

IVPNSDWGA: AN ENHANCED DOMBI-BASED INTERVAL-VALUED PYTHAGOREAN NEUTROSOPHIC AGGREGATION MODEL FOR SDG-DRIVEN SUSTAINABLE INVESTMENT DECISION

(IVPNSDWGA: Model Pengagregatan Nueutrosifik Pythagoras Selang Nilai Berasaskan Dombi yang Dipertingkat untuk Keputusan Pelaburan Lestari Berteraskan SDG)

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ABSTRACT

Decision-making in the area of sustainable and green finance often depends on information that is uncertain and difficult to express using conventional numerical methods. This issue becomes more apparent when evaluating environmental, social, and governance (ESG) indicators, as expert judgements in this context are typically imprecise and may overlap with one another. To address this limitation, the present study proposed a novel aggregation mechanism for Interval Valued Pythagorean Neutrosophic Sets (IVPNS), referred to as the Interval-Valued Pythagorean Neutrosophic Dombi Weighted Geometric Average (IVPNSDWGA) aggregation operator. The proposed aggregation operator incorporates Dombi t-norm and t-conorm functions into the IVPNS framework. It enables a more realistic representation of interval values based on linguistic information and the inherent uncertainty in expert evaluations. The mathematical properties of the operator, including idempotency, monotonicity, and boundedness, are examined to ensure its consistency and suitability in multi-attribute decision-making environments. The proposed operator is applied to demonstrate the practical applicability in the sustainable investment problem involving the evaluation of green-oriented companies. The case study evaluates the green sector and alignment with selected Sustainable Development Goals (SDGs). The findings demonstrate that the proposed operator produces consistent, stable rankings even when the input data are imprecise. The IVPNSDWGA offers better alternative discrimination than current aggregation techniques, particularly when expert evaluations involve overlapping or uncertain information. These results indicate that the proposed operator offers a practical and reliable tool for sustainability-driven financial decisions, where uncertainty plays a central role.

Keywords: interval-valued Pythagorean neutrosophic set; sustainable investment; ESG decision-making; Dombi t-norm; Dombi t-conorm; multi-attribute decision-making; uncertainty modeling

ABSTRAK

Pembuatan keputusan dalam kewangan Lestari dan pembiayaan hijau lazimnya sering kali bergantung kepada maklumat yang tidak pasti dan sukar dinyatakan menggunakan kaedah berangka konvensional. Kesukaran ini lebih ketara apabila penilaian melibatkan indikator Alam Sekitar, Sosial dan Tadbir Urus (ESG), kerana pertimbangan pakar sering mengandungi tahap ketaksaan serta pertindihan penilaian. Bagi menangani kekangan tersebut, kajian ini membangunkan satu mekanisme pengagregatan baharu bagi Set Selang Nilai Pythagoras Neutrosophic (IVPNS), iaitu Selang Nilai Purata Geometri Berwajaran Pythagoras Neutrosophic Dombi (IVPNSDWGA) operator. Operator pengagregatan yang dicadangkan ini menggabungkan fungsi T-norm dan t-conorm Dombi dalam kerangka IVPNS bagi membolehkan maklumat linguistic selang nilai dan ketidakpastian pakar dimodelkan dengan lebih realistic. Tingkah laku teori pengendali diperiksa melalui beberapa sifat matematik termasuk *idempotency*, *monotonicity* dan *boundedness*, bagi memastikan ianya konsisten dan

kesesuaiannya dalam persekitaran pembuatan keputusan pelbagai atribut. Operator yang dicadangkan ini diaplikasikan untuk menunjukkan kebolehpakaian praktikal dalam masalah pelaburan Lestari yang melibatkan penilaian syarikat berorientasikan hijau. Kajian kes ini menilai sektor hijau dan penjarangannya dengan Matlamat Pembangunan Mampan (SDG) terpilih. Penemuan menunjukkan bahawa operator yang dicadangkan menghasilkan keputusan kedudukan yang tekal dan stabil walaupun data input adalah bersifat kabur. IVPNSDWGA menawarkan diskriminasi alternatif yang lebih baik berbanding teknik pengagregatan semasa, terutamanya apabila penilaian pakar melibatkan maklumat yang bertindih atau tidak pasti. Dapatan ini membuktikan bahawa operator yang dicadangkan merupakan instrumen praktikal dan berpotensi bagi pembuatan keputusan kewangan berpaksikan kemampanan, terutamanya dalam mendepani unsur ketidakpastian yang kritikal.

Kata kunci: selang nilai set Pythagoras neutrosophic; pelaburan mampan; pembuatan keputusan ESG; Dombi t-norm, Dombi t-conorm; operator pengagregatan; pembuatan keputusan pelbagai atribut; permodelan ketidakpastian

1. Introduction

1.1. *The Role of Multi-Attribute Decision-Making (MADM) in Sustainable and Green Finance*

In recent years, researchers have shown growing enthusiasm for applying Multi-Attribute Decision-Making (MADM) techniques within the domains of green investment and sustainable finance. These tools provide systematic approaches to evaluating complex sustainability-related factors, and numerous studies between 2018 and 2024 have explored their application in assessing low-carbon initiatives, green investment alternatives, and Environmental, Social, and Governance (ESG) performance. Among the most frequently applied frameworks are the Analytic Hierarchy Process (AHP), the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), VIKOR, and other fuzzy-based Multi-Criteria Decision-Making (MCDM) models. More broadly, mathematical optimization methods continue to be developed to improve computational efficiency and solution performance when addressing complex, large-scale problems (Alhawarat *et al.* 2021).

Recent studies have shown that MADM techniques play an increasingly important role in examining ESG dimensions within sustainable finance. Li *et al.* (2023), for example, integrated fuzzy AHP and fuzzy DEMATEL to analyse ESG factors in green investment decision-making. Their study showed that the environmental, social and governance dimensions are closely connected, and this interplay forms a key part of China's sustainable development efforts. When researchers must balance dozens of intricate ESG indicators with the subjective judgements of experts, hybrid models are very useful. These hybrid solutions allow us the flexibility to capture different perspectives without losing the strict structure of a formal assessment. In 2024, Ristanović *et al.* show that Multi-Criteria Decision Making (MCDM) isn't just a theoretical tool; it's a practical way to rank sustainability goals effectively. Their research highlights how these methods help decision-makers navigate the messy trade-offs that usually come with complex environmental objectives.

AHP and TOPSIS are structured tools that have become staples in sustainability research precisely because they ensure transparency. Whether researchers are evaluating environmental compliance, planning carbon cuts, or selecting green energy projects, MCDM methods reliably turn complicated data into actionable decisions. For example, Zhang *et al.* (2022) combined the AHP and TOPSIS approach to guide green investment decisions. They demonstrated how the two methods can be aligned to balance financial priorities with environmental objectives. Similarly, Reig-Mullor *et al.* (2022) used a neutrosophic version of the AHP-TOPSIS approach to evaluate

corporate ESG performance, highlighting how inconsistencies in sustainability reporting standards make the assessment process more challenging.

Following these earlier efforts, Zeng *et al.* (2020) showed that incorporating fuzzy logic into the decision-making processes can improve the interpretation of information that is uncertain or may conflict with other inputs. Table 1 summarises several significant contributions published between 2018 and 2024 that applied MCDM and fuzzy-based techniques within green finance and sustainable investment evaluations.

Table 1: Summary of recent studies employing MCDM techniques in green finance and sustainability evaluation

References	MCDM Method	Application Context	Key Findings
Zeng <i>et al.</i> (2020)	Fuzzy MCDM	Supplier selection in sustainable chains	Emphasizes the importance of multi-criteria in supply management
Shahabi <i>et al.</i> (2022)	Fuzzy MADM	Investment risk prioritization	Highlights the challenges of managing subjective criteria
Li <i>et al.</i> (2018)	Fuzzy AHP/DEMATEL	ESG evaluation in green finance	Explain how ESG dimensions are linked to one another and cannot be isolated.
Reig-Mullor <i>et al.</i> (2022)	AHP/TOPSIS	Investment optimization	Provides a more organised way to examine the feasibility of planning and decision alternatives.
Meng and Shaikh (2023)	Fuzzy AHP	Green finance strategies	Illustrates prioritization complexity within ESG factors
Ristanović <i>et al.</i> (2024)	MCDM Framework	Green investments assessment	Empirical evidence that strengthens decision-making in green investment

In sustainability and green finance research, MADM approaches have helped bring more structure to situations where many criteria are linked to one another. Over time, the use of fuzzy-based MCDM methods, including AHP and TOPSIS, has made ESG evaluations easier to interpret. These methods are particularly useful because they allow researchers to handle uncertainty and subjective elements that normally appear when assessing sustainability indicators.

1.2. Aggregation Challenges under Uncertainty and the Rise of IVPNS

In decision-making studies, aggregation operators are essential because they bring together information from different criteria and allow the alternatives to be evaluated in a more organized way. Within the wide range of operators introduced in the literature, those based on Dombi weighting have gained attention. When dealing with uncertain data, Dombi formulations are incredibly useful. Their inherent flexibility directly drives more dependable outcomes in decision-making. The Dombi operator itself traces back to Dombi's (1990) foundational paper, "Membership Function as an Evaluation". He structured the operator around a parametric t-norm and t-conorm pair, which successfully bridges the gap between conjunctive and disjunctive reasoning. Crucially, the operator maintains standard mathematical properties like continuity, associativity, commutativity, and monotonicity. However, the real advantages lie in its adjustability through the parameter p , which offers intuitive control over the aggregation process. Owing to this flexibility, Dombi-based operators have been effectively used not only in fuzzy and intuitionistic-fuzzy systems but also among multiple attributes.

Intuitionistic Fuzzy Hybrid Geometric (IFHG), Intuitionistic Fuzzy Weighted Geometric (IFWG), and Intuitionistic Fuzzy Ordered-Weighted Geometric (IFOWG) operators are three geometric aggregation models for intuitionistic fuzzy sets based on this foundation that has been introduced by Das *et al.* (2014). All these models demonstrated strong discrimination capabilities in MADM problems that require sensitivity parameter variation. Liu and Tang (2018) integrated the Power Average (PA) and Generalized Weighted Average (GWA) concepts to design a new family

of aggregation methods for interval neutrosophic sets, including the Interval Neutrosophic Power Generalized Aggregation (INPGA), Interval Neutrosophic Power Generalized Weight Aggregation (INPGWA), and Interval neutrosophic Power Generalized Ordered-Weight Aggregation (INPGOWA). Their work broadened the theoretical foundation of uncertainty-based aggregation by improving the ability to capture complex attribute interactions.

Subsequent developments were reported by Liu and You (2017), who introduced interval-neutrosophic Muirhead mean operators to capture the interrelationships among criteria within group decision-making contexts. Later, Chen and Ye (2017) proposed the Single-Valued Neutrosophic Dombi Weighted Arithmetic (SVNDWAA) and Geometric (SVNDWGA) operators. By incorporating the Dombi t-norm and t-conorm to single-valued neutrosophic numbers, their approach proved effective for handling investment selection problems. Khan *et al.* (2018) handled the problem of complex data by introducing interval-neutrosophic Dombi power Bonferroni mean operators. Their approach is significant because it gives researchers a better way to capture both higher-order uncertainty and attribute interaction to be represented more accurately. According to Fahmi *et al.* (2020), they combined Dombi logic with hybrid weighted geometry, proposing linguistic interval-valued intuitionistic neutrosophic geometric operators, LIVINDFWG, LIVINDFOWG, and LIVINDFHWG, which enhance linguistic decision-making flexibility.

In a related contribution, Seikh and Mandal (2021) introduced a family of aggregation operators, including Dombi-based averaging and geometric aggregation models, and applied Dombi operations within the intuitionistic fuzzy domain. This study highlights the adaptability of Dombi functions to diverse decision contexts through parameter turning. Jana and Pal (2021) integrated Dombi operations with the power-averaging concept to form a set of single-valued neutrosophic Dombi power operators; specifically, the single-valued neutrosophic Dombi power weighted averaging (SVNDPWA) and single-valued neutrosophic Dombi power weighted geometric averaging (SVNDPWGA) operators. It offers an improvement in responsiveness to uncertainty and attribute weighting. Niu and Ye (2024) revisited work and explored a different angle by incorporating Tangent-Dombi t-norm and t-conorm ideas into linguistic neutrosophic numbers. His study produced several new weighted averaging and geometric operators that are better suited to situations where multiple experts and multiple attributes must be considered at the same time.

In a different area of study, Mahmood *et al.* (2021) proposed the Bipolar Neutrosophic Dombi Prioritized Weighted Geometric Aggregation (BNDPWGA) and the Bipolar Neutrosophic Dombi Prioritized Ordered Weighted Geometric Aggregation (BNDPOWGA), where there are two aggregation operators tailored for bipolar neutrosophic sets. Both operators allow them to process positive and negative membership values at the same time by using Dombi-based t-norms and t-conorms. For Pythagorean fuzzy data, Garg (2016) introduced interval-valued Pythagorean fuzzy aggregation operators, the weighted averaging (IVPFWA) and weighted geometric (IVPFWG) operators, that apply interval truth and falsity membership degrees to strengthen decision accuracy. Later, this field of work was developed by Peng and Yang (2016), who proposed additional interval-valued Pythagorean fuzzy averaging and geometric models to better represent uncertainty in multi-criteria settings. Akram *et al.* (2019) further advanced this direction by developing several Dombi-based Pythagorean fuzzy aggregators, including Pythagorean Dombi fuzzy weighted arithmetic averaging (PDFWAA) and geometric averaging (PDFWGA) operators and their ordered-weighted variants, especially for use in group decision-making contexts. More recently, Deb and Biswas (2021) integrated the geometric Bonferroni mean and its weighted variants with Dombi operations, significantly improving the modelling of inter-attribute relationships in complex decision contexts. Shahabi *et al.* (2022) highlighted that assessing qualitative risks, particularly in sectors such as mining, remains difficult due to the challenge of quantifying intangible and non-measurable factors. Furthermore, Meng and Shaikh (2023) noted that environmental factors received more attention

compared with governance and social aspects of ESG. They argued that this imbalance highlights the need for more advanced analytical models, especially those that can deal with the vagueness and uncertainty that are frequently present in ESG evaluations.

1.3. Limitations of Conventional Aggregation Operators in Sustainability and Green Finance Decision-Making

In many decision-making studies, especially those focused on green finance and sustainability, it is common practice to use basic methods such as geometric means or weighted averaging (OWA). While these approaches provide a straightforward way to integrate with the multiple criteria, they frequently fail to deal with the conflicting nature of environmental, social, and economic objectives.

One of the main drawbacks of these traditional operators is the assumption that all input values are precise and remain constant. In the real world, sustainability assessments are rarely straightforward. ESG evaluations are frequently driven by qualitative judgments and stakeholder expectations, making exact quantification incredibly difficult. This limitation can influence the outcome of decisions. Luo *et al.* (2022) observed that it often results in decisions that favour short-term financial returns over long-term social value. Joshipura *et al.* (2024) highlighted that the conventional aggregation approaches often struggle to capture the complex trade-offs involved in sustainability issues, which are multidimensional and closely interrelated.

Due to these constraints, more recent research makes the case for aggregation methods that can adjust to a greater range of sustainability indicators and take into account variations in decision-maker preferences (Yang & Li 2020). Methods with adjustable parameters allow the model to more accurately link the relationship between financial, social, and environmental factors and the outcomes, supporting the making of more context-sensitive decisions. The Dombi t-norm and t-conorm functions have attracted a lot of attention during this transition. The degree of interaction between criteria can be easily changed because of their parametric structure. A decision-maker can decide how strongly or weakly the criteria should impact one another by adjusting their control parameters, which will provide an aggregation process that is more responsive to the current issue. This makes Dombi-based operators particularly useful in sustainable finance applications, where information tends to be uncertain and stakeholder expectations may change over time (Jana *et al.* 2019).

The use of Dombi operators in fuzzy and neutrosophic environments has further shown that they are capable of representing ambiguity more effectively. Yang and Li (2020), for instance, demonstrated that the embedding of Dombi operators within neutrosophic sets can make decisions under uncertainty clearer and easier to interpret. However, despite these developments, their application within the IVPNS framework remains limited. Much of existing research has focused on interval-valued intuitionistic or traditional neutrosophic models (He 2019), leaving the richer expressive potential of IVPNS largely unexplored. As a result, decision-makers in sustainability-oriented finance may not yet be benefiting fully from what Dombi-based aggregation can offer within a more advanced neutrosophic environment. Existing models are generally limited to fuzzy, intuitionistic fuzzy, or interval neutrosophic settings, leaving a considerable gap in the integration of Dombi operations with more expressive uncertainty representation. Its integration within the IVPNS framework remains relatively unexplored.

Recognizing this gap in the existing literature, this study takes a step further by adapting the Dombi operator to the IVPNS setting. This research introduced a new operator in the IVPNS environment, the Interval-Valued Pythagorean Neutrosophic Dombi Weighted Geometric Average (IVPNSDWGA) operator, to support aggregation in situations where uncertainty is more complex. The model formally establishes and verifies key mathematical properties of the operator, evaluates

its effectiveness in capturing multidimensional uncertainty, and demonstrates its practical value in multi-attribute decision-making (MADM) through several illustrative case studies.

2. Preliminaries

This section presents the framework of IVPNS, describes the outlines of the associated algebraic operations, and discusses the key mathematical properties.

2.1. Interval Valued Pythagorean Neutrosophic Set (IVPNS)

The way IVPNS handles the interaction between its membership components is one of its advantages. Within this framework, indeterminacy operates independently, even as the truth and falsity membership degrees remain dependent on one another. The IVPNS was originally based on the integration of Interval Value Pythagorean sets (IVPS) and Interval Valued Neutrosophic sets (IVNS); it has been further developed in recent studies by Razak *et al.* (2024). This feature enhances its capability framework to capture complex uncertainty patterns that cannot be adequately represented using conventional approaches. Standard models often struggle with the inconsistent and linguistic evaluations common in sustainable finance. However, IVPNS is capable of handling real-world decision-making problems under vague and uncertain conditions. Because it captures uncertainty more comprehensively, it allows decision-makers to build models that feel realistic rather than oversimplified. This enhanced modeling capability makes IVPNS particularly suitable for integration with flexible aggregation operators such as the Dombi operator. In this section, the fundamental concept of IVPNS is briefly introduced, followed by its basic algebraic operations and associated mathematical properties, as established by Razak *et al.* (2024).

Definition 2.1. Let U be a universe of discourse (objects) consisting of generic elements in U denoted by x . Then the Interval Valued Pythagorean Neutrosophic Sets (IVPNS) $\tilde{A}$ is an object having the form

$$\tilde{A} = \left\{ \langle x, \left(\left[\tilde{b}_A^L(x), \tilde{b}_A^U(x) \right], \left[\tilde{I}_A^L(x), \tilde{I}_A^U(x) \right], \left[\tilde{s}_A^L(x), \tilde{s}_A^U(x) \right] \right) \mid x \in X \right\},$$

Then, the IVPNS $\tilde{A}$ in x is characterized by an interval valued of truth-membership function $\tilde{b}(x)$, interval valued of indeterminacy membership function $\tilde{I}(x)$, and an interval valued falsity-membership function $\tilde{s}(x)$. For every element $x \in X$, the lower and upper intervals are $\tilde{b}_A(x) = [\tilde{b}_A^L, \tilde{b}_A^U] \subseteq [0, 1]$, $\tilde{I}_A(x) = [\tilde{I}_A^L, \tilde{I}_A^U] \subseteq [0, 1]$ and, $\tilde{s}_A(x) = [\tilde{s}_A^L, \tilde{s}_A^U] \subseteq [0, 1]$ respectively. The $\tilde{b}_A(x)$ and $\tilde{s}_A(x)$ are dependent on Pythagorean components and $\tilde{I}_A(x)$ is an independent components on neutrosophic. These components satisfy the following conditions:

$$\text{i.} \quad 0 \leq \tilde{b}_A(x) + \tilde{s}_A(x) \leq 1 \quad (1)$$

$$\text{ii.} \quad 0 \leq \left(\tilde{b}_A^U(x) \right)^2 + \left(\tilde{s}_A^U(x) \right)^2 \leq 1 \quad (2)$$

$$\text{iii.} \quad 0 \leq \tilde{b}_A^U(x) + \tilde{I}_A^U(x) + \tilde{s}_A^U(x) \leq 2 \quad (3)$$

$$\text{iv.} \quad 0 \leq \left(\tilde{b}_A^U(x) \right)^2 + \left(\tilde{I}_A^U(x) \right)^2 + \left(\tilde{s}_A^U(x) \right)^2 \leq 2 \quad (4)$$

For every, $x \in X$, $\tilde{\pi}_A = [\tilde{\pi}_A^U(x), \tilde{\pi}_A^L]$ is called the IVPNS index of x to $\tilde{P}$, where

$$\tilde{\pi}_A(x) = \left(1 - \left(\tilde{b}_A^U(x) \right)^2 - \left(\tilde{I}_A^U(x) \right)^2 - \left(\tilde{s}_A^U(x) \right)^2 \right)^{\frac{1}{2}}.$$

In this case, the algebraic operations of IVPNS are presented in Definition 2.2 as follows:

Definition 2.2. Let $C = ([\tilde{b}_C^L, \tilde{b}_C^U], [I_C^L, I_C^U], [\tilde{s}_C^L, \tilde{s}_C^U])$ and $D = ([\tilde{b}_D^L, \tilde{b}_D^U], [I_D^L, I_D^U], [\tilde{s}_D^L, \tilde{s}_D^U])$ be two IVPNS, and k is any scalar multiplication. Consequently, the algebraic operations are defined as follows:

$$\text{i. } C \oplus D = \left[\left(\left(\tilde{b}_C^L \right)^2 + \left(\tilde{b}_D^L \right)^2 - \left(\tilde{b}_C^L \right)^2 \left(\tilde{b}_D^L \right)^2 \right)^{\frac{1}{2}}, \left(\left(\tilde{b}_C^U \right)^2 + \left(\tilde{b}_D^U \right)^2 - \left(\tilde{b}_C^U \right)^2 \left(\tilde{b}_D^U \right)^2 \right)^{\frac{1}{2}} \right],$$

$$[I_C^L I_D^L, I_C^U I_D^U], [\tilde{s}_C^L \tilde{s}_D^L, \tilde{s}_C^U \tilde{s}_D^U] \quad (5)$$

$$\text{ii. } C \otimes D = [\tilde{b}_C^L \tilde{b}_D^L, \tilde{b}_C^U \tilde{b}_D^U], [I_C^L + I_D^L - I_C^L I_D^L, I_C^U + I_D^U - I_C^U I_D^U],$$

$$\left[\left(\left(\tilde{s}_C^L \right)^2 + \left(\tilde{s}_D^L \right)^2 - \left(\tilde{s}_C^L \right)^2 \left(\tilde{s}_D^L \right)^2 \right)^{\frac{1}{2}}, \left(\left(\tilde{s}_C^U \right)^2 + \left(\tilde{s}_D^U \right)^2 - \left(\tilde{s}_C^U \right)^2 \left(\tilde{s}_D^U \right)^2 \right)^{\frac{1}{2}} \right] \quad (6)$$

$$\text{iii. } kC = \left[\left(1 - \left(1 - \left(\tilde{b}_C^L \right)^2 \right)^k \right)^{\frac{1}{2}}, \left(1 - \left(1 - \left(\tilde{b}_C^U \right)^2 \right)^k \right)^{\frac{1}{2}} \right], [(I_C^L)^k, (I_C^U)^k], [(\tilde{s}_C^L)^k, (\tilde{s}_C^U)^k],$$

$$(k > 0) \quad (7)$$

$$\text{iv. } C^k = [\left(\tilde{b}_C^L \right)^k, \left(\tilde{b}_C^U \right)^k], [1 - I_C^L, 1 - I_C^U], \left[\left(1 - \left(1 - \left(\tilde{s}_C^L \right)^2 \right)^k \right)^{\frac{1}{2}}, \left(1 - \left(1 - \left(\tilde{s}_C^U \right)^2 \right)^k \right)^{\frac{1}{2}} \right],$$

$$(k > 0) \quad (8)$$

2.2. IVPNS Scoring Method

Definition 2.3. (Khan *et al.* 2018) Let $\tilde{c} = ([\tilde{b}_c^L, \tilde{b}_c^U], [I_c^L, I_c^U], [\tilde{s}_c^L, \tilde{s}_c^U])$ be IVPNS, then its score $S(\tilde{c})$ and accuracy $A(\tilde{c})$ function is defined as follows:

$$S(\tilde{c}) = \frac{\tilde{b}_c^L + \tilde{b}_c^U}{2} + 1 - \frac{\tilde{I}_c^L + \tilde{I}_c^U}{2} + 1 - \frac{\tilde{s}_c^L + \tilde{s}_c^U}{2} \quad (9)$$

$$A(\tilde{c}) = \frac{\tilde{b}_c^L + \tilde{b}_c^U}{2} + 1 - \frac{\tilde{I}_c^L + \tilde{I}_c^U}{2} + \frac{\tilde{s}_c^L + \tilde{s}_c^U}{2}, \quad (10)$$

The procedure for comparing two IVPNS follows the approach described by Khan *et al.* (2018). In this method, comparison is first made through their respective score functions. When the score values are identical, further distinction between the two sets can still be achieved by examining their accuracy or secondary functions.

Definition 2.4. Let $\tilde{c}_1 = \left(\left[\tilde{b}_{\tilde{c}_1}^L, \tilde{b}_{\tilde{c}_1}^U \right], \left[I_{\tilde{c}_1}^L, I_{\tilde{c}_1}^U \right], \left[\tilde{s}_{\tilde{c}_1}^L, \tilde{s}_{\tilde{c}_1}^U \right] \right)$ and $\tilde{c}_2 = \left(\left[\tilde{b}_{\tilde{c}_2}^L, \tilde{b}_{\tilde{c}_2}^U \right], \left[I_{\tilde{c}_2}^L, I_{\tilde{c}_2}^U \right], \left[\tilde{s}_{\tilde{c}_2}^L, \tilde{s}_{\tilde{c}_2}^U \right] \right)$

be two IVPNS. Then, we have:

- i. If $S(\tilde{c}_1) \succ S(\tilde{c}_2)$, then c_1 is better than c_2 , and denoted by $c_1 \succ c_2$;
- ii. If $S(\tilde{c}_1) = S(\tilde{c}_2)$, and $A(c_1) \succ A(c_2)$, then c_1 is better than c_2 , and denoted by $c_1 \succ c_2$;
- iii. If $S(\tilde{c}_1) = S(\tilde{c}_2)$, and $A(c_1) = A(c_2)$, then c_1 is equal to c_2 , and denoted by $c_1 = c_2$.

2.3. Linguistic Variable for IVPNS

Conversion scales are used to transform linguistic assessment into IVPNS representations. Through this process, each alternative is expressed on a normalized scale ranging from the interval $[0,1]$, where subintervals are carefully assigned so that all five linguistic categories correspond to appropriate truth, indeterminacy, and falsity membership intervals within the IVPNS framework (Razak *et al.* 2025). The linguistic terms adopted in this study, along with their IVPNS interval mappings, are presented in Table 2, which serves as the basis for the numerical example that follows.

Table 2: The linguistic characteristics of the IVPNS concept

Linguistic Characteristic	IVPNS Set		
No Influence (NI)	[0.05,0.10]	[0.70,0.80]	[0.70,0.90]
Very Low Influence (VLI)	[0.20,0.40]	[0.60,0.70]	[0.50,0.60]
Medium Influence (MI)	[0.45,0.60]	[0.40,0.50]	[0.30,0.40]
High Influence (HI)	[0.60,0.70]	[0.30,0.40]	[0.20,0.30]
Absolutely Influence (AI)	[0.80,0.90]	[0.20,0.30]	[0.05,0.10]

2.4. Dombi Operation

The Dombi product (often referred to as the Dombi T-Norm) and Dombi sum (the corresponding Dombi T-Conorm) are fundamental operations introduced by Dombi (1982). For any two real numbers $\tilde{m}$ and $\tilde{n}$, these operations provide a flexible mechanism for modelling conjunction and disjunction in aggregation processes. The Dombi t-conorm and t-norm are defined as follows:

$$Dom(\tilde{m}, \tilde{n}) = \frac{1}{1 + \left\{ \left(\frac{1-\tilde{m}}{\tilde{m}} \right)^\gamma + \left(\frac{1-\tilde{n}}{\tilde{n}} \right)^\gamma \right\}^{1/\gamma}} \quad (11)$$

$$Dom^*(\tilde{m}, \tilde{n}) = 1 - \frac{1}{1 + \left\{ \left(\frac{\tilde{m}}{1-\tilde{m}} \right)^\gamma + \left(\frac{\tilde{n}}{1-\tilde{n}} \right)^\gamma \right\}^{1/\gamma}} \quad , \quad (12)$$

where $\gamma > 0$ and $\tilde{m}, \tilde{n} \in [0,1]$.

2.5. Operations of IVPNS Based on Dombi t-Conorm and t-Norm

This section provides the formal definitions of the Dombi t-conorm and t-norm operations within the IVPNS framework and discusses their underlying properties in detail.

Definition 2.5. Let $P = \{p_1, p_2, \dots, p_n\}$ be an IVPNS, where $p_k = ([\tilde{b}_{p_k}^L, \tilde{b}_{p_k}^U], [\tilde{I}_{p_k}^L, \tilde{I}_{p_k}^U], [\tilde{s}_{p_k}^L, \tilde{s}_{p_k}^U])$ and $p_2 = ([\tilde{b}_{p_2}^L, \tilde{b}_{p_2}^U], [\tilde{I}_{p_2}^L, \tilde{I}_{p_2}^U], [\tilde{s}_{p_2}^L, \tilde{s}_{p_2}^U])$ be two IVPNS, $k \in \mathfrak{N}$, p_1 and p_2 are denoted by $p_1 \otimes p_2$ and $p_1 \oplus p_2$ respectively defined as follows:

$$p_1 \oplus p_2 = \left[\left(\left(1 - \frac{1}{1 + \left\{ \left(\frac{(\tilde{b}_1^L)^2}{1 - (\tilde{b}_1^L)^2} \right)^\gamma + \left(\frac{(\tilde{b}_2^L)^2}{1 - (\tilde{b}_2^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}}, \left(1 - \frac{1}{1 + \left\{ \left(\frac{(\tilde{b}_1^U)^2}{1 - (\tilde{b}_1^U)^2} \right)^\gamma + \left(\frac{(\tilde{b}_2^U)^2}{1 - (\tilde{b}_2^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}}, \left(\frac{1}{1 + \left\{ \left(\frac{(1 - I_1^L)}{(I_1^L)} \right)^\gamma + \left(\frac{(1 - I_2^L)}{(I_2^L)} \right)^\gamma \right\}^{1/\gamma}}, \frac{1}{1 + \left\{ \left(\frac{(1 - I_1^U)}{(I_1^U)} \right)^\gamma + \left(\frac{(1 - I_2^U)}{(I_2^U)} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} \right] \quad (13)$$

$$p_1 \otimes p_2 = \left[\left(\left(\frac{1}{1 + \left\{ \left(\frac{1 - (\tilde{b}_1^L)^2}{(\tilde{b}_1^L)^2} \right)^\gamma + \left(\frac{1 - (\tilde{b}_2^L)^2}{(\tilde{b}_2^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}}, \left(\frac{1}{1 + \left\{ \left(\frac{1 - (\tilde{b}_1^U)^2}{(\tilde{b}_1^U)^2} \right)^\gamma + \left(\frac{1 - (\tilde{b}_2^U)^2}{(\tilde{b}_2^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}}, \left(1 - \frac{1}{1 + \left\{ \left(\frac{(I_1^L)}{(1 - I_1^L)} \right)^\gamma + \left(\frac{(I_2^L)}{(1 - I_2^L)} \right)^\gamma \right\}^{1/\gamma}}, 1 - \frac{1}{1 + \left\{ \left(\frac{(I_1^U)}{(1 - I_1^U)} \right)^\gamma + \left(\frac{(I_2^U)}{(1 - I_2^U)} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} \right]$$

$$\left(\left(1 - \frac{1}{1 + \left\{ \left(\frac{(\tilde{s}_1^L)^2}{1 - (\tilde{s}_1^L)^2} \right)^\gamma + \left(\frac{(\tilde{s}_2^L)^2}{1 - (\tilde{s}_2^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}, \left(1 - \frac{1}{1 + \left\{ \left(\frac{(\tilde{s}_1^U)^2}{1 - (\tilde{s}_1^U)^2} \right)^\gamma + \left(\frac{(\tilde{s}_2^U)^2}{1 - (\tilde{s}_2^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right) \right] \quad (14)$$

$$\begin{aligned} kp_1 = & \left[\left(\left(1 - \frac{1}{1 + \left\{ k \left(\frac{(\tilde{b}_1^L)^2}{1 - (\tilde{b}_1^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}, \left(1 - \frac{1}{1 + \left\{ k \left(\frac{(\tilde{b}_1^U)^2}{1 - (\tilde{b}_1^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right) \right. \\ & \left. , \frac{1}{1 + \left\{ k \left(\frac{(1 - I_1^L)}{(I_1^L)} \right)^\gamma \right\}^{1/\gamma}}, \frac{1}{1 + \left\{ k \left(\frac{(1 - I_1^U)}{(I_1^U)} \right)^\gamma \right\}^{1/\gamma}} \right) \\ & \left. \left(\left(\frac{1}{1 + \left\{ k \left(\frac{1 - (\tilde{s}_1^L)^2}{(\tilde{s}_1^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}, \left(\frac{1}{1 + \left\{ k \left(\frac{1 - (\tilde{s}_1^U)^2}{(\tilde{s}_1^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right) \right] \quad (15) \end{aligned}$$

$$\begin{aligned} p_1^k = & \left[\left(\left(1 - \frac{1}{1 + \left\{ k \left(\frac{1 - (\tilde{b}_1^L)^2}{(\tilde{b}_1^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}, \left(1 - \frac{1}{1 + \left\{ k \left(\frac{1 - (\tilde{b}_1^U)^2}{(\tilde{b}_1^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right) \right. \\ & \left. \left(1 - \frac{1}{1 + \left\{ k \left(\frac{(I_1^L)}{(1 - I_1^L)} \right)^\gamma \right\}^{1/\gamma}}, 1 - \frac{1}{1 + \left\{ k \left(\frac{(I_1^U)}{(1 - I_1^U)} \right)^\gamma \right\}^{1/\gamma}} \right) \right] \end{aligned}$$

$$\left[\left(\frac{1}{1 + \left\{ k \left(\frac{(\tilde{s}_1^L)^2}{1 - (\tilde{s}_1^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}, \left(\frac{1}{1 + \left\{ k \left(\frac{(\tilde{s}_1^U)^2}{1 - (\tilde{s}_1^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right] \quad (16)$$

3. Interval Valued Pythagorean Neutrosophic Set based on Dombi Weight Aggregation Operators

Building on Dombi-based IVPNS operations presented in Definition 2.5, the Dombi Geometric operator is based on the Dombi product (often referred to as the Dombi T-Norm). This study introduces the Interval-Valued Pythagorean Neutrosophic Weighted Geometric Average (IVPNSWGA) operator. The key mathematical properties associated with this newly formulated operator are also examined in the subsequent discussion.

Definition 3.1. Let $p_j = ([\tilde{b}_j^L, \tilde{b}_j^U], [I_j^L, I_j^U], [\tilde{s}_j^L, \tilde{s}_j^U])$, where $j = (j=1, 2, \dots, n)$ be a collection of IVPNS and $w = (w_1, w_2, \dots, w_n)$, be the corresponding weight vector for p_j with, such that $w_l > 0$ and $\sum_{j=1}^n w_j = 1$. The IVPNSWGA operators using the Dombi product are consequently specified as follows:

$$IVPNSDWGA(p_1, p_2, \dots, p_n) = \bigotimes_{j=1}^n p_j^{w_j} \quad (17)$$

3.1. Interval Valued Pythagorean Neutrosophic Set Dombi Weight Geometric Average

Theorem 3.1. Let $p_j = ([\tilde{b}_j^L, \tilde{b}_j^U], [I_j^L, I_j^U], [\tilde{s}_j^L, \tilde{s}_j^U])$ $j = (j=1, 2, 3, \dots, n)$ be a collection of IVPNS and $w = (w_1, w_2, \dots, w_n)$ be the weight vector for p_j with $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$. Hence, the result generated by the IVPNSDWGA operator remains an IVPNS and is obtained using the following equation:

$$\begin{aligned} IVPNSDWGA(p_{i1}, p_{i2}, \dots, p_{ij}) &= w_1 p_{i1} \otimes w_2 p_{i2} \otimes \dots \otimes w_{ij} p_{ij} \\ &= \left[\left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_{ij}^L)^2}{(\tilde{b}_{ij}^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}, \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_{ij}^U)^2}{(\tilde{b}_{ij}^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right] \end{aligned}$$

$$\left(\frac{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{ij}^L)}{(1 - I_{ij}^L)} \right)^\gamma \right\}^{1/\gamma}}}{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{ij}^U)}{(1 - I_{ij}^U)} \right)^\gamma \right\}^{1/\gamma}}}, \frac{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{s}_{ij}^L)^2}{1 - (\tilde{s}_{ij}^L)^2} \right)^\gamma \right\}^{1/\gamma}}}{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{s}_{ij}^U)^2}{1 - (\tilde{s}_{ij}^U)^2} \right)^\gamma \right\}^{1/\gamma}}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} \quad (18)$$

By mathematical induction, Theorem 3.1 can be established.

Proof. For the base case where $n=2$, and by applying the Dombi operations for IVPNS as defined in Definition 2.5, which are denoted by the operation of IVPNS based on Dombi product $(p_1 \otimes p_2)$ (often referred to as the Dombi T-Norm), the following result is obtained:

$$IVPNSDWGA(p_1, p_2) = w_1 p_1 \otimes w_2 p_2$$

$$p_1 \otimes p_2 = \left[\left(\frac{1 - \frac{1}{1 + \left\{ w_1 \left(\frac{1 - (\tilde{b}_1^L)^2}{(\tilde{b}_1^L)^2} \right)^\gamma + w_2 \left(\frac{1 - (\tilde{b}_2^L)^2}{(\tilde{b}_2^L)^2} \right)^\gamma \right\}^{1/\gamma}}}{1 - \frac{1}{1 + \left\{ w_1 \left(\frac{1 - (\tilde{b}_1^U)^2}{(\tilde{b}_1^U)^2} \right)^\gamma + w_2 \left(\frac{1 - (\tilde{b}_2^U)^2}{(\tilde{b}_2^U)^2} \right)^\gamma \right\}^{1/\gamma}}}, \frac{1 - \frac{1}{1 + \left\{ w_1 \left(\frac{(I_1^L)}{(1 - I_1^L)} \right)^\gamma + w_2 \left(\frac{(I_2^L)}{(1 - I_2^L)} \right)^\gamma \right\}^{1/\gamma}}}{1 - \frac{1}{1 + \left\{ w_1 \left(\frac{(I_1^U)}{(1 - I_1^U)} \right)^\gamma + w_2 \left(\frac{(I_2^U)}{(1 - I_2^U)} \right)^\gamma \right\}^{1/\gamma}}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} \right]$$

$$\begin{aligned}
 &= \left[\left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_1^L)^2}{(\tilde{b}_1^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}, \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_1^U)^2}{(\tilde{b}_1^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right] \\
 &\quad \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_1^L)}{(1 - I_1^L)} \right)^\gamma \right\}^{1/\gamma}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_1^U)}{(1 - I_1^U)} \right)^\gamma \right\}^{1/\gamma}} \right), \\
 &\quad \left[\left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{s}_1^L)^2}{1 - (\tilde{s}_1^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}, \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{s}_1^U)^2}{1 - (\tilde{s}_1^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right] \right] \quad (19)
 \end{aligned}$$

Suppose that Eq. (19) holds for $n=k$, the resulting expression is as follows:

$$\begin{aligned}
 &IVPNSDWGA(p_1, p_2, \dots, k) = w_1 p_1 \times w_2 p_2 \times \dots \times w_k p_k \\
 &= \left[\left(\frac{1}{1 + \left\{ \sum_{j=1}^k w_j \left(\frac{1 - (\tilde{b}_1^L)^2}{(\tilde{b}_1^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}, \left(\frac{1}{1 + \left\{ \sum_{j=1}^k w_j \left(\frac{1 - (\tilde{b}_1^U)^2}{(\tilde{b}_1^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right] \\
 &\quad \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^k w_j \left(\frac{(I_1^L)}{(1 - I_1^L)} \right)^\gamma \right\}^{1/\gamma}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^k w_j \left(\frac{(I_1^U)}{(1 - I_1^U)} \right)^\gamma \right\}^{1/\gamma}} \right), \\
 &\quad \left[\left(\frac{1}{1 + \left\{ \sum_{j=1}^k w_j \left(\frac{(\tilde{s}_1^L)^2}{1 - (\tilde{s}_1^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}, \left(\frac{1}{1 + \left\{ \sum_{j=1}^k w_j \left(\frac{(\tilde{s}_1^U)^2}{1 - (\tilde{s}_1^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right] \right] \quad (20)
 \end{aligned}$$

Suppose that Eq. (19) holds for $n = k + 1$, the following result can be obtained.

$$\begin{aligned}
 IVPNSDWGA(p_1, p_2, \dots, k, k+1) &= w_1 p_1 \times w_2 p_2, \dots, \times w_k p_k \times w_{k+1} p_{k+1} \\
 &= \left[\left(\frac{1}{1 + \left\{ \sum_{j=1}^{k+1} w_j \left(\frac{1 - (\tilde{b}_1^L)^2}{(\tilde{b}_1^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \left(\frac{1}{1 + \left\{ \sum_{j=1}^{k+1} w_j \left(\frac{1 - (\tilde{b}_1^U)^2}{(\tilde{b}_1^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right. \\
 &\quad \left. \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^{k+1} w_j \left(\frac{(I_1^L)}{(1 - I_1^L)} \right)^\gamma \right\}^{1/\gamma}} \right), 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{k+1} w_j \left(\frac{(I_1^U)}{(1 - I_1^U)} \right)^\gamma \right\}^{1/\gamma}} \right) \right. \\
 &\quad \left. \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^{k+1} w_j \left(\frac{(\tilde{s}_1^L)^2}{1 - (\tilde{s}_1^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^{k+1} w_j \left(\frac{(\tilde{s}_1^U)^2}{1 - (\tilde{s}_1^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right) \right] \quad (21)
 \end{aligned}$$

Hence, Theorem 3.1 holds for $n = k + 1$; accordingly, Eq. (18) also holds for every n . $\square$

3.1.1. Properties of interval-valued Pythagorean neutrosophic set Dombi weight geometric average

To further establish the reliability and mathematical soundness of the proposed IVPNSDWGA operator in Eq. (18), several fundamental properties are examined in this section. These properties characterise the behaviour of the operator under different conditions and ensure that it functions consistently within the IVPNS framework. Based on the established definitions and operations, the IVPNSDWGA operator satisfies the following properties:

Theorem 3.2. (Reducibility): when $w = (1/n, 1/n, \dots, 1/n)$ it is obvious that it exists

$$= \left[\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \frac{1}{n} \left(\frac{1 - (\tilde{b}_{ij}^L)^2}{(\tilde{b}_{ij}^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \left(\frac{1}{1 + \left\{ \sum_{j=1}^n \frac{1}{n} \left(\frac{1 - (\tilde{b}_{ij}^U)^2}{(\tilde{b}_{ij}^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right.$$

$$\left(\frac{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \frac{1}{n} \left(\frac{(I_1^L)}{(1-I_1^L)} \right)^\gamma \right\}^{1/\gamma}}}}{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \frac{1}{n} \left(\frac{(I_1^U)}{(1-I_1^U)} \right)^\gamma \right\}^{1/\gamma}}}} \right)^{\frac{1}{2}} \left(\frac{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \frac{1}{n} \left(\frac{(\tilde{s}_1^L)^2}{1-(\tilde{s}_1^L)^2} \right)^\gamma \right\}^{1/\gamma}}}}{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \frac{1}{n} \left(\frac{(\tilde{s}_1^U)^2}{1-(\tilde{s}_1^U)^2} \right)^\gamma \right\}^{1/\gamma}}}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}}$$

Theorem 3.3. (Idempotency). For any IVPNS $p_j = ([\tilde{b}_j^L, \tilde{b}_j^U], [I_j^L, I_j^U], [\tilde{s}_j^L, \tilde{s}_j^U])$, where $j=1, 2, \dots, n$, we have $IVPNSDWGA(p_1, p_2, \dots, p_j) = p$

$$= \left(\left(\frac{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1-(\tilde{b}_{ij}^L)^2}{(\tilde{b}_{ij}^L)^2} \right)^\gamma \right\}^{1/\gamma}}}}{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1-(\tilde{b}_{ij}^U)^2}{(\tilde{b}_{ij}^U)^2} \right)^\gamma \right\}^{1/\gamma}}}} \right)^{\frac{1}{2}} \left(\frac{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{ij}^L)}{(1-I_{ij}^L)} \right)^\gamma \right\}^{1/\gamma}}}}{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{ij}^U)}{(1-I_{ij}^U)} \right)^\gamma \right\}^{1/\gamma}}}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}}$$

$$IVPNSDWGA(p_1, p_2, \dots, p_n) =$$

$$= \left[\left(\frac{1}{1 + \left\{ \left(\frac{1 - (\tilde{b}_{ij}^L)^2}{(\tilde{b}_{ij}^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}, \left(\frac{1}{1 + \left\{ \left(\frac{1 - (\tilde{b}_{ij}^U)^2}{(\tilde{b}_{ij}^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right],$$

$$\left(1 - \frac{1}{1 + \left\{ \left(\frac{(I_{ij}^L)}{(1 - I_{ij}^L)} \right)^\gamma \right\}^{1/\gamma}}, 1 - \frac{1}{1 + \left\{ \left(\frac{(I_{ij}^U)}{(1 - I_{ij}^U)} \right)^\gamma \right\}^{1/\gamma}} \right),$$

$$\left[\left(\frac{1}{1 + \left\{ \left(\frac{(\tilde{s}_{ij}^L)^2}{1 - (\tilde{s}_{ij}^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}, \left(\frac{1}{1 + \left\{ \left(\frac{(\tilde{s}_{ij}^U)^2}{1 - (\tilde{s}_{ij}^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right]$$

Theorem 3.4. (Monotonocity). Let $\tilde{p} = (\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \dots, \tilde{p}_n)$ and $\tilde{B} = (\tilde{B}_1, \tilde{B}_2, \tilde{B}_3, \dots, \tilde{B}_n)$ be two collections of IVPNS sets, where $\tilde{p}_j = \left([\tilde{b}_{\tilde{p}_j}^L, \tilde{b}_{\tilde{p}_j}^U], [\tilde{I}_{\tilde{p}_j}^L, \tilde{I}_{\tilde{p}_j}^U], [\tilde{s}_{\tilde{p}_j}^L, \tilde{s}_{\tilde{p}_j}^U] \right)$ ($i=1,2,\dots,n$) and $\tilde{B}_j = \left([\tilde{b}_{\tilde{B}_j}^L, \tilde{b}_{\tilde{B}_j}^U], [\tilde{I}_{\tilde{B}_j}^L, \tilde{I}_{\tilde{B}_j}^U], [\tilde{s}_{\tilde{B}_j}^L, \tilde{s}_{\tilde{B}_j}^U] \right)$. If for any j , there are $\tilde{b}_{\tilde{B}_j}^L \geq \tilde{b}_{\tilde{p}_j}^L, \tilde{I}_{\tilde{B}_j}^L \geq \tilde{I}_{\tilde{p}_j}^L, \tilde{s}_{\tilde{B}_j}^L \geq \tilde{s}_{\tilde{p}_j}^L, \tilde{b}_{\tilde{B}_j}^U \leq \tilde{b}_{\tilde{p}_j}^U, \tilde{I}_{\tilde{B}_j}^U \leq \tilde{I}_{\tilde{p}_j}^U, \tilde{s}_{\tilde{B}_j}^U \leq \tilde{s}_{\tilde{p}_j}^U$, that is $\tilde{A}_i \leq \tilde{B}_i$, then there is $IVPNSDWGA^{s,t,r}(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \dots, \tilde{p}_n) \leq IVPNSDWGA^{s,t,r}(\tilde{B}_1, \tilde{B}_2, \tilde{B}_3, \dots, \tilde{B}_n)$.

$$\left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_{B_j}^L)^2}{(\tilde{b}_{B_j}^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \leq \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_{p_j}^L)^2}{(\tilde{b}_{p_j}^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}},$$

$$\begin{aligned}
 & \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_{Bj}^U)^2}{(\tilde{b}_{Bj}^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \leq \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_{pj}^U)^2}{(\tilde{b}_{pj}^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \\
 & 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{Bj}^L)^2}{(1 - I_{Bj}^L)^2} \right)^\gamma \right\}^{1/\gamma}} \geq 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{pj}^L)^2}{(1 - I_{pj}^L)^2} \right)^\gamma \right\}^{1/\gamma}} \\
 & 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{Bj}^U)^2}{(1 - I_{Bj}^U)^2} \right)^\gamma \right\}^{1/\gamma}} \leq 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{pj}^U)^2}{(1 - I_{pj}^U)^2} \right)^\gamma \right\}^{1/\gamma}}, \\
 & \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{s}_{Bj}^L)^2}{1 - (\tilde{s}_{Bj}^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \leq \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{s}_{pj}^L)^2}{1 - (\tilde{s}_{pj}^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}, \\
 & \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{s}_{Bj}^U)^2}{1 - (\tilde{s}_{Bj}^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \leq \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{s}_{pj}^U)^2}{1 - (\tilde{s}_{pj}^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}},
 \end{aligned}$$

Theorem 3.5. (Boundedness): Consider the collection $P = \{p_1, p_2, \dots, p_n\}$, $\tilde{p}_j = \left([\tilde{b}_{\tilde{p}_j}^L, \tilde{b}_{\tilde{p}_j}^U], [\tilde{I}_{\tilde{p}_j}^L, \tilde{I}_{\tilde{p}_j}^U], [\tilde{s}_{\tilde{p}_j}^L, \tilde{s}_{\tilde{p}_j}^U] \right)$, $j, (j=1, 2, \dots, n)$ of IVPNS, such that $P_{\min} = \min\{p_1, p_2, \dots, p_n\}$ and $P_{\max} = \max\{p_1, p_2, \dots, p_n\}$. Then $P_{\min} \leq \text{IVPNSDWGA}(p_1, p_2, \dots, p_n) \leq P_{\max}$.

Proof. Assume that $p_{\min} = \min\{p_1, p_2, \dots, p_n\} = \left([\tilde{b}_j^L, \tilde{b}_j^{U^-}], [I_j^L, I_j^{U^-}], [\tilde{s}_j^L, \tilde{s}_j^{U^-}] \right)$ and $p_{\max} = \max\{p_1, p_2, \dots, p_n\} = \left([\tilde{b}_j^{L^+}, \tilde{b}_j^{U^+}], [I_j^{L^+}, I_j^{U^+}], [\tilde{s}_j^{L^+}, \tilde{s}_j^{U^+}] \right)$. Consequently, the truth-membership function $\tilde{b}_{\tilde{A}}(x)$ satisfies the following inequalities:

$$\left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_{ij}^L)^{2^-}}{(\tilde{b}_{ij}^L)^{2^-}} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \leq \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_{ij}^L)^2}{(\tilde{b}_{ij}^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \leq \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_{ij}^L)^{2^+}}{(\tilde{b}_{ij}^L)^{2^+}} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}$$

$$\left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_{ij}^U)^{2^-}}{(\tilde{b}_{ij}^U)^{2^-}} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \leq \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_{ij}^U)^2}{(\tilde{b}_{ij}^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \leq \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_{ij}^U)^{2^+}}{(\tilde{b}_{ij}^U)^{2^+}} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}$$

Likewise, for Indeterminacy membership ($\tilde{I}_A(x)$) grade:

$$1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{ij}^L)^-}{(1 - I_{ij}^L)^-} \right)^\gamma \right\}^{1/\gamma}} \leq 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{ij}^L)}{(1 - I_{ij}^L)} \right)^\gamma \right\}^{1/\gamma}} \leq 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{ij}^L)^+}{(1 - I_{ij}^L)^+} \right)^\gamma \right\}^{1/\gamma}}$$

$$1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{ij}^U)^-}{(1 - I_{ij}^U)^-} \right)^\gamma \right\}^{1/\gamma}} \leq 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{ij}^U)}{(1 - I_{ij}^U)} \right)^\gamma \right\}^{1/\gamma}} \leq 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{ij}^U)^+}{(1 - I_{ij}^U)^+} \right)^\gamma \right\}^{1/\gamma}}$$

For Falsity non-membership ($\tilde{S}_A(x)$) grade:

$$\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{S}_{ij}^L)^{2^-}}{1 - (\tilde{S}_{ij}^L)^{2^-}} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \leq \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{S}_{ij}^L)^2}{1 - (\tilde{S}_{ij}^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \leq \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{S}_{ij}^L)^{2^+}}{1 - (\tilde{S}_{ij}^L)^{2^+}} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}$$

$$\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{S}_{ij}^U)^{2^-}}{1 - (\tilde{S}_{ij}^U)^{2^-}} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \leq \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{S}_{ij}^U)^2}{1 - (\tilde{S}_{ij}^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \leq \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{S}_{ij}^U)^{2^+}}{1 - (\tilde{S}_{ij}^U)^{2^+}} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}}$$

Therefore, $P_{\min} \leq IVPNSDWAA(p_1, p_2, \dots, p_n) \leq p_{\max}$, completing the proof.

4. Multi-Criteria Decision-Making for Sustainable Investment Using the IVPNSDWGA Operator

This section presents the decision-making framework used to assess sustainable investment alternatives through the IVPNSDWGA operator. In the proposed decision-making process, both the criterion weights and the evaluations of each alternative are expressed using IVPNS values. This approach makes it easier to capture the uncertainty and indeterminate elements that frequently appear in sustainability and green-finance assessments.

The integration of Dombi-based aggregation into the framework means the method is able to handle the subjective and sometimes inconsistent views expressed by experts in ESG evaluations. To show how the method works in real decision scenarios, a numerical example involving several potential investment options is included. The example illustrated that the operator can still produce clear and consistent rankings even when the decision setting is complex or the information provided is not fully precise.

Consider an MCDM problem characterised by IVPNS information. Let $A = \{A_1, A_2, \dots, A_m\}$ be a discrete set of m possible alternatives and $C = \{C_1, C_2, \dots, C_n\}$ be the finite set of criteria. Assume that the corresponding weight vector of each criterion is given as $w = (w_1, w_2, \dots, w_n)$, such that $w_j > 0$ and $\sum_{j=1}^n w_j = 1$. If a decision maker is required to compare the available alternatives $A = \{(i=1, 2, \dots, m)\}$ with respect to criteria $c_i = \{(i=1, 2, \dots, n)\}$ by the IVPNS $A_{ij} = \left([\tilde{b}_{ij}^L, \tilde{b}_{ij}^U], [\tilde{I}_{ij}^L, \tilde{I}_{ij}^U], [\tilde{s}_{ij}^L, \tilde{s}_{ij}^U] \right)$, $(i=1, 2, \dots, m; j=1, 2, \dots, n)$, then we can be presented with an MCDM problem that elicits an IVPNS decision matrix $\tilde{P} = (\tilde{p}_{ij})_{m \times n}$ in the following form:

$$\tilde{P} = \begin{matrix} & \begin{matrix} C_1 & C_2 & \dots & C_n \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \begin{pmatrix} (\tilde{b}_{11}^L, \tilde{b}_{11}^U), (\tilde{I}_{11}^L, \tilde{I}_{11}^U), (\tilde{s}_{11}^L, \tilde{s}_{11}^U) & (\tilde{b}_{12}^L, \tilde{b}_{12}^U), (\tilde{I}_{12}^L, \tilde{I}_{12}^U), (\tilde{s}_{12}^L, \tilde{s}_{12}^U) & \dots & (\tilde{b}_{1n}^L, \tilde{b}_{1n}^U), (\tilde{I}_{1n}^L, \tilde{I}_{1n}^U), (\tilde{s}_{1n}^L, \tilde{s}_{1n}^U) \\ (\tilde{b}_{21}^L, \tilde{b}_{21}^U), (\tilde{I}_{21}^L, \tilde{I}_{21}^U), (\tilde{s}_{21}^L, \tilde{s}_{21}^U) & (\tilde{b}_{22}^L, \tilde{b}_{22}^U), (\tilde{I}_{22}^L, \tilde{I}_{22}^U), (\tilde{s}_{22}^L, \tilde{s}_{22}^U) & \dots & (\tilde{b}_{2n}^L, \tilde{b}_{2n}^U), (\tilde{I}_{2n}^L, \tilde{I}_{2n}^U), (\tilde{s}_{2n}^L, \tilde{s}_{2n}^U) \\ \vdots & \vdots & \ddots & \vdots \\ (\tilde{b}_{m1}^L, \tilde{b}_{m1}^U), (\tilde{I}_{m1}^L, \tilde{I}_{m1}^U), (\tilde{s}_{m1}^L, \tilde{s}_{m1}^U) & (\tilde{b}_{m2}^L, \tilde{b}_{m2}^U), (\tilde{I}_{m2}^L, \tilde{I}_{m2}^U), (\tilde{s}_{m2}^L, \tilde{s}_{m2}^U) & \dots & (\tilde{b}_{mn}^L, \tilde{b}_{mn}^U), (\tilde{I}_{mn}^L, \tilde{I}_{mn}^U), (\tilde{s}_{mn}^L, \tilde{s}_{mn}^U) \end{pmatrix} \end{matrix}$$

The proposed methodology for addressing the MCDM using the IVPNSDWGA operator, which is developed using IVPNS information described through the following steps:

Step 1: Construct a decision matrix in terms of IVPNS. If IVPNS is denoted as $A_{ij} = \left\{ [\tilde{b}_{ij}^L, \tilde{b}_{ij}^U], [\tilde{I}_{ij}^L, \tilde{I}_{ij}^U], [\tilde{s}_{ij}^L, \tilde{s}_{ij}^U] \right\} (i=1, 2, \dots, m; j=1, 2, \dots, n)$, then the generic layout of the decision matrix, columns representing criteria and rows representing alternative is presented below:

$$\tilde{P}_{mn} = \begin{pmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ p_{21} & p_{22} & \dots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \dots & p_{mn} \end{pmatrix}$$

Step 2: Standardize the decision matrix. Normally, in real problems, criteria are typically divided into two types: benefit criteria and cost criteria. To ensure consistency, cost criteria are

converted into benefit criteria. The decision matrix $\tilde{p} = (\tilde{p}_{ij})_{m \times n}$, is then normalized form $\tilde{R} = (\tilde{r}_{ij})_{m \times n}$ using the formula provided (Khan *et al.* 2018):

$$\tilde{R}_{ij} = \left(\left[\tilde{s}_{ij}^L, \tilde{s}_{ij}^U \right], \left[1 - \tilde{I}_{ij}^U, 1 - \tilde{I}_{ij}^L \right], \left[\tilde{b}_{ij}^L, \tilde{b}_{ij}^U \right] \right) \quad (22)$$

Step 3: Aggregate all the attribute values $\tilde{R} = (\tilde{r}_{ij})_{m \times n}$ by using the IVPNSDWGA operator:

$$\tilde{d}_i = IVPNSDWGA(d_{i1}, d_{i2}, \dots, d_{ij})$$

$$= \left[\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_{ij}^L)^2}{(\tilde{b}_{ij}^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \left(\frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{1 - (\tilde{b}_{ij}^U)^2}{(\tilde{b}_{ij}^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right. \right. \\ \left. \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{ij}^L)}{(1 - I_{ij}^L)})^\gamma \right\}^{1/\gamma}} \right), 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(I_{ij}^U)}{(1 - I_{ij}^U)})^\gamma \right\}^{1/\gamma}} \right) \right. \right. \\ \left. \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{s}_{ij}^L)^2}{1 - (\tilde{s}_{ij}^L)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n w_j \left(\frac{(\tilde{s}_{ij}^U)^2}{1 - (\tilde{s}_{ij}^U)^2} \right)^\gamma \right\}^{1/\gamma}} \right)^{\frac{1}{2}} \right) \right] \quad (23)$$

Step 4: Determine the score values corresponding to the aggregated by using Eq. (9).

Step 5: Rank the alternatives and find the best alternative.

The decision technique is also described in Figure 1 below.

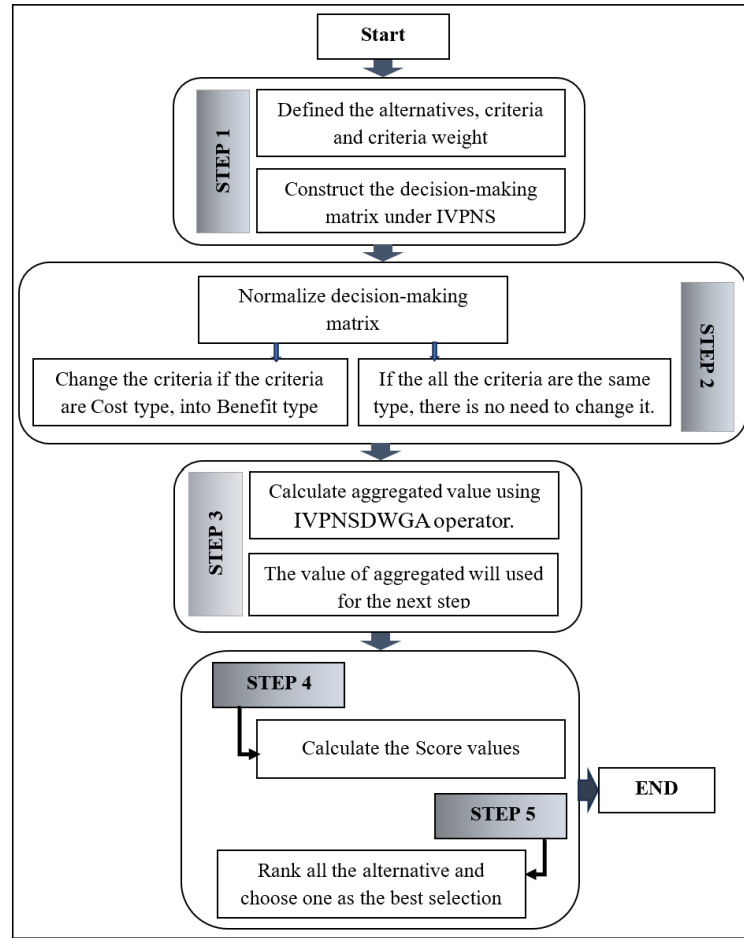


Figure 1: Framework of the proposed IVPNS method

5. Sustainable Investment Evaluation application

To illustrate how the proposed IVPNSDWGA method can be applied within a practical green finance setting, this section presents a numerical example adapted from Khan *et al.* (2018) and reinterpreted for sustainable investment evaluation. In this scenario, a financial institution assesses four potential companies for green investment allocation. The alternatives represent different sectors that align with sustainability objectives: a renewable energy provider, a sustainable food enterprise, a green technology firm, and a low-carbon defence company. The evaluation is based on three criteria:

- C1: Risk level – indicating financial and operational stability (benefit type),
- C2: Long-term growth potential – reflecting scalability and future value (benefit type),
- C3: Environmental impact – representing ecological footprint (cost type).

As these criteria are largely based on subjective and uncertain judgments, particularly when evaluating environmental impacts and long-term outcomes, expert opinions are represented using IVPNS. This approach allows decision-makers to express uncertainty more clearly by using intervals for truth, indeterminacy, and falsity, providing a more detailed representation compared to traditional crisp or fuzzy scoring methods. The Linguistic evaluations are first transformed into IVPNS linguistic values to construct the decision matrix, while the criterion

weights are determined based on strategic priorities. The overall evaluation follows a structured procedure according to the five steps comprising normalisation, aggregation using the IVPNSDWGA operator, computation of the score values, and ranking the final alternatives. The overall decision workflow is presented in Figure 2, in order to demonstrate how the proposed approach supports a more consistent, flexible, and robust analysis for sustainable investment decision-making.

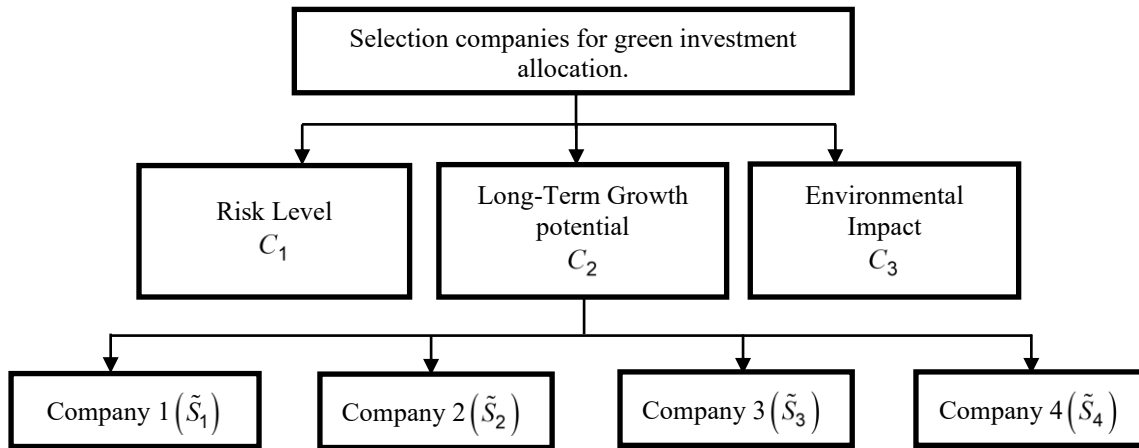


Figure 2: The selection of companies for investment allocation

Step 1: The decision matrix process of selection the best investment company involved only one decision maker (DM), and his rating by using linguistic variable IVPNS according to Table 1 is presented in Table 3 below:

Table 3: Interval-valued Pythagorean neutrosophic decision matrix given by DM

	C1	C2	C3
$\tilde{S}_1$	[0.45,0.60] [0.40,0.50] [0.30,0.40]	[0.80,0.90] [0.20,0.30] [0.05,0.10]	[0.60,0.70] [0.30,0.40] [0.20,0.30]
$\tilde{S}_2$	[0.45,0.60] [0.40,0.50] [0.30,0.40]	[0.20,0.40] [0.60,0.70] [0.50,0.60]	[0.45,0.60] [0.40,0.50] [0.30,0.40]
$\tilde{S}_3$	[0.20,0.40] [0.60,0.70] [0.50,0.60]	[0.60,0.70] [0.30,0.40] [0.20,0.30]	[0.80,0.90] [0.20,0.30] [0.05,0.10]
$\tilde{S}_4$	[0.45,0.60] [0.40,0.50] [0.30,0.40]	[0.45,0.60] [0.40,0.50] [0.30,0.40]	[0.45,0.60] [0.40,0.50] [0.30,0.40]

Step 2: Since C_3 is a cost criterion, it will change into a benefit type by using Eq. (22), and C_1 and C_2 are benefit criteria. Table 4 presents the normalized decision matrix.

Table 4: The normalize decision matrix of Interval valued Pythagorean neutrosophic decision matrix

	C ₁	C ₂	C ₃
$\tilde{S}_1$	[0.45,0.60]	[0.80,0.90]	[0.20,0.30]
	[0.40,0.50]	[0.20,0.30]	[0.60,0.70]
	[0.30,0.40]	[0.05,0.10]	[0.60,0.70]
$\tilde{S}_2$	[0.45,0.60]	[0.20,0.40]	[0.30,0.40]
	[0.40,0.50]	[0.60,0.70]	[0.50,0.60]
	[0.30,0.40]	[0.50,0.60]	[0.45,0.60]
$\tilde{S}_3$	[0.20,0.40]	[0.60,0.70]	[0.05,0.10]
	[0.60,0.70]	[0.30,0.40]	[0.70,0.80]
	[0.50,0.60]	[0.20,0.30]	[0.80,0.90]
$\tilde{S}_4$	[0.45,0.60]	[0.45,0.60]	[0.30,0.40]
	[0.40,0.50]	[0.40,0.50]	[0.50,0.60]
	[0.30,0.40]	[0.30,0.40]	[0.45,0.60]

Table 5 shows the criteria weight value used to identify the optimal investment company.

Table 5: The criteria weight of the selection investment company

Weight	C ₁	C ₂	C ₃
w_j	0.35	0.4	0.25

Step 3: Aggregate the comprehensive value of all the attributes by using the IVPNSDWGA operator, using Eq. (23) for $\gamma=1$ as follows:

$$\tilde{S}_1 = \langle (0.287, 0.418) (0.473, 0.582) (0.454, 0.559) \rangle$$

$$\tilde{S}_2 = \langle (0.284, 0.446) (0.502, 0.605) (0.424, 0.549) \rangle$$

$$\tilde{S}_3 = \langle (0.077, 0.153) (0.610, 0.721) (0.675, 0.811) \rangle$$

$$\tilde{S}_4 = \langle (0.367, 0.490) (0.444, 0.545) (0.372, 0.503) \rangle$$

Step 4: By using Eq. (9), calculate the score values for the value in step 3, corresponding to the aggregated. The results are shown in Table 6 below:

$$\tilde{S}_1 = \frac{0.287+0.418}{2} + 1 - \left(\frac{0.473+0.582}{2} \right) + 1 - \left(\frac{0.454+0.559}{2} \right) = 1.319$$

Table 6: The score values of suppliers by using IVPNSDWGA

	IVPNSDWGA	Score Function
$\tilde{S}_1$	$\langle [0.287, 0.418] [0.473, 0.582] [0.454, 0.559] \rangle$	1.319
$\tilde{S}_2$	$\langle [0.284, 0.446] [0.502, 0.605] [0.424, 0.549] \rangle$	1.324
$\tilde{S}_3$	$\langle [0.077, 0.153] [0.610, 0.721] [0.675, 0.811] \rangle$	0.706
$\tilde{S}_4$	$\langle [0.367, 0.490] [0.444, 0.545] [0.372, 0.503] \rangle$	1.496

Step 5: Rank the alternatives

Applying the method arranges the alternatives from highest to lowest, allowing the most favorable option to be selected.

Table 7: Ranking results IVPNSDWGA

Ranking Alternatives	
IVPNSDWAA	$\tilde{S}_4 \succ \tilde{S}_2 \succ \tilde{S}_1 \succ \tilde{S}_3$

The ranking order, determined by the score values using IVPNSDGA, indicates that the best company for investment, as shown in Table 7, is the Arm company ($\tilde{S}_4$).

5.1. Analysis of the Effect on Parameter γ on Decision-Making Results

This subsection provides an in-depth evaluation of the parameter γ . The influence of this parameter γ on the IVPNSDGA operator is outlined below.

Table 8: Ranking orders with parameter γ from 1 to 10 of the IVPNSDWGA operator

γ	S(c)1	S(c)2	S(c)3	S(c)4	Rank
1	1.319	1.324	0.706	1.496	$\tilde{S}_4 \succ \tilde{S}_2 \succ \tilde{S}_1 \succ \tilde{S}_3$
2	1.178	1.294	0.612	1.452	$\tilde{S}_4 \succ \tilde{S}_2 \succ \tilde{S}_1 \succ \tilde{S}_3$
3	1.110	1.259	0.572	1.417	$\tilde{S}_4 \succ \tilde{S}_2 \succ \tilde{S}_1 \succ \tilde{S}_3$
4	1.072	1.232	0.550	1.392	$\tilde{S}_4 \succ \tilde{S}_2 \succ \tilde{S}_1 \succ \tilde{S}_3$
5	1.048	1.212	0.537	1.373	$\tilde{S}_4 \succ \tilde{S}_2 \succ \tilde{S}_1 \succ \tilde{S}_3$
6	1.032	1.197	0.527	1.359	$\tilde{S}_4 \succ \tilde{S}_2 \succ \tilde{S}_1 \succ \tilde{S}_3$
7	1.020	1.185	0.520	1.349	$\tilde{S}_4 \succ \tilde{S}_2 \succ \tilde{S}_1 \succ \tilde{S}_3$
8	1.011	1.176	0.514	1.340	$\tilde{S}_4 \succ \tilde{S}_2 \succ \tilde{S}_1 \succ \tilde{S}_3$
9	1.004	1.168	0.510	1.334	$\tilde{S}_4 \succ \tilde{S}_2 \succ \tilde{S}_1 \succ \tilde{S}_3$
10	0.999	1.162	0.506	1.328	$\tilde{S}_4 \succ \tilde{S}_2 \succ \tilde{S}_1 \succ \tilde{S}_3$

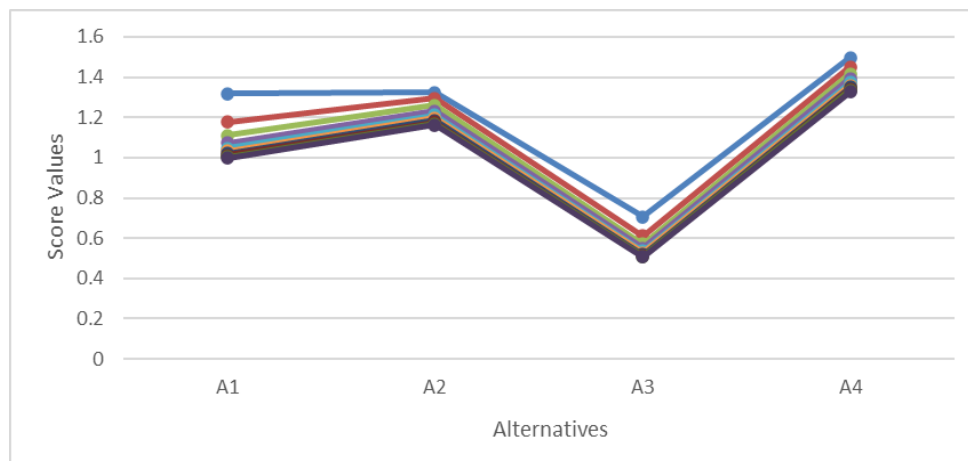


Figure 3: IVPNSDWGA operator

An examination of Table 8 shows that the score values of all four companies gradually decline as the parameter γ increases from 1 to 10; however, the relative ranking of the alternatives remains unchanged throughout. This trend is further illustrated in Figure 3, which

plots the score trajectories of each company across the full range of γ values. The ranking produced by the IVPNSDWGA operator remains entirely stable for all parameter settings, consistently following the order $\tilde{S}_4 \prec \tilde{S}_2 \prec \tilde{S}_1 \prec \tilde{S}_3$. Consequently, the most attractive investment option across all scenarios is the Arm company ($\tilde{S}_4$).

The stability observed in the results can be attributed to the geometric nature of the IVPNSDWGA operator. Because the final ranking is unaffected by changes in the parameters, this analysis shows how robust the aggregation operator is. As a result, the chosen range of γ from 1 to 10 is adequate to represent the operator's behavior under various aggregation situations while guaranteeing accurate decision-making outcomes. In 2014, Das *et al.* noted that the geometric aggregation approaches are inherently more sensitive to extreme values than their arithmetic counterparts, which are comparatively less affected by such fluctuations. It also offers a fair and significant variation for sensitivity analysis; this range is frequently used in studies utilizing the Dombi aggregation operator (Jana *et al.* 2019).

5.2. Comparing with Another Method

This section presents a comparative based on the same MCDM problem that was evaluated by using different data structures and aggregation operators. There are several operators considered in the comparative analysis, including the IN-Weight Averaging operator, the Dombi Power Bonferroni operator, the Geometric Bonferroni operator, and the SVN Dombi Weighted Arithmetic operator, as well as the IVPNS-based Dombi Weighted Geometric operator. The result of the comparative analysis, as shown in Table 9, is as follows.

Table 9: Ranking order of the alternatives produced by the various aggregation operators

Study from	Aggregation Operator	Parameter	Score Values	Ranking Order
(Zhang <i>et al.</i> 2014)	INWGA Operator	No	$S(R_1) = 1.7286, S(R_2) = 2.0991$ $S(R_3) = 1.7751, S(R_4) = 2.1608$	$\tilde{s}_4 \prec \tilde{s}_2 \prec \tilde{s}_3 \prec \tilde{s}_1$
(Liu & You 2017)	INWDPBM Operator $x = 1, y = 0, \gamma = 3$	Yes	$S(R_1) = 1.5032, S(R_2) = 1.7934$ $S(R_3) = 1.5136, S(R_4) = 1.8037$	$\tilde{s}_4 \prec \tilde{s}_2 \prec \tilde{s}_3 \prec \tilde{s}_1$
(Liu & You 2017)	INWDPGBM Operator $x = 1, y = 0, \gamma = 3$	Yes	$S(R_1) = 1.8761, S(R_2) = 2.2866$ $S(R_3) = 1.9254, S(R_4) = 2.1781$	$\tilde{s}_4 \prec \tilde{s}_2 \prec \tilde{s}_3 \prec \tilde{s}_1$
(Chen & Ye 2017)	SVNDW Operator	Yes	$S(R_1) = 0.5898, S(R_2) = 0.5361$ $S(R_3) = 0.4363, S(R_4) = 0.5286$	$\tilde{s}_4 \prec \tilde{s}_2 \prec \tilde{s}_3 \prec \tilde{s}_1$
In this paper	IVNSDWGA Operator $\gamma = 5$	Yes	$S(R_1) = 1.048, S(R_2) = 1.212$ $S(R_1) = 0.537, S(R_2) = 1.373$	$\tilde{s}_4 \prec \tilde{s}_2 \prec \tilde{s}_3 \prec \tilde{s}_1$

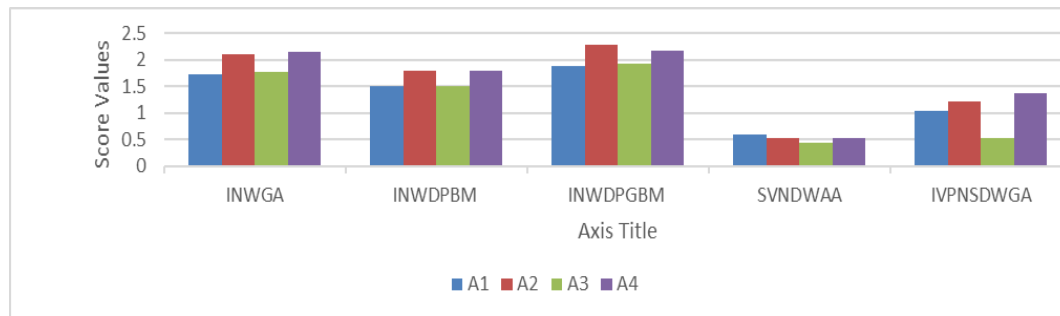


Figure 4: Comparing of various aggregation operators

5.3. Theoretical Implications of Dombi Operations in Interval-Valued Pythagorean Neutrosophic Sets (IVPNS)

In sustainability and ESG evaluations, experts rarely agree, and the data is almost never perfectly aligned. We addressed this by extending Dombi operations into the IVPNS environment. The resulting IVPNSDWGA operator is intentionally flexible; by adjusting its parameters, you can fine-tune how it handles the vague or inconsistent information that usually makes these decisions so difficult (Khan *et al.* 2018).

The method enhances previous neutrosophic MCDM techniques by integrating Dombi-based operations into the IVPNS framework. It also provides a more adaptable mechanism to adjust how various criteria interact. This added adaptability improves the accuracy of models used in demanding areas such as green finance (Yaacob *et al.* 2024; Jana *et al.* 2019). The development and assessment of the IVPNSDWGA operators also create a useful starting point for future research on hybrid decision frameworks that blend neutrosophic reasoning with other intelligent computational tools. These advances pave the way for broader analyses that can include trade-off evaluations, parameter adjustment, and nonlinear aggregation across multiple sustainability dimensions (Li *et al.* 2018; Kavitha *et al.* 2022).

5.4. Managerial Implications of the IVPNSDWGA Framework

The IVPNSDWGA framework gives decision-makers practical ways to evaluate green investment options that involve financial, policy, and environmental dimensions. Its ability to combine expert opinions, which are often subjective or uncertain, with interval-valued ESG ratings is one of its advantages. These situations help the evaluation process remain sensitive to the specific context in which the decision is made, rather than relying on rigid numerical inputs (Pai & Prabhu Gaonkar 2021). The approach is also suitable for situations where both benefit and cost criteria must be considered simultaneously, which is often the case in environments shaped by changing regulations, long-term climate uncertainty, and expectations beyond financial performance. (Sarkar & Biswas 2021; Kavitha *et al.* 2022).

In practice, the IVPNSDWGA operator can assist with various tasks such as investment screening, ranking projects, evaluating green suppliers, and carrying out more general sustainability evaluation tasks. It helps decision-makers understand the data more clearly and make judgements with greater confidence. Although sustainability decisions can be complicated, the framework offers a set of tools that remain mathematically sound but still workable in real applications (Mahmood *et al.* 2021; Qiyas *et al.* 2019).

In summary, introducing the IVPNSDWGA operators represents a significant value to the existing work in decision science, especially in sustainability and green finance. By combining Dombi-based operations with the IVPNS framework, the method is able to handle decision-making problems where the information is uncertain or given in terms of interval form. It offers a practical approach that fits real-world needs while remaining grounded in a strong theoretical foundation.

6. Conclusion

We developed the IVPNSDWGA operator by integrating Dombi T-norm and T-conorm operations directly into the IVPNS framework. Rather than just proposing a new formula, we focused on stress-testing its mathematical properties to ensure it actually holds up under pressure. This is a critical step because, in the real world, especially with ESG and sustainability data is almost never “clean.” Usually, it is a disorganized combination of incomplete knowledge

and subjective expert opinions. Our operator was designed to navigate this exact kind of fuzziness, turning inconsistent inputs into a reliable basis for a final decision.

We applied the operator to a practical, real-world case study, including assessments of sustainable investment in order to go beyond theory. The results were telling: the IVPNSDWGA operator remains remarkably stable, meaning you can adjust the internal parameters without causing a chaotic or unpredictable shift in how alternatives are ranked. This stability is its greatest strength. By reflecting multi-dimensional uncertainty without becoming fragile, the method provides the kind of clear, dependable support that decision-makers in green finance actually need to move forward with confidence.

Ultimately, these findings show that the IVPNSDWGA operator is a significant upgrade for decision science, particularly where sustainability and human judgment collide. It bridges the gap between complex math and the “gray areas” of expert feedback. Looking ahead, there is huge potential to push this method further by testing it in high-stakes environments, such as auditing carbon-heavy industries, planning renewable energy infrastructure, or building more nuanced ESG benchmarking systems.

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