records directional consistency across expert preferences. Subsequently, WOWA allows flexible integration of weighting and ordering effects, IOWA facilitates induced-preference ordering, and WAM provides a stable baseline aggregation procedure. Combining these methods enhances the resilience, flexibility, and dependability of the proposed decision-making framework.

Based on these considerations, this paper proposes a hybrid MCDM framework that integrates objective weighting using CRITIC, similarity-based preference modelling and aggregation operators such as the WAM, IOWA and WOWA. The framework is used to select a hybrid energy system based on technological, economic, and environmental criteria. The suggested workflow proceeds systematically from decision matrix construction to objective weighting, similarity computation, aggregation, and final ranking, providing a transparent and resilient approach to complex MCDM.

Specifically, Phase 1 contributes to Phase 2 by providing objectively derived criterion weights obtained using the CRITIC method. These weights are incorporated into the preference-similarity-based aggregation process in Phase 2 to ensure that the evaluation of alternatives reflects each criterion's relative importance. Therefore, Phase 1 establishes the weighting structure, while Phase 2 performs collective decision aggregation and final alternative ranking. The CRITIC method determines objective criterion weights based on contrast intensity and inter-criteria correlation. These weights are then integrated into the preference-similarity-based GDM model to reflect the relative importance of each criterion during aggregation and ranking. Consequently, the alternatives are evaluated based on expert preferences and on the objectively derived importance of the criteria.

The IOWA procedure is described in Section 3 (Implementation), Step 11, where the preference centrality index of each expert is calculated using Equation 15. In particular, these values represent each expert's relative influence and are subsequently arranged in descending order to guide the aggregation process.

This paper is structured as follows: Section 1 presents the introduction; Section 2 describes the methodology; Section 3 presents the implementation; Section 4 discusses the results and comparative analysis; and Section 5 concludes the study.

2. Methodology

The proposed work in this research consists of two phases. In Phase 1, the criteria weights are determined using the CRITIC method. In Phase 2, the rating of alternatives is performed using the preference similarity model proposed by Kamis *et al.* (2018). In this phase, the methodology was enhanced by implementing three different similarity measures, namely Cosine, Manhattan, and ARPS, to measure the closeness of experts' opinions. Another improvement is proposed, where three different aggregation operators, IOWA, WAM, and WOWA, are utilised for the purpose of unifying all individual experts' preferences into a collective one.

The robustness of the decision-making framework is enhanced by the inclusion of various similarity indicators, which enable comparison of their effectiveness. In particular, ARPS accounts for reciprocal preference consistency, Manhattan similarity assesses absolute differences in preference values, and cosine similarity focuses on directional agreement between experts' preferences Suhaeri *et al.* (2021). The use of these metrics enables a more thorough examination of expert agreement patterns and of whether various similarity structures affect the final ranking results. To investigate the effects of various aggregation behaviours on collective decision outcomes, this study also includes the WAM and WOWA operators as alternative aggregation methods. The WOWA operator simultaneously accounts for importance weights and ordered preference positions, providing greater flexibility in handling uncertainty and the diversity of preferences among experts. In contrast, the WAM operator offers a simple weighted-average procedure based solely on criterion importance. The suggested hybrid MCDM framework's adaptability and reliability are enhanced by integrating these aggregation operators, which also enable a comparative examination of aggregation effects.

In general, the proposed methodology is presented in Figure 1 below:

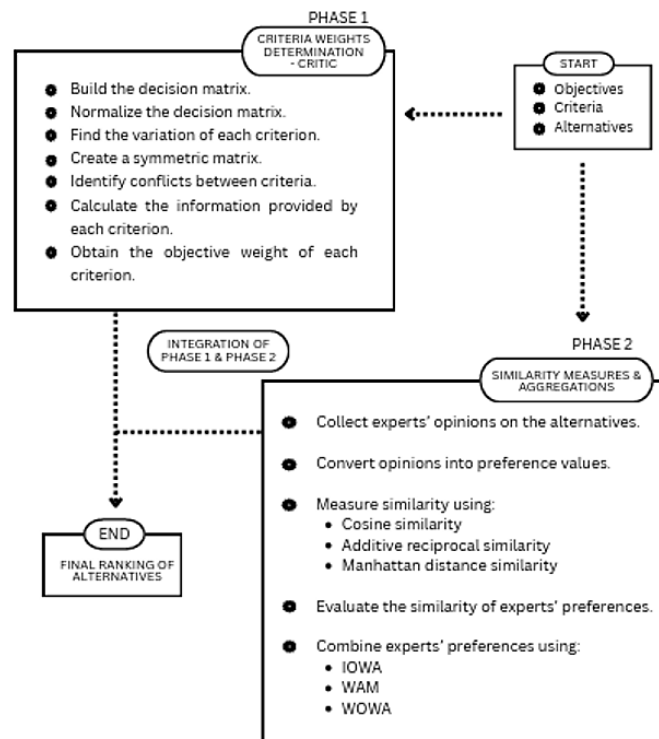


Figure 1: A unified critic-similarity hybrid weighting and aggregation model for multi-criteria decision making

Phase 1: Criteria weight determination using the CRITIC method

Consider a group of experts, $E = \{e_1, e_2, \dots, e_m\}$, who provide their judgements on a finite set of options, $X = \{X_1, X_2, \dots, X_n\}$ ($n > 2$).

The determination of criteria weights based on the CRITIC approach (Diakoulaki *et al.* 1995) can be demonstrated as follows:

Step 1: Construct a decision matrix, X .

$$X = X_{ij} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & x_{m3} \end{bmatrix}, \quad (1)$$

where X_{ij} ($X_{ij} \geq 0$) denotes the performance of the i^{th} alternative on the j^{th} criterion. The row indicates the i^{th} alternative and the column indicates the j^{th} criterion.

Step 2: Normalise the decision matrix, $\bar{x}_{ij}$.

$$\text{For beneficial criteria, } \bar{x}_{ij} = \frac{x_{ij} - x_j^{Worst}}{x_j^{Best} - x_j^{Worst}}. \quad (2)$$

$$\text{For non-beneficial criteria, } \bar{x}_{ij} = \frac{x_{ij} - x_j^{Best}}{x_j^{Worst} - x_j^{Best}}.$$

Step 3: Estimate the standard deviation (σ_j) for a sample of each criterion using:

$$\sigma = \sqrt{\frac{\sum (X - \bar{X})^2}{n-1}}, \quad (3)$$

where n is the number of criteria.

Step 4: Create a symmetric matrix of $n \times n$, γ , with a generic factor r_{jk} , which is the linear correlation coefficient vector between x_i and x_j , σ based on:

$$\gamma = \frac{\sum ((x - \bar{x})(y - \bar{y}))}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}, \quad (4)$$

where y is the criterion in the column and $\bar{y}$ is the correlation of the criteria in the column.

Step 5: Determine the measure of conflict caused by criterion j in the decision situation defined by the remaining criteria:

$$\sum_{k=1}^m (1 - r_{jk}), \quad (5)$$

where r_{jk} is the linear correlation coefficient between the vectors x_i and x_j .

Step 6: Calculate the quantity of information concerning each criterion, C_i :

$$C_j = \sigma_j \sum_{k=1}^m (1 - r_{jk}), \quad (6)$$

where σ_j is the standard deviation of each criterion.

Step 7: Estimate the objective weights of each criterion, w_j :

$$w_j = \frac{C_j}{\sum_{k=1}^m C_j}. \quad (7)$$

2.1. Rating of Alternatives Based on Preference Similarity Decision-Making Model (Kamis et al. 2018) Using Different Similarity Measures and Aggregation Operators

The rating of alternatives was performed to systematically evaluate and compare each option against the established criteria, thereby identifying the most suitable alternative. In this phase, Steps 8 through 16 are described as follows:

Step 8: A group of experts, $E = \{e_1, e_2, \dots, e_m\}$, give their opinions towards a set of alternatives, $A = \{A_1, A_2, \dots, A_n\}$ in the form of the Reciprocal Preference Relation (RPR), (Definition 1).

Definition 1 (Kamis et al. 2018). An RPR on Y is a fuzzy binary relation P where the experts' preference intensity of alternative i over alternative j , $P(y_i, y_j) = p_{ij}$, verifies that $P(y_i, y_j) = 0.5 \forall y_i \in$ and $\sum P_{ij} = 1, \forall y_i, y_j \in Y$.

Let $P_{n \times n}$ refer to the set of $n \times n$ matrices P constructed from all the RPRs on Y :

$$\begin{bmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{bmatrix}, \quad (8)$$

verify that $0 \leq p_{ij} \leq 1$ and $\sum P_{ij} = 1$ for $i, j \in \{1, 2, \dots, n\}$.

Step 9: Extract the RPR entries into the Intensity Preference Vector (IV) (Definition 2).

Definition 2 (Kamis et al. 2018). The experts' IV of an RPR $P (P_{ij})_{n \times n} \in P_{n \times n}$ is the vector of dimension $n(n-1)/2$, IV , with components of the vector that consist of the elements above the main diagonal:

$$IV = (p_{12}, p_{13}, p_{1n}, p_{23}, \dots, p_{2n}, \dots, p_{n(n-1)}). \quad (9)$$

Step 10: Calculate the similarity index based on:

(a) Cosine Similarity Measure

The cosine similarity between the preferences of expert e^p and e^q is calculated using the formula proposed by Kamis et al. (2018):

$$CS^{pq} = cs(iv^p, iv^q) = \frac{\sum_{i=1}^{n(n-1)/2} (iv^p \cdot iv^q)}{\sqrt{\sum_{i=1}^{n(n-1)/2} (iv^p)^2 \cdot \sum_{i=1}^{n(n-1)/2} (iv^q)^2}}, \quad (10)$$

where iv^p = the IV of expert e^p , and iv^q = the IV of expert e^q .

(b) Manhattan Distance Similarity measure (Minkowski 1896)

The Manhattan distance is expressed mathematically as:

$$d_{ij} = \sum_{l=1}^m |e_{il} - e_{jl}|, \quad (11)$$

where e are expert preference vectors. A smaller value indicates greater similarity, while a larger value reflects higher disagreement.

$$S_{pq}^{(Man)} = 1 - \frac{d_{pq}}{2}. \quad (12)$$

(c) Additive Reciprocal Preference Similarity (ARPS) (Herrera *et al.* 1996)

The ARPS function is presented as follows:

$$S_{pq}^{ARPS} = 1 - \frac{D_{pq}}{x(x-1)}, \quad (13)$$

where $S_{pq}^{ARPS} \in [0,1]$ and x is the number of alternatives. D_{pq} is the absolute divergence between expert p and q ,

where

$$D_{pq} = \sum_{p \neq q} |x_{ij} + x_{ji}|, \quad (14)$$

in which x_{ij} is the preference value assigned by expert p , indicating how much alternative i is preferred over alternative j . Meanwhile, x_{ji} is the preference value assigned by expert q , denoting how much alternative j is preferred over alternative i .

Step 11: Measure the experts' preference similarity centrality index using the formula by Kamis *et al.* (2018):

$$A(e^p) = \frac{1}{e-1} \sum_{q=1}^e CS^{pq}, \quad (15)$$

where e is the number of experts, and CS^{pq} = cosine similarity measure of expert e^p and expert e^q .

Step 12: Aggregate all individual preferences into a collective one using:

(a) IOWA operator (Yager 1986)

The weights of the quantifier linguistic IOWA operator are defined as:

$$w_i = Q\left(\frac{i}{e}\right) - Q\left(\frac{i-1}{e}\right), i = 1, \dots, e, \quad (16)$$

where i is the index indicating the position of each element and e is the number of experts.

The final IOWA weighting vector is further determined by the weighting vector using the formula below (Yager 1986):

$$w_i = Q\left(\frac{\sum_{k=1}^i \mu_{\sigma}(k)}{D}\right) - Q\left(\frac{\sum_{k=1}^{i-1} \mu_{\sigma}(k)}{D}\right), \quad (17)$$

where D = the total sum of importance, and $\mu_{\sigma}(k)$ = permutation employed to obtain the ordering values.

(b) Weighted Arithmetic Mean (WAM) (Keeney & Raiffa 1976)

The WAM is presented as:

Compute group score for each alternative:

$$WAM(x) = \sum_{i=1}^n w_i x_i, \quad (18)$$

where x_i are the inputs (e.g., experts' evaluations) and w_i are their associated weights, with $\sum w_i = 1$.

WAM is easy to interpret, ensures fairness when equal weights are used, and provides a straightforward way to aggregate information in GDM.

(c) Weighted Ordered Weighted Averaging (WOWA) (Torra 2000)

The WOWA formula is designed as:

$$WOWA(x): \sum_{i=1}^n w_i x_i, \quad (19)$$

where x_i is the alternative scores x_{il} sorted in decreasing order to produce ordered values $b_l \geq \dots \geq b_n$, and ω_i are WOWA weights derived from both criteria weights and order weights.

This makes WOWA more flexible than WAM, especially in decision contexts requiring a balance between importance and rank sensitivity.

Step 13: Determine the average of alternative weights by:

$$Avg = \frac{\sum_{j=1}^n p_{ij}^n}{n}, \quad (20)$$

where n = number of alternatives, and p_{ij}^n = collective preference relation of alternatives.

Step 14: Calculate the weighted alternative rating using the formula:

$$W = \sum_{i=1}^m w_i A_i, \quad (21)$$

where $w_i A_i$ = the weight of alternative rating.

Step 15: Aggregate the weighted expert ratings into a collective rating:

$$Z = \frac{1}{m} \sum_{i=1}^m w_i A_i, \quad (22)$$

where m is the number of experts.

Step 16: Rank the alternatives in descending order.

The alternatives are ranked based on the average of aggregated weighted preference similarity. Note that the higher the average value, the higher the ranking position.

2.2. Implementation of the Proposed Model

Secondary data from Ali *et al.* (2020) were used to illustrate the suggested model and verify the findings. In this case study, 13 criteria and six experts are presented. The data on the alternatives with respect to the criteria are presented in Table 1 below.

Table 1: Evaluated criteria data values (Ali *et al.* 2020)

Criteria for a hybrid energy system	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀	C ₁₁	C ₁₂	C ₁₃
e_1	75688	21464	323510	0.28	60	4	4	65193.2	3	3	3	4	4
e_2	96188	21343	320399	0.29	61	5	5	3489434	4	2	2	5	5
e_3	49375	22468	360834	0.28	56	3	2	39018.1	5	4	4	3	2
e_4	81813	32851	618339	0.41	12	3	3	78060	4	5	5	4	3
e_5	267438	11151	48737	0.28	100	2	3	643.24	3	3	3	4	3
e_6	37350	33869	689442	0.4	0	3	2	87036	2	5	5	3	1

The decision matrix, X , can be constructed (Step 1) and normalised (Step 2). C_5, C_6, C_{11}, C_{12} , and C_{13} are beneficial criteria, whereas the rest are non-beneficial criteria.

Since the criteria may have different measurement ranges, the CRITIC technique normalises the decision matrix to a common scale, typically 0-1. For benefit criteria, where higher values are preferred, normalisation is calculated by subtracting the worst value from each criterion value and dividing it by the difference between the best and worst values. For cost criteria, where lower values are preferred, the normalisation process is reversed to ensure that lower criterion values produce higher normalised scores. This standardisation enables the CRITIC framework to accurately determine objective criterion weights and consistently evaluate all criteria. As demonstrated in Step 3, the standard deviation σ_j for each criterion is computed using Equation 3.

In Step 4, a symmetric correlation matrix, γ , is developed by applying Equation 4 to compute the Pearson linear correlation coefficients between each pair of criteria. This matrix provides insight into the degree of redundancy or conflict among criteria.

$$\gamma = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 & C_{10} & C_{11} & C_{12} & C_{13} \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \\ C_{12} \\ C_{13} \end{matrix} & \begin{bmatrix} 1 & -0.78 & -0.79 & -0.38 & -0.76 & 0.46 & 0.16 & -0.05 & -0.15 & -0.42 & 0.42 & -0.35 & -0.24 \\ -0.78 & 1 & 1 & 0.88 & 1 & -0.12 & -0.33 & -0.13 & -0.14 & -0.77 & -0.77 & -0.35 & -0.44 \\ -0.79 & 1 & 1 & 0.87 & 1 & -0.11 & -0.35 & -0.13 & -0.17 & -0.77 & -0.77 & 0.39 & 0.47 \\ -0.38 & 0.88 & 0.87 & 1 & 0.89 & 0.20 & -0.39 & -0.24 & -0.33 & -0.82 & -0.82 & 0.28 & 0.49 \\ 0.76 & 1 & 1 & 0.89 & 1 & -0.08 & -0.36 & -0.15 & -0.21 & -0.78 & -0.78 & 0.39 & 0.49 \\ 0.46 & -0.12 & -0.11 & 0.2 & -0.08 & 1 & -0.77 & -0.8 & -0.18 & -0.53 & -0.53 & 0.6 & 0.68 \\ 0.16 & -0.33 & -0.35 & -0.39 & -0.36 & -0.77 & 1 & 0.77 & 0.08 & 0.8 & 0.8 & -0.95 & -0.97 \\ -0.05 & -0.13 & -0.13 & -0.24 & -0.15 & -0.8 & 0.77 & 1 & 0.23 & 0.66 & 0.66 & -0.76 & -0.69 \\ -0.15 & -0.14 & -0.17 & -0.33 & -0.21 & -0.18 & 0.08 & 0.23 & 1 & 0.16 & 0.16 & -0.13 & -0.27 \\ -0.42 & -0.77 & 0.77 & 0.82 & 0.78 & 0.53 & -0.8 & -0.66 & -0.16 & 1 & 1 & 0.73 & 0.82 \\ 0.42 & 0.77 & -0.77 & -0.82 & -0.78 & -0.53 & 0.8 & 0.66 & 0.16 & 1 & 1 & -0.73 & -0.82 \\ -0.354 & -0.45 & -0.39 & 0.28 & 0.39 & 0.6 & -0.95 & -0.76 & -0.13 & -0.73 & -0.73 & 1 & 0.94 \\ -0.24 & -0.44 & -0.49 & 0.49 & 0.49 & 0.68 & -0.97 & -0.69 & -0.27 & -0.82 & -0.82 & 0.94 & 1 \end{bmatrix} \end{bmatrix}$$

Following Step 4, Step 5 determines the conflict generated by each criterion in relation to the remaining criteria using Equation 5. The resulting pairwise conflict values are presented in Table 2, with the row sums indicating the total conflict intensity associated with each criterion.

Table 2: Conflict caused by the criterion

Criteria for a hybrid energy system	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀	C ₁₁	C ₁₂	C ₁₃	Sum of all criteria
C ₁	0.00	1.78	1.79	1.38	1.76	0.54	0.84	1.05	1.15	1.42	0.58	1.35	1.24	14.867
C ₂	1.78	0.00	0.00	0.12	0.00	1.12	1.33	1.13	1.14	0.23	1.77	0.65	0.56	9.840
C ₃	1.79	0.00	0.00	0.13	0.00	1.11	1.35	1.13	1.17	0.23	1.77	0.61	0.53	9.828
C ₄	1.38	0.12	0.13	0.00	0.11	0.80	1.39	1.24	1.33	0.18	1.82	0.72	0.51	9.731
C ₅	1.76	0.00	0.00	0.11	0.00	1.08	1.36	1.15	1.21	0.22	1.78	0.61	0.51	9.797
C ₆	0.54	1.12	1.11	0.80	1.08	0.00	1.77	1.80	1.18	0.47	1.53	0.40	0.32	12.120
C ₇	0.84	1.33	1.35	1.39	1.36	1.77	0.00	0.23	0.92	1.80	0.20	1.95	1.97	15.106
C ₈	1.05	1.13	1.13	1.24	1.15	1.80	0.23	0.00	0.77	1.66	0.34	1.76	1.69	13.949
C ₉	1.15	1.14	1.17	1.33	1.21	1.18	0.92	0.77	0.00	1.16	0.84	1.13	1.27	13.274
C ₁₀	1.42	0.23	0.23	0.18	0.22	0.47	1.80	1.66	1.16	0.00	2.00	0.27	0.18	9.813
C ₁₁	0.58	1.77	1.77	1.82	1.78	1.53	0.20	0.34	0.84	2.00	0.00	1.73	1.82	16.188
C ₁₂	1.35	0.65	0.61	0.72	0.61	0.40	1.95	1.76	1.13	0.27	1.73	0.00	0.06	11.238
C ₁₃	1.24	0.56	0.53	0.51	0.51	0.32	1.97	1.69	1.27	0.18	1.82	0.06	0.00	10.656

According to Step 6, the information content of each criterion is computed by multiplying its standard deviation by the sum of its conflict values, as presented in Table 2. These values reflect each criterion's overall contribution to the decision environment. As demonstrated in Step 7, the objective CRITIC weights (Table 3) are obtained by normalising the information content values using Equation 7. The resulting weights for the 13 criteria are presented below:

Table 3: The objective weights of each criterion

Criteria for a hybrid energy system	STD	SUM	STD*SUM	Objective weights
C ₁	0.366	14.867	5.448	0.097
C ₂	0.373	9.840	3.665	0.065
C ₃	0.362	9.828	3.554	0.063
C ₄	0.487	9.731	4.739	0.085
C ₅	0.366	9.797	3.578	0.064
C ₆	0.345	12.120	4.186	0.075
C ₇	0.390	15.106	5.899	0.105
C ₈	0.270	13.949	3.771	0.067
C ₉	0.253	13.274	3.356	0.060
C ₁₀	0.390	9.813	3.834	0.068
C ₁₁	0.405	16.188	6.546	0.117
C ₁₂	0.376	11.238	4.231	0.076
C ₁₃	0.303	10.656	3.228	0.058
Sum C _j			56.034	

After Phase 1 is completed, Phase 2 can proceed. In Step 8, the analysis considers the set of alternatives = $\{A_1, A_2, A_3\}$ and the group of six experts $E = \{e_1, \dots, e_6\}$ and evaluations of alternatives by each expert, $P = \{p_1, p_2, \dots, p_6\}$.

A_1, A_2 , and A_3 consists of three influencing alternatives, and a group of six experts stating their opinions on each one.

$$\begin{array}{ccc}
 A_1 & A_2 & A_3 \\
 p_1 = \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} \begin{bmatrix} 1 & \mathbf{0.2} & \mathbf{0.4} \\ 0.8 & 1 & \mathbf{0.9} \\ 0.6 & 0.1 & 1 \end{bmatrix}, & & \begin{matrix} A_1 & A_2 & A_3 \\ P^4 = \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} \begin{bmatrix} 1 & 0.5 & 0.6 \\ 0.5 & 1 & 0.9 \\ 0.4 & 0.1 & 1 \end{bmatrix}, \\
 \\
 A_1 & A_2 & A_3 \\
 p_2 = \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} \begin{bmatrix} 1 & 0.4 & 0.7 \\ 0.6 & 1 & 0.8 \\ 0.3 & 0.2 & 1 \end{bmatrix}, & & \begin{matrix} A_1 & A_2 & A_3 \\ P^5 = \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} \begin{bmatrix} 1 & 0.8 & 0.2 \\ 0.2 & 1 & 0.7 \\ 0.8 & 0.3 & 1 \end{bmatrix}, \\
 \\
 A_1 & A_2 & A_3 \\
 P^3 = \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} \begin{bmatrix} 1 & 0.9 & 0.8 \\ 0.1 & 1 & 0.4 \\ 0.2 & 0.6 & 1 \end{bmatrix}, & & \begin{matrix} A_1 & A_2 & A_3 \\ P^6 = \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} \begin{bmatrix} 1 & 0.1 & 0.9 \\ 0.9 & 1 & 0.4 \\ 0.1 & 0.6 & 1 \end{bmatrix}.
 \end{array}$$

As outlined in Step 9, the reciprocal preference matrices from Step 8 are transformed into IVs. These vectors reflect each expert's degree of preference across the three alternatives.

$$\begin{array}{lll}
 IV_1 = (0.2, 0.4, 0.9), & IV_2 = (0.4, 0.7, 0.8), & IV_3 = (0.9, 0.8, 0.4), \\
 IV_4 = (0.5, 0.6, 0.9), & IV_5 = (0.8, 0.2, 0.7), & IV_6 = (0.1, 0.9, 0.4).
 \end{array}$$

Moreover, based on Step 10(a), cosine similarity values are computed between all pairs of IVs using Equation 10. This step produces the cosine similarity matrix, which quantifies the alignment of the expert judgement matrix as presented below.

$$CS = \begin{bmatrix} 1 & 0.9462 & 0.6744 & 0.9603 & 0.8003 & 0.744 \\ 0.9462 & 1 & 0.860 & 0.990 & 0.830 & 0.880 \\ 0.6744 & 0.860 & 1 & 0.853 & 0.845 & 0.772 \\ 0.9603 & 0.990 & 0.853 & 1 & 0.892 & 0.805 \\ 0.800 & 0.830 & 0.845 & 0.892 & 1 & 0.504 \\ 0.744 & 0.880 & 0.772 & 0.805 & 0.504 & 1 \end{bmatrix}.$$

The implementation of the different similarity measures (Steps 10(b) and 10(c)) can be performed as described in the methodology section. The results from these measures are used in Steps 11-16.

In Step 11, the preference centrality index of each expert is calculated using Equation 15. These values indicate each expert's relative influence and are then arranged in descending order to guide the aggregation process.

$$\begin{array}{l}
 A(e^1) = 0.82504, A(e^2) = 0.901, A(e^3) = 0.8011, A(e^4) = 0.9002, A(e^5) = 0.774, \\
 A(e^6) = 0.741.
 \end{array}$$

According to Step 12(a), cumulative centrality values are derived to produce the collective preference relation. Using these values, the expert weighting vector is normalised and applied within the IOWA operator to obtain the aggregated reciprocal fuzzy preference relation.

The weighting vector is:

$$(0.1823, 0.3644, 0.5313, 0.6933, 0.8503, 1.0).$$

Using the cumulative centrality values, the normalised expert weighting vector is obtained. Notably, these weights indicate the relative contribution of each expert during the aggregation process, in which larger values represent a stronger influence on the collective group preference relation, as defined by the formula below:

$$w_i = \frac{cc_i}{\sum_{i=1}^n cc_i},$$

where cc_i = expert importance

Following this, the collective preference relation can be determined using the IOWA operator: $\emptyset_{\omega}^{\sigma}$,

$$w_1 = 0.1823, w_2 = 0.1821, w_3 = 0.1669, w_4 = 0.1620, w_5 = 0.1570, w_6 = 0.1497.$$

The collective preference relation, Z , of all experts in pairwise comparisons, is:

$$Z = \begin{bmatrix} 1 & 0.481 & 0.597 \\ 0.519 & 1 & 0.691 \\ 0.403 & 0.309 & 1 \end{bmatrix}.$$

Another two aggregation operators, IOWA and WOWA, can also be implemented. The steps are continued as described in the methodology section.

As described in Step 13, the average preference for each alternative is computed by averaging the aggregated RPR values. The results are 0.693, 0.734, and 0.571 for the respective alternatives, A_1 , A_2 , and A_3 . Notably, these averages provide the baseline preference levels for Alternatives 1, 2, and 3.

Steps 14-16 are implemented, and the final ranking of alternatives is obtained, as in Table 4.

Table 4: Rank of the alternatives

Alternative	Raw score	Normalised score	Ranking of alternatives
A_1	0.2310	0.3466	2
A_2	0.2457	0.3686	1
A_3	0.1901	0.2850	3

The ranking order of the alternatives is $A_2 > A_1 > A_3$, as presented in Table 4. This result reveals that A_2 is the most preferred alternative, followed by A_1 , with A_3 being the least preferred.

3. Results and Comparative Analysis

The cumulative results of the proposed work with three different similarity measures, cosine similarity, Manhattan similarity, and ARPS and three different aggregation operators, IOWA, WAM, and WOWA, are presented in Table 5. The results reveal that the numerical scores vary

significantly across techniques, yet all nine configurations consistently produce the same ranking order: $A_2 > A_1 > A_3$.

Table 5: The final ranking of alternatives by different similarity measures and aggregation operators

Similarity Measure	Aggregation Operator	A ₁	A ₂	A ₃	Final Ranking
Cosine	WAM	0.34	0.367	0.293	$A_2 > A_1 > A_3$
	IOWA	0.345	0.365	0.29	$A_2 > A_1 > A_3$
	WOWA	0.338	0.37	0.292	$A_2 > A_1 > A_3$
ARPS	WAM	0.33	0.39	0.28	$A_2 > A_1 > A_3$
	IOWA	0.335	0.388	0.277	$A_2 > A_1 > A_3$
	WOWA	0.332	0.392	0.276	$A_2 > A_1 > A_3$
Manhattan	WAM	0.342	0.368	0.29	$A_2 > A_1 > A_3$
	IOWA	0.34	0.37	0.29	$A_2 > A_1 > A_3$
	WOWA	0.339	0.372	0.289	$A_2 > A_1 > A_3$

A sensitivity analysis was conducted to evaluate the robustness and stability of the proposed hybrid MCDM framework under different similarity measures and aggregation operators.

The average score, $\bar{x}_{A_i}$ for each alternative is calculated using:

$$\bar{x}_{A_i} = \frac{\sum x_i}{n},$$

where x_i is the ranking score of alternative A_i with respect to 3 different similarity measures and 3 different aggregation operators and n is the total no. of ranking scores measured.

The sensitivity range is calculated using:

$$\text{Sensitivity Range} = \max(x_i) - \min(x_i)$$

Table 6 presents the average scores, maximum scores, minimum scores and sensitivity range of 3 alternatives.

Table 6: Sensitivity range of 3 alternatives

Alternative	Average Score, $\bar{x}_{A_i}$	Maximum Score, max (x_i)	Minimum Score, min (x_i)	Sensitivity Range
A ₁	0.338	0.330	0.345	0.015
A ₂	0.376	0.365	0.392	0.027
A ₃	0.286	0.276	0.293	0.017

The stability of the proposed hybrid MCDM framework across various similarity measures and aggregation operators was assessed through a sensitivity analysis. Additionally, three similarity measures, cosine similarity, ARPS, and Manhattan similarity, as well as three aggregation operators, which are WAM, IOWA, and WOWA, were used in the analysis to compare the final ranking results, as indicated in Table 6. The findings demonstrate that the ranking trend remained consistent across all combinations, with A_2 consistently ranking first, followed by A_1 and A_3 . The overall ranking order remained the same despite minor variations in the numerical points. Therefore, this finding suggests that the proposed model is robust and stable even when aggregation operators and similarity measures are modified.

This consistent result indicates the strength of the CRITIC similarity aggregation framework. The Cosine and Manhattan similarity measures produce very mild score differences. However, the ARPS demonstrates a slightly greater differentiation between options, especially when paired with the WOWA operator. The dispersion of final scores is also

influenced by aggregation operators, with WAM producing the most balanced outputs, IOWA reflecting expert influence based on preference centrality, and WOWA increasing contrast via nonlinear weighting.

Figure 2 compares the performance of three alternatives (A_1 , A_2 , A_3) using three similarity measures: Cosine, ARPS as well as Manhattan and aggregation operators, which are WAM, IOWA, and WOWA.

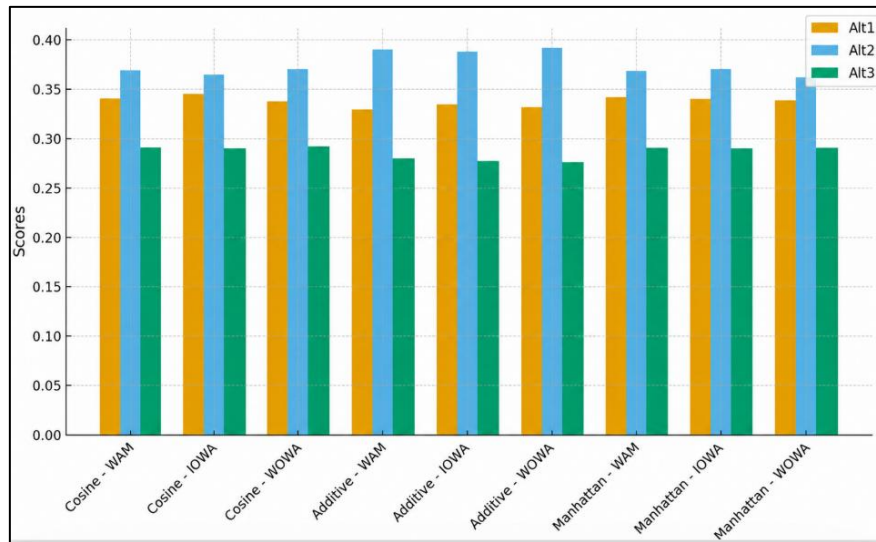


Figure 2: Bar chart Comparison of alternative performance scores under different similarity measures and aggregation operators

The grouped bar chart clearly depicts the final weighted scores for each method configuration, enabling a straightforward comparison of how each alternative performs across different analytical settings. Alternative A_2 consistently outperforms the other alternatives in all nine combinations, regardless of approach. A_1 is the second-highest option, whereas A_3 consistently has the lowest score. Furthermore, the graphic demonstrates the resilience of the decision-making conclusion, as the ranking order remains constant across different similarity measures and aggregation operators. This visual depiction lends weight to the conclusion that the proposed hybrid CRITIC-similarity-aggregation framework yields stable, reliable decisions across a range of methodological contexts.

The alternative scores obtained using the CRITIC weighting approach with various similarity measures and aggregation operators are illustrated in Figure 2. By accounting for both criterion variability and interactions, the CRITIC technique reduces reliance on subjective expert judgement and enables objective criterion weighting. According to the findings, A_2 consistently received the highest ratings across all combinations of similarity measures and operators, demonstrating its resilience as the recommended option. Although A_1 was still relatively competitive, A_3 had the lowest scores. The comparison of WAM, IOWA, and WOWA operators indicates that different aggregation algorithms have a minor impact on ranking intensity and the final score distribution. Specifically, this is because it can account for both importance weights and ordered positions simultaneously. The WOWA operator reflected greater discrimination across alternatives, whereas WAM produced more balanced aggregate outcomes. Overall, the results reveal that the flexibility and dependability of the decision-making framework are improved by integrating CRITIC with different aggregation operators.

Figure 3 illustrates a radar chart comparing the performance of three options, A_1 , A_2 , and A_3 , using three similarity measures. Notably, the radar visualisation provides a comprehensive perspective on how each alternative performs across the three similarity assessment methodologies, mapping their aggregated scores onto multiple axes.

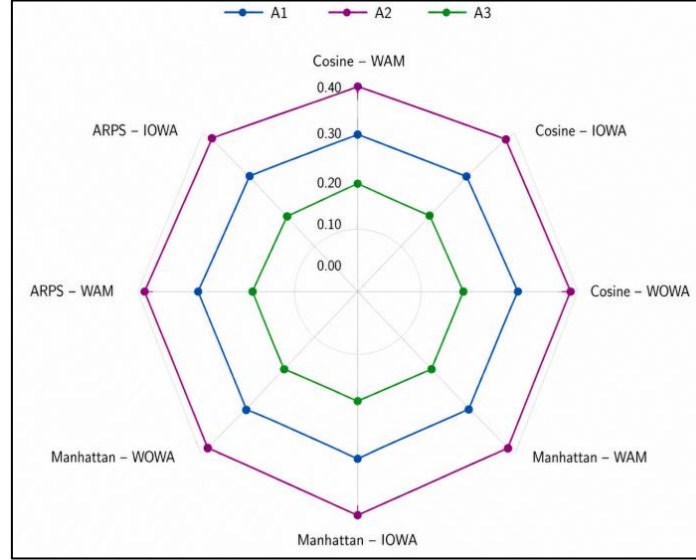


Figure 3: Radar chart comparing alternative performance across different similarity measures and aggregation operators

The radar map supports the numerical results by highlighting the stability of the ranking pattern, with A_2 outperforming A_3 in all similarity techniques. This visualisation improves the interpretability of the comparison study and supports the robustness of the findings.

The radar chart of alternative scores derived from various similarity metrics and aggregation operators is demonstrated in Figure 3. According to the chart, A_2 is the most stable and favoured option within the suggested framework since it consistently produced the best results across all assessed combinations. However, while A_1 performed moderately, A_3 had the lowest results. The rather stable pattern across the radar dimensions implies that reliable decision-making outcomes are obtained when the CRITIC weighting method is combined with various similarity metrics and aggregation operators. Furthermore, small changes in the radar shape indicate that the aggregation operators affect the final ranking intensity by influencing the distribution and discrimination of alternative scores.

3. Conclusion

The research presents a hybrid MCDM framework that integrates CRITIC objective weighting with a preference-similarity-based decision-making model, involving different similarity measures and aggregation operators. In particular, the proposed model produced consistent rankings across different similarity measures and aggregation operators when applied to hybrid energy system selection, demonstrating its strong reliability. The framework addresses the limitations of previous models by combining criteria weight determination and preference merging based on the similarity of experts' opinions. Therefore, in the future, other hybrid models can be introduced and implemented in large-scale decision-making environments with diverse real-world applications.

Acknowledgements

The authors sincerely thank the reviewers for the valuable comments and suggestions. The author also thanks everyone who supported this research.

References

- Ali T., Ma, H. & Nahian, A. J. (2020). A multi-criteria decision-making approach to determine the optimal hybrid energy system in coastal off-grid areas: A case study of Bangladesh. *Process Integration and Optimisation for Sustainability* 4(3): 265–277.
- Alhawarat, A., Alolaiyan, H., Masmali, I. A. H., Salleh, Z., & Ismail, S. (2021). A descent four-term of Liu and Storey conjugate gradient method for large scale unconstrained optimization problems. *European Journal of Pure and Applied Mathematics*, 14(4): 1429–1456.
- Diakoulaki, D., Mavrotas, G. & Papayannakis, L. (1995). Determining objective weights in multiple criteria problems: The CRITIC method. *Computers & Operations Research* 22(7): 763–770.
- Herrera F., Herrera-Viedma E. & Verdegay J.L. 1996. A model of consensus in group decision making under linguistic assessments. *Fuzzy Sets and Systems* 78(1): 73–87.
- Kamis N.H., Chiclana F. & Levesley J. 2018. Preference similarity network structural equivalence clustering-based consensus group decision making model. *Applied Soft Computing* 67: 706-720.
- Keeney R.L. & Raiffa H. 1976. *Decision Making with Multiple Objectives: Preferences and Value Tradeoffs*. New York: Wiley.
- Liu P., Li Y., Zhang X. & Pedrycz W. 2022. A multiattribute group decision-making method with probabilistic linguistic information based on an adaptive consensus reaching model and evidential reasoning. *IEEE Transactions on Cybernetics* 53(3): 1905–1919.
- Minkowski H. 1896. *Geometrie der Zahlen*. Leipzig: Teubner.
- Suhaeri M.E., Alimudin Javaid A., Ismail M.T. & Ali M.K.M. 2021. Evaluation of clustering approach with euclidean and Manhattan distance for outlier detection. *AIP Conference Proceedings*, p. 070025.
- Torra V. 2000. The WOWA operator and the interpolation function W^* : Chen and Otto's interpolation method revisited. *Fuzzy Sets and Systems* 113(3): 389-396.
- Yager R.R. 1986. On the theory of bags. *International Journal of General Systems* 13(1): 23–37.
- Zhu X., Meng X. & Zhang M. 2021. Application of multiple criteria decision making methods in construction: A systematic literature review. *Journal of Civil Engineering and Management* 27(6): 372-403.

Department of Mathematical Sciences
Faculty of Computer and Mathematical Sciences
Universiti Teknologi MARA (UiTM)
40450 Shah Alam
Selangor, MALAYSIA
E-mail: faqihahnurulzulkifli@gmail.com, norhanimah@fskm.uitm.edu.my*

Received: 27 January 2026

Accepted: 24 July 2026

*Corresponding author